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A DECENTRALIZED PRODUCTION–DISTRIBUTION SCHEDULING PROBLEM: SOLUTION AND ANALYSIS

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Abstract. In modern production–distribution supply chains, decentralization has increased significantly, due to increasing production network efficiency. This study investigates a production scheduling and vehicle routing problem in a make-to-order context under a decentralized decision-making structure. Specifically, two different decision makers hierarchically decide the production and distribution schedules to minimize their incurred costs and we formulate the problem as a bi-level mixed-integer optimization model as a static Stackelberg game between manufacturer and distributor. At the upper level, the manufacturer decides its best scheduling under a flexible job-shop manufacturing system, and at the lower level, the distributor decides its distribution scheduling (routing) which influences the upper-level decisions. The model derives the best production–distribution scheduling scheme, with the objective of minimizing the cost of the manufacturer (leader) at the lowest possible cost for the distributor (follower). As the lower level represents a mixed-integer programming problem, it is challenging to solve the resulting bi-level model. Therefore, we extend an efficient decomposition algorithm based on Duplication Method and Column Generation. Finally, to discuss the decentralization value, the results of the presented bi-level model are compared with those of the centralized approach.

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1. Introduction

In today's competitive market, production companies focus on one or a limited number of custom-made products in order to become more efficient. A popular manufacturing system in which production is done according to the customers' requirements is named make-to-order (MTO). Although MTO system provides comparative advantage through reducing the inventory level, high customization, and matching to instantly-changing of customer behavior, it is faced the challenges of not only producing customized products but also timely delivery. So, in MTO supply chains, scheduling of production and distribution operations in addition to employing appropriate production and distribution configurations becomes much more important for the business owners [30]. It is even more challenging when production and distribution decisions are made by different entities.

Keywords. Supply chain management, decentralized production–distribution scheduling, flexible job-shop, bi-level mixed-integer programming, decomposition algorithm.

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Traditionally, the production–distribution supply chain is managed in a centralized way in which the authority of decision-making is concentrated at a unique entity where a single decision-maker undertakes both production and distribution decisions, simultaneously. However, such a system needs to be reconsidered under the new supply chain configurations. Indeed, with the increasing specialization, many companies have typically outsourced some of their supply chain activities, such as distribution. Nowadays, many practical production–distribution supply chains consist of a decentralized system where the manufacturer focuses on production activities, whereas the distribution of final products to the customers is managed by a different company. In such a decentralized system, the decision process involves two decision makers and each of them undertakes a subset of decisions to optimize its own (local) objective, while affecting and being affected by the decisions of the other.

In this paper, we are concerned with an MTO supply chain optimization problem under a decentralized decision-making structure. As mentioned above, timely delivery, as well as high customization, are the main goals of an MTO business. However, it is not economical if timely delivery leads to increased production and logistics costs. So, to meet the requirements of the MTO businesses, it is important to gain insight into not only timely delivery but also cost. In this way, adopting suitable production and delivery configurations as well as designing a scheduling scheme becomes an inevitable part of the MTO business.

Mohammadi *et al.* [30] studied an MTO optimization problem under a centralized decision-making structure and showed that joint optimization of production and distribution schedules leads to a timelier delivery compared to the separate situation while the total operational cost remains fixed. However, these challenges are more complicated when scheduling decisions for the production and delivery operations are made by separate entities in a decentralized way. Despite the importance of "decentralization" in logistics and supply chain, the question that how the opposite interests of the MTO supply chain decision makers in a hierarchical game-based situation can be optimized has not yet been supported by the supply chain professionals.

To solve these challenges, we propose a bi-level production scheduling-vehicle routing problem (VRP) in a flexible job-shop production environment where the manufacturer's problem is the upper-level optimization model and the distributor's is the lower-level one. In this paper, we investigate how a manufacturer who acts as the leader and tends to fast pickup due to making lower level of final product inventory can decide its ideal production scheduling decision while affects and is affected by the distribution scheduling (routing) decisions of the distributor who as the follower doesn't prefer fast pickup due to rising delivery cost. More importantly, our proposed bi-level model aims to find the most preferred production scheduling of the manufacturer by considering the best distribution scheduling and routing of the distributor so that the total cost of the manufacturer at the lowest possible cost of the distributor is minimized. Indeed, our framework provides us to derive the best schedule and process routing in a flexible job-shop environment while also considering the optimal configuration of trips including vehicles, pickups, and routes decided by the distributor.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. In Section 3, the proposed bi-level problem is described and formulated. In Section 4, we develop the solving approach. We dedicate Section 5 to numerical experiments and applications. In Section 6, we compare the decentralized approach to the centralized one and discuss the results. We conclude and give future research directions in Section 7.

2. Literature review

Production and distribution operations as two strategic functions play a key role in practical supply chains, where their optimization can significantly improve operational performance. Recently, significant attention has been devoted to the production–distribution scheduling problem (PDSP) by both academic researchers and industry practitioners [14,28]. However, the decentralized decision of the PDSP is rarely discussed in the supply chain literature. Table 1 provides an overview of the extant literature related to our framework. Different criteria are used to describe and classify the relevant studies.

Observing Table 1, we deduce that in the production part, most of the literature on the PDSP studied single machine (*e.g.*, [16]), identical parallel machines (*e.g.*, [26]), and unrelated parallel machines (*e.g.*, [22]).

TABLE 1. Summary of related PDSP studies.

Author(s)	Decentralized setting	Flexible job shop	VRP	Process routing	General purpose machines	Nonhomogeneous vehicles	Solution approach	Other features
Garcia and Lozano [17]	–	–	–	–	–	–	Tabu search, exact	–
Soukhal *et al.* [38]	–	–	–	–	–	1 vehicle	–	–
Li and Vairaktarakis [25]	–	–	–	–	–	–	Heuristic	–
Stecke and Zhao [39]	–	–	–	–	–	✓	Heuristic	MTO supply chain
Armstrong *et al.* [3]	–	–	–	–	–	–	Heuristic, B&B	MTO supply chain, time window
Chen and Pundoor [11]	–	–	–	–	–	–	Heuristic	MTO supply chain
Cakici *et al.* [8]	–	–	–	–	–	✓	Heuristic	–
Chang *et al.* [10]	–	–	–	–	–	–	Ant colony algorithm	MTO supply chain
Ullrich [40]	–	–	–	–	–	✓	Decomposition, genetic algorithm	Time window
Low *et al.* [27]	–	–	–	–	–	✓	Genetic algorithm	Time window
Guo *et al.* [19]	–	–	–	–	–	–	Harmony search based memetic	MTO supply chain, multiple transportation modes
Hassanzadeh *et al.* [21]	–	–	–	–	–	–	MOPSO, NSGA-II	–
Joo and Kim [22]	–	–	–	–	–	✓	Genetic algorithm	–
Guo *et al.* [20]	✓	–	–	–	–	–	Bi-level evolutionary algorithm	Multiple transportation modes
Kergosien *et al.* [24]	–	–	–	–	–	–	Benders decomposition-based	–
Saglam and Banerjee [33]	–	–	–	–	–	–	Exact	–
Marandi and Zegordi [29]	–	–	–	–	–	✓	PSO and genetic algorithm	Perishable products
Noroozi *et al.* [32]	–	–	–	–	–	–	Genetic algorithm	Third-party logistic (3pl) provider
Wang *et al.* [42]	–	–	✓	–	–	✓	Tabu search hybrid algorithm	Sustainability
Mohammadi *et al.* [30]	–	✓	✓	✓	✓	✓	Hybrid PSO, ε-constraint method	MTO supply chain, time window
Ganji *et al.* [16]	–	–	✓	–	–	✓	NSGA-II, MOPSO, MOACO	Green supply chain
Yağmur and Kesen [45]	–	–	–	–	–	–	Memetic algorithm	–
Wang [41]	–	–	–	–	–	–	Hybrid augmented lagrangian relaxation and PSO	–
Solina and Mirabelli [37]	–	–	–	–	–	✓	Heuristic algorithm	Perishable products
This study	✓	✓	✓	✓	✓	✓	Decomposition algorithm	MTO supply chain

More importantly, very few works have been devoted to some rather complicated production configurations such as flow-shop, job-shop, and bundling operation systems where jobs are completed through processing a fixed sequence of operations on dedicated machines. For example, Soukhal *et al.* [38] studied a variation of the production–distribution scheduling problem in a flow-shop environment with only two processing machines. Li and Vairaktarakis [25] investigated an PDSP problem where finished products are processed by two different machines and bundled together for delivery. Hassanzadeh *et al.* [21] proposed a bi-objective PDSP model in which production is performed in a flow-shop manufacturing system. Marandi and Zegordi [29] dealt with a variation of PDSP for perishable products in a flow-shop environment by consideration of vehicle routing with a fleet of limited-capacity trucks. Mohammadi *et al.* [30] addressed a bi-objective PDSP problem where jobs are processed by flexible machines in a job-shop environment. Yağmur and Kesen [45] studied an integrated flow-shop scheduling and delivery problem by a single capacitated vehicle.

In the delivery part, the majority of the papers addressed some classic methods such as single delivery (*e.g.*, [17]), direct delivery (*e.g.*, [8]), and split delivery (*e.g.*, [19]). However, according to Table 1, the routing delivery as the most complicated method is addressed by a limited number of the previous papers on the PDSP [4, 16, 30, 42, 45].

None of the mentioned PDSP papers concerning multi-stage production systems deals with a real-world production system and delivery method except Mohammadi *et al.* [30] where the authors addressed a joint scheduling of production and distribution operations in a flexible job-shop production system by considering process routing and VRP. Although Mohammadi *et al.* [30] studied a flexible job-shop system but they dealt with a centralized scheduling model while, in many practical situations, scheduling decisions of the production and distribution operations are made by different entities under a decentralized decision-making process. Bi-level optimization, as an analytical tool for hierarchical decentralized decision-making problems [34,35], has not been analyzed and supported by the PDSP researchers, except Guo *et al.* [20] in which the authors proposed a bi-level model with unrelated parallel machines and split delivery. As shown in Table 1, the majority of the existing PDSP papers used heuristic or meta-heuristic algorithms, except Ullrich [40], Saglam and Banerjee [33], and Kergosien *et al.* [24] who adopted exact methods. Although these researches used exact methods, they studied a single-level problem and have not addressed the more challenging bi-level problem. It is also important to note that Guo *et al.* [20], who studied a PDSP problem in a bi-level framework, adopted a bi-level evolutionary algorithm.

In this paper, we developed a decomposition algorithm that includes several methods as duplication technique, column and constraint approach, and Karush–Kuhn–Tucker (KKT) which is significantly efficient in a normal time. So, to the best of our knowledge in all variants of the PDSP, we could not find any paper to study a joint optimization of production-scheduling and VRP with a fleet of heterogeneous vehicles (different capacity, fixed and variable costs) under decentralized decision-making in a flexible job shop system and also, adopting an efficient exact solution method.

This study contributes to the previous research on the PDSP by: (1) considering a novel mathematical model that deals with a decentralized setting and integrates many real-world complexities in both manufacturer's and distributor's problems using a game theory approach. In terms of modeling, we consider JSS problem with flexible machines which refers to one of the most applied scheduling methods, in which each job is composed of a number of operations, where each operation can be processed by a set of available multi-purpose machines (FJSS). Regarding the distributor's problem, we adopt realistic assumptions for the VRP, as we consider a general size of orders, a heterogeneous fleet (different capacity, fixed and variable costs), and a limited number of capacitated vehicles; (2) developing an efficient resolution approach for the resulting bi-level mixed-integer optimization model, which belongs to a class of models that are known as difficult to solve; and (3) deriving insights into the applicability of the model and the optimal strategy of each actor (manufacturer and distributor).

3. Problem statement

In this section, we first describe the characteristics of the investigated decentralized PDSP. Second, we introduce the notation. Finally, we present the proposed bi-level optimization model.

3.1. Problem description

We consider two independent companies operating in MTO under a decentralized setting: a manufacturer acting as a leader and a distributor acting as a follower. At the beginning of the planning horizon, the manufacturer receives orders from different customers and handles production operations. Each customer makes exactly one order with specific characteristics such as size. The distributor is responsible for the delivery of completed products from the manufacturing site to customers. The manufacturer moves first and decides the production scheduling, with the objective of minimizing the total scheduling, production and inventory holding cost of products, while anticipating the reaction of the distributor. The distributor considers the completion times set by the manufacturer and optimizes its distribution scheduling to minimize delivery cost. If the distributor picks up completed orders immediately after production, no inventory holding cost for the finished products will be incurred for the manufacturer. However, quick pickups increase the delivery cost for the distributor since a higher number of shipments will be required. This interaction between the manufacturer and distributor resembles the well-known bi-level optimization. Finding the best schedule of production and distribution operations to achieve the minimal cost of both companies is the major challenge in this system.

The production company produces orders in a flexible job-shop manufacturing system by a number of flexible machines which are able to process more than one type of operation. Each order has a predefined arrangement of operations which must be processed continuously by one of the available multi-purpose machines in a unique processing time (each of which has its own processing time and cost). The challenge is that each task must be operated *via* one of the existing machines, while each machine is able to run just an individual operation at a time. Once an order is completed, it is held in stock waiting delivery. Therefore, on the production side, the manufacturer decides about assignment and scheduling the operations through a given set of machines such that the cost of machine processing plus the inventory holding cost of finished items are minimized.

The distribution company delivers finished products using a fleet of heterogeneous vehicles with various capacity, fixed and variable costs. Completed products are batched with each other and delivered to respective customers in their specific routes, where each batch is shipped by one vehicle. The first challenging issue in the delivery part is batching the completed orders in a group and assigning each batch to each vehicle so that the vehicle capacity constraint is satisfied. All vehicles are located in the manufacturer location and, after taking their assigned route, come back to it (production company). It is also worthwhile to note that the start time of each vehicle in each shipment is equal to the longest production fulfilment time of those orders assigned in a batch. Therefore, the distributor in response to the producer, should specify which customer should be visited in which trip, how many tours of each vehicle type to form, the sequence of customers in each trip, and the departure and arrival time of each cargo from the company so that the total transportation cost of using vehicles is minimized.

3.2. Notation

The notation used throughout the problem under study is as follows:

Sets

$P = \{1, \ldots, \rho\}$	Set of parts (orders, jobs, customers, indexed by i, j
$O_j = \{1, 2, \ldots, \phi_j\}$	Set of operations of job j indexed by r, s
$M = \{1, 2, \ldots, \omega\}$	Set of machines, indexed by m
$V = \{1, 2, \ldots, \upsilon\}$	Set of available vehicles, indexed by k

Parameters

pc_m	Processing cost of machine m per unit of time
fc_k	Fixed transportation cost of vehicle type k
vc_k	Variable transportation cost of vehicle type k per unit of time
h_j	Inventory holding cost of job j
pt_{jsm}	Processing time of operation s of job j on machine m
β_{jsm}	Takes the value 1 if machine m enables to process operation s of job j, and 0 otherwise
t_{ijk}	Transportation time from customer i to customer j by vehicle k
l_j	Size of job j
q_k	Loading capacity of vehicle k

Upper-level (manufacturer) decision variables

S_{js}	Start time of operation s of job j
CC_{js}	Completion time of operation s of job j
C_j	Completion time of job j
Z_{jsm}	Takes the value 1 if operation s of job j is processed by machine m, and 0 otherwise
X_{irjsm}	Takes the value 1 if operation s of job j is processed immediately after the operation r of job i, both on machine m, and 0 otherwise

Lower-level (distributor) decision variables

T_j	Delivery time of order j
TT_{jk}	Delivery time of order j by vehicle k (*i.e.*, visiting time of customer j by vehicle k)
SS_k	Start time of vehicle k from production facility
PV_{jk}	Pickup time of job j by vehicle k
E_k	End time of vehicle k
U_{jk}	Takes the value 1 if job j is delivered by vehicle k, and 0 otherwise
Y_{ijk}	Takes the value 1 if job j is delivered after job i, both are distributed by vehicle k, and 0 otherwise
W_k	Takes the value 1 if vehicle k is assigned for delivery, and 0 otherwise

3.3. Modeling

The proposed PDSP problem is modeled under a MINLP, and formulated as follows:

$$\text{Min } f_1 = \left(\sum_{j=1}^{\rho}\sum_{s=1}^{\phi_j}\sum_{m=1}^{\omega}(pc_m \times pt_{jsm} \times Z_{jsm}) + \sum_{j=1}^{\rho} h_j\left(\sum_{k=1}^{\upsilon}\mathrm{PV}_{jk} - C_j\right)\right) \tag{1}$$

s.t.

$$\sum_{m=1}^{\omega} Z_{jsm} = 1 \qquad \forall_j \in P, \quad \forall_s \in O_j \tag{2}$$

$$Z_{jsm} \leq \beta_{jsm} \qquad \forall_j \in P, \quad \forall_s \in O_j, \quad \forall_m \in M \tag{3}$$

$$Z_{irm} = \sum_{j=1}^{\rho+1}\sum_{s=1}^{\phi_j} X_{irjsm} \qquad \forall_i \in P, \quad \forall_r \in O_i, \quad \forall_m \in M \tag{4}$$

$$Z_{jsm} = \sum_{i=0}^{\rho}\sum_{r=1}^{\phi_j} X_{irjsm} \qquad \forall_j \in P, \quad \forall_s \in O_j, \quad \forall_m \in M \tag{5}$$

$$S_{js} \geq \max\left\{\mathrm{CC}_{j(s-1)}, \sum_{i=0}^{\rho}\sum_{r=1}^{\phi_i}\sum_{m=1}^{\omega}\mathrm{CC}_{ir} \times X_{irjsm}\right\} \qquad \forall_j \in P, \quad \forall_s \in O_j \tag{6}$$

$$\mathrm{CC}_{js} = S_{js} + \sum_{m=1}^{\omega} pt_{jsm} \times Z_{jsm} \qquad \forall_j \in P, \quad \forall_s \in O_j \tag{7}$$

$$C_j = \mathrm{CC}_{j\phi_j} \qquad \forall_j \in P \tag{8}$$

$$\text{Min } f_2 = \sum_{k=1}^{\upsilon}(fc_k W_k + (E_k - \mathrm{SS}_k)vc_k) \tag{9}$$

s.t.

$$\sum_{k=1}^{\upsilon} U_{jk} = 1 \qquad \forall_j \in P \tag{10}$$

$$U_{jk} = \sum_{i=0}^{\rho} Y_{ijk} \qquad \forall_j \in P, \quad \forall_k \in V \tag{11}$$

$$\sum_{i=0}^{\rho} Y_{ijk} = \sum_{i=1}^{\rho+1} Y_{jik} \leq 1 \qquad \forall_j \in P, \quad \forall_k \in V \tag{12}$$

$$\sum_{j=1}^{\rho} Y_{[0]jk} = \sum_{i=1}^{\rho} Y_{i[\rho+1]k} \leq 1 \qquad \forall_k \in V \tag{13}$$

$$\sum_{j=1}^{\rho} U_{jk} l_j \leq q_k \qquad \forall_k \in V \tag{14}$$

$$\mathrm{TT}_{0k} = \mathrm{SS}_k = \max_{j \in P} U_{jk} C_j \qquad \forall_k \in V \tag{15}$$

$$\mathrm{PV}_{jk} = \mathrm{SS}_k U_{jk} \qquad \forall_j \in P, \quad \forall_k \in V \tag{16}$$

$$\mathrm{TT}_{jk} = \sum_{i=0}^{\rho} Y_{ijk}(\mathrm{TT}_{ik} + t_{ijk}) \qquad \forall_j \in P, \quad \forall_k \in V \tag{17}$$

$$T_j = \sum_{k=1}^{v} U_{jk} \mathrm{TT}_{jk} \qquad \forall_j \in P \tag{18}$$

$$E_k = \mathrm{SS}_k + \sum_{i=0}^{\rho} \sum_{j=1}^{\rho+1} Y_{ijk} t_{ijk} \qquad \forall_k \in V \tag{19}$$

$$W_k = \max_{j \in P} U_{jk} \qquad \forall_k \in V \tag{20}$$

$$Z_{jsm}, X_{irjsm}, U_{jk}, Y_{ijk}, W_k \in \{0,1\} \qquad \forall_{i,j} \in P, \quad \forall_{r,s} \in O_j, \quad \forall_m \in M, \quad \forall_k \in V \tag{21}$$

$$S_{js}, \mathrm{CC}_{js}, C_j, T_j, \mathrm{TT}_{jk}, \mathrm{SS}_k, \mathrm{PV}_{jk}, E_k \geq 0 \qquad \forall_{i,j} \in P, \quad \forall_{r,s} \in O_j, \quad \forall_m \in M, \quad \forall_k \in V. \tag{22}$$

Equation (1) is the objective function of the leader (manufacturer), which is twofold: the cost of production scheduling incurred by processing the operations using machines (machine processing cost) and the inventory cost for holding finished products. Constraint (2) is the machine assignment equation and means that operations of jobs should only be processed by one of the available machines. Constraint (3) makes sure that assignment of each operation to each machine is according to the processing ability of each machine. As previously mentioned, in the production facility, processing machines are multi-purpose, which makes it important to assign operations to those machines according to their abilities. Constraints (4) and (5) specify the succession of the operation processing on machines and determine which operation is processed before and after a specific operation, respectively. In order to present a professional model, we have used two dummy orders as 0 and $\rho+1$ with the following assumptions: $pt_{[0]sm} = 0, pt_{[\rho+1]sm} = 0$. Indeed, when each machine starts its operation, first, operation s of job 0 is completed, and operation s of job $\rho+1$ at the end of the processing operation. We also note that the starting time and completion time of operation s of orders 0 and $\rho+1$ are 0 ($\mathrm{CC}_{0s} = 0$, $S_{0s} = 0$, $\mathrm{CC}_{[\rho+1]s} = 0$, $S_{[\rho+1]s} = 0$). Constraint (6) makes sure that the processing of the operation s of job j can start after the fulfillment of its previous operation ($\mathrm{CC}_{j(s-1)}$) on any machine, when the respective machine is not involved. Constraints (7) and (8) compute each operation's- and each job's completion time, respectively. The former calculates the completion time of operation s of job j by adding the processing time of the respective operation on any machine to its starting time and the later represents that each job's completion time is equal to the completion time of the last operation of the related job. Equation (9) is the objective function of the follower (distributor), including a fixed cost and a variable transportation cost, where the first part belongs to the total fixed cost of the vehicles used and the second part incurred by the total routes of trips serve by the vehicles. Equation (10) is the vehicle assignment constraint and indicates that each job should be only assigned to one of the available vehicles. Constraint (11) deals with the consequences of vehicles in each tour and defines which customer is served in which tour and after which customer. Constraint (12) is the flow conservation constraint which makes sure that each customer is only visited one time in each tour. In other words, constraint (12) implies that each vehicle should leave instantly a customer after serving it. Constraint (13) guarantees that each vehicle departs from the manufacturer location and comes back to it only one time immediately after serving its associated route. Exactly like the production scheduling, in the transportation part orders 0 and $\rho+1$ as two dummy orders are used to show the start and the end of each tour taken by each vehicle, respectively. Indeed, for each vehicle, the first order 0 leaves the manufacturer location and at the end of the associated tour order $\rho+1$ comes back to it. Constraint (14) ensures that the total size of jobs in each batch does not exceed the capacity of the vehicle. Constraint (15) signifies that the start time of each vehicle in each tour is equal to the longest manufacturing fulfillment time of those jobs delivered in a batch. Constraint (16) shows the pickup time of finished jobs in each batch is equal to the departure time of each vehicle. Constraint (17) specifies the visiting time of customer j by vehicle k by adding the distance between its former customer i (in the same batch k) and customer j to the receipt time of customer i. Equation (18) calculates the visit time of each customer by each vehicle in each tour. Constraint (19) computes the end time of a tour by adding its start time to the total traveling time of the tour. Constraint (20) determines which vehicles were employed for the delivery of finished products. Equations (21) and (22) represent the domains of variables.

3.4. Linearization

As the proposed bi-level model is a nonlinear mixed integer mathematical framework, first, it is essential to make it linear, due to the more tractability of the linear models. So, here we apply some theatrical methods to transform the primary nonlinear model into the linear one. In constraints (6), (15) and (20), we employed the *maximum operator* which is a nonlinear term. Although IBM ILOG CPLEX by default linearizes the *max (min)* operator by using *maxl (minl)* function, in order to make the model more capable we linearize them as follows:

Assume a generic nonlinear term as $\max(y_1, y_2, \ldots, y_n)$ which can be transformed to an equivalent linear form by applying a new positive variable Y and a set of binary variables z_i, and by adding the following constraints.

$$\max(y_1, y_2, \ldots, y_n) \longrightarrow Y \tag{23}$$

$$Y \geq y_i \qquad \forall_i = 1, \ldots, n \tag{24}$$

$$Y \leq y_i + M \times z_i \qquad \forall_i = 1, \ldots, n \tag{25}$$

$$\sum_{i=1}^{n} z_i \leq n-1 \tag{26}$$

where M called Big-M is an arbitrary large positive value. Constraint (24) denotes that Y is the maximum of y_i and hence is greater than each y_i. Constraints (25) and (26) state that at least for each i, Y should be less than or equal to y_i and hedge Y from approaching infinity.

Besides, in the proposed mixed nonlinear integer model, we use some bilinear terms such as $\mathrm{CC}_{ir}X_{irjsm}$, $U_{jk}\mathrm{TT}_{jk}$, which are explicitly nonlinear. Assume that there is a bilinear term as $y.z$, where y is a positive variable and z is a binary variable. Again, by adopting a new variable Y and adding the following constraints the model is transformed to an equivalent linear form.

$$y.z \longrightarrow Y \tag{27}$$

$$y - (1-z) \times M \leq Y \leq y \tag{28}$$

$$Y \leq M \times z \tag{29}$$

where M is an arbitrary large number.

(For more details please refer to [5]).

4. Solution method

Bi-level programming as an optimization tool for solving decentralized decision-making problems, arises in many practical situations, such as logistics (*e.g.*, [46]), manufacturing (*e.g.*, [9]), economics (*e.g.*, [1]), government policy-making (*e.g.*, [6]), and revenue management (*e.g.*, [7]). Most of these real-world problems involve integer decision variables. However, it is difficult to solve a bi-level optimization model with integer variables. Even for the simplest case, where the objective functions of both upper- and lower-level models and their constraints are all linear, the problem would be NP-hard (*e.g.*, [15]). For a bi-level problem with linear programming (LP) lower-level model, the KKT approach which is based on single level reformulation, is used as a common solution method in most of the bi-level models [2]. However, for a bi-level mixed-integer programming (BLMIP) problem with an MIP lower-level problem, the KKT condition would be invalid for a single-level formulation. Hitherto, some algorithms were developed to deal with BLMIP with an MIP lower-level problem (*e.g.*, [13, 31, 43, 44]). Nevertheless, existing solution methods either have employed the branch and bound (B&B) technique for pure integer programming lower-level problems or involve complex operations. Consequently, there is no single approach applicable to compute a general type of BLMIP.

Since the upper-level and the lower-level problems are formulated as MIP models in our proposed BLMIP model, it is challenging to solve them. Moreover, due to the NP-hard nature of the FJSS problem [18, 23] and VRP [12], our presented model which is a combination of these two sub-problems, would definitely be strongly NP-hard. Based on reformulations and decomposition, we employ in this study a novel solution scheme, which is applicable to every BLMIP. Before we explain the procedure, it is obvious from the problem structure that it is decomposable into two sub-problems, producer and distributor. If the problem is decomposed into sub-problems, it is promising to solve it in a more efficient manner. But decomposition itself disjoins the rational relations of the two levels of the problem. So, we need to add some feasibility cuts to make sure that the global optimal don't be excluded from the search space. On the other hand, since we want to make use of an iteration-based structure, so we must guarantee that in each new iteration, the objective should not become worse, which is why we add some optimality cuts.

Due to the paper length limitation, and in order to make the algorithm straightforward to follow, we explain the procedure based on a generic form, but the equivalent models of each part are given in Appendices A–C. Let us consider the following BLMIP. Here, we present the steps of the solution algorithm.

BLMIP (→ Equivalent to Eqs. (1)–(22))

$$\theta^* = \text{minimize}(ax_1 + bx_2 + cx_3) \rightarrow \text{Equivalent to equation (1)} \tag{30}$$

s.t.

$$Ax_1 \leq B \qquad x_1 \in R_+^{m_c} \times Z_+^{m_d} \rightarrow \text{Equivalent to equations (2)–(8)} \tag{31}$$

$$(x_2, x_3) \in \vartheta(x_1) \equiv \arg\max\left\{fx_2 + gx_3 : Dx_2 + Ex_3 \leq R - Lx_1, x_2 \in R_+^{n_c}, x_3 \in Z_+^{n_d}\right\} \rightarrow \text{Equivalent to equations (9)–(20)} \tag{32}$$

where x_1 refers to the upper-level decision variables, x_2 refers to the continuous lower-level decision variables and x_3 refers to the integer lower-level decision variables. Since the KKT condition cannot be straightforwardly utilized for bi-level models in which the lower-level model has integer variables, relaxation techniques should be used to overcome the challenging lower-level MIP. However, relaxation is pointless if it significantly enlarges the feasible region of the bi-level MIP. Therefore, to avoid weak relaxations, we use a duplication technique and replicate the lower-level's variables (and constraints) in the upper-level model. Please refer to Scheel and Scholtes [36] and Zeng and An [47] and for a comprehensive overview of the duplication technique. Indeed, such a technique provides a stronger relaxation and is more suitable to make the analysis and design algorithm for the general bi-level MIP. Hence, we obtain the equivalent form for duplicated bi-level mixed-integer programming (BLMIPd) problem.

BLMIPd

$$\theta^* = \text{minimize}\left(ax_1 + bx_2^d + cx_3^d\right) \tag{33}$$

s.t.

$$Ax_1 \leq B \qquad x_1 \in R_+^{m_c} \times Z_+^{m_d} \tag{34}$$

$$Dx_2^d + Ex_3^d \le R - Lx_1 \qquad x_2^d \in R_+^{n_c}, x_3^d \in Z_+^{n_d} \tag{35}$$

$$fx_2^d + gx_3^d \ge \max\{fx_2 + gx_3 : Dx_2 + Ex_3 \le R - Lx_1, x_2 \in R_+^{n_c}, x_3 \in Z_+^{n_d}\}. \tag{36}$$

Proposition. *The BLMIP and BLMIPd are equivalent.*

Proof. We provide the two following statements to show that the equivalence between BLMIP and BLMIPd is straightforward. (1) Equations (35) and (36) denote that $\left(x_2^d, x_3^d\right)$ is an optimal solution to the lower-level problem under a given x_1. Similar to the solution concept of the conventional BLMIP, we consider the three-tuple $\left(x_1, x_2^d, x_3^d\right)$ to represent a solution for the BLMIPd and show that an optimal solution to the lower-level model, under a given x_1, can be obtained by setting $(x_2, x_3) = \left(x_2^d, x_3^d\right)$. By applying the duplication strategy, $\left(x_1, x_2^d, x_3^d\right)$ is formed as a solution for the BLMIPd, which involves the upper-level variables x_1 and the lower-level variables are controlled by the upper-level duplicated model and are denoted by $\left(x_2^d, x_3^d\right)$. Conceptually, this strategy allows the upper-level duplicated model to make decisions by simulating the reaction of the lower-level duplicated model. (2) Mathematically, equation (36) shows that the feasible set of $\left(x_1, x_2^d, x_3^d\right)$ is the feasible region of the BLMIP. □

Now, to make the proposed model more tractable we utilize a decomposition technique and separate the main model to the master- and sub-problems (Appendices A–C). The essential steps of the applied decomposition algorithm are depicted in Figure 1 and are described as follows.

Below, the routine of the applied decomposition algorithm is described in detail.

Step 1. Initialization of feasible solutions

Step 1.1. The first iteration ($l = 0$) solves the upper-level model of the BLMIPd:

$$\varnothing^* = \text{minimize}\left(ax_1 + bx_2^d + cx_3^d\right) \tag{37}$$

s.t.

$$Ax_1 \le B \qquad x_1 \in R_+^{m_c} \times Z_+^{m_d} \tag{38}$$

$$Dx_2^d + Ex_3^d \le R - Lx_1 \qquad x_2^d \in R_+^{n_c}, x_3^d \in Z_+^{n_d}. \tag{39}$$

The optimal solutions for the upper-level model, x_1^*, duplicated variables, $\left(x_2^{d^*}, x_3^{d^*}\right)$, and $\varnothing^*$ as the optimal value of the lower bound (LB) are derived.

Step 1.2. For a given upper-level solution x_1^*, the lower-level model of *sub-problem 1,* is solved. We note that since the completion time $\left(C_j^*\right)$ is the only common variable between the first and second levels, only the optimal value of this variable is required to be set as a parameter in the lower-level model.

Sub-problem 1 (→ Equivalent to Appendix A: Eqs. (A.1)–(A.14))

$$\varphi_1^* = \max fx_2 + gx_3 \tag{40}$$

s.t.

$$Dx_2 + Ex_3 \le R - Lx_1^* \qquad x_2 \in R_+^{n_c}, x_3 \in Z_+^{n_d}. \tag{41}$$

The value of φ_1^* as the objective function of *sub-problem 1* is derived.

Step 1.3. Now, to optimize those lower-level variables controlled by the upper level, the following *sub-problem 2*, under the given $x_1^*\left(C_j^*\right)$ and φ_1^*, is solved.

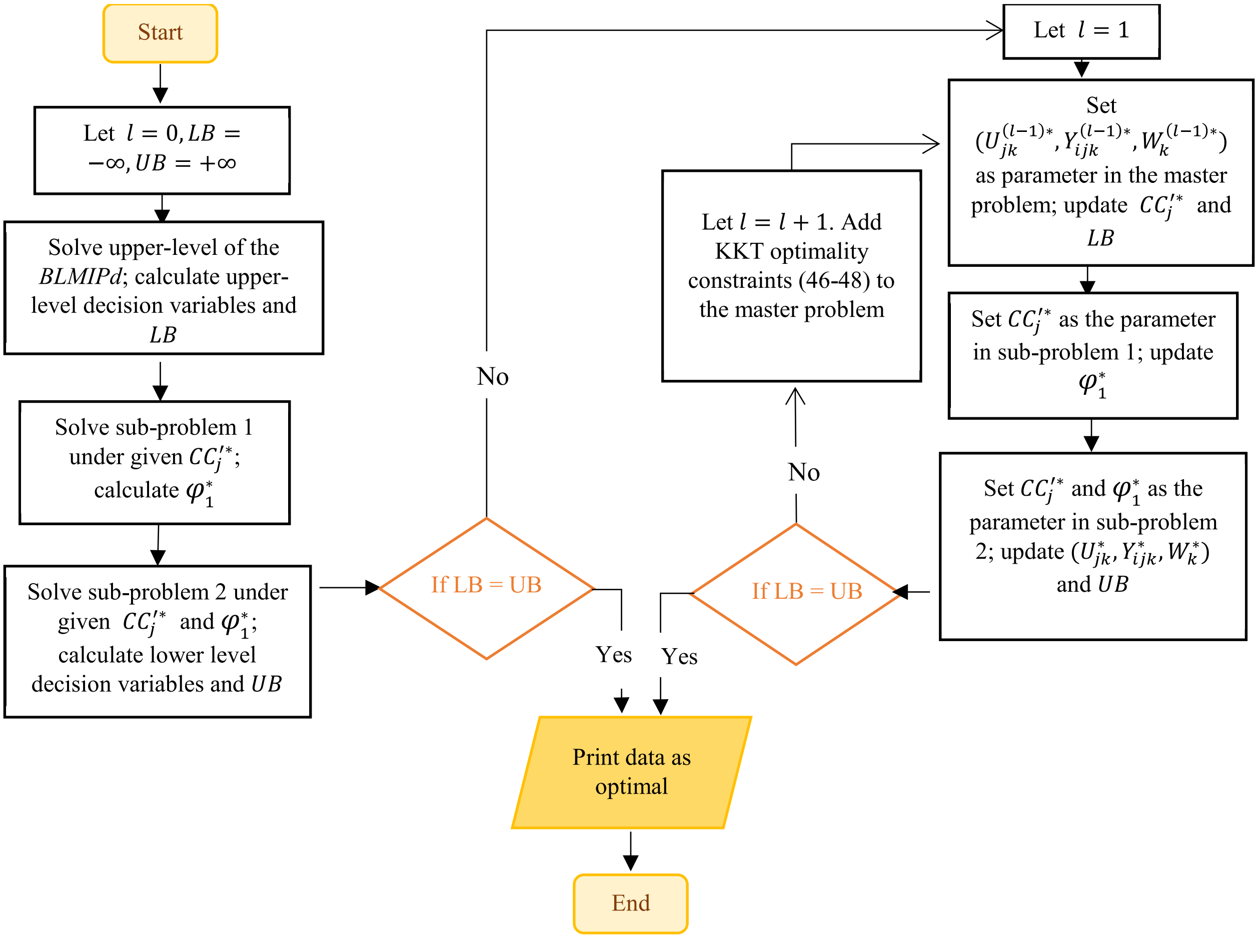

FIGURE 1. Flowchart of the proposed decomposition algorithm.

Sub-problem 2 (→ Equivalent to Appendix B: Eqs. (B.1)–(B.15))

$$\varphi_2^* = \min bx_2 + cx_3 \tag{42}$$

s.t.

$$Dx_2 + Ex_3 \le R - Lx_1^* \qquad x_2 \in R_+^{n_c}, x_3 \in Z_+^{n_d} \tag{43}$$

$$fx_2 + gx_3 \ge \varphi_1^*. \tag{44}$$

Equation (44) denotes that, by optimizing the variables of the lower-level model, which are controlled by the upper level, the optimality of the lower-level problem or *sub-problem 1* is ensured. More importantly, *sub-problem 2* allows the selection of an optimal solution for the lower-level problem, x_3^*, and x_2^* is in favor of the upper-level decision-maker.

Step 1.4. Since (x_1^*, x_2^*, x_3^*) is a feasible solution, its associated value, $ax_1^* + bx_2^* + cx_3^*$, gives an upper bound (UB) to the BLMIPd. Now, we calculate $\text{UB} = \min\{\text{UB}, ax_1^* + bx_2^* + cx_3^*\}$.

Step 1.5. If LB = UB, stop; otherwise, go to step 2.1.

Step 2. Decomposition algorithm

Step 2.1. Set $l = 1$. In this step, the optimal integer values of the lower-level model derived in *sub-problem 2* of the prior iteration should be set as the parameter in the master problem, which is formed through expanding the BLMIPd.

Master problem (→ Equivalent to Appendix C: Eqs. (C.1)–(C.36))

$$\emptyset^* = \min ax_1 + bx_2^d + cx_3^d \tag{45}$$

$$fx_2^d + gx_3^d \geq fx_2^l + gx_3^l \quad 1 \leq l \leq L \rightarrow \text{Equivalent to equations (C.9)–(C.20)} \tag{46}$$

$$Dx_2^l \leq R - Lx_1 - Ex_3^l, D^t\pi^t \geq f^t \quad 1 \leq l \leq L \rightarrow \text{Equivalent to equations (C.26)–(C.30)} \tag{47}$$

$$\pi^t\left(R - Lx_1 - Ex_3^l - Dx_2^l\right), x_2^l\left(D^t\pi - f^t\right) = 0 \quad 1 \leq l \leq L \rightarrow \text{Equivalent to equations (C.31)–(C.36)} \tag{48}$$

$$x_2^l \in R_+^{n_c}, \pi^l \in R_+^{n_1} \quad 1 \leq l \leq L. \tag{49}$$

The *master problem* is formed by expanding equation (36), under a finite optimal value of x_3. Conceptually, this formation is a relaxation of the BLMIPd, and its associated objective function provides a lower bound to BLMIPd.

Step 2.1.1. Based on the enumeration strategy and considering a finite set of x_3, continuous and separate variables are separated in the right-hand-side of equation (36) to restructure the lower-level problem as follows:

$$fx_2^d + gx_3^d \geq \max_{x_3 \in Z} gx_3 + \max\left\{fx_2 : Dx_2 + Ex_3 \leq R - Lx_1, x_2 \in R_+^{n_c}\right\} \tag{50}$$

where Z denotes the collection of all possible x_3.

Step 2.1.2. As the second maximization problem in (40) is a linear programming problem, it can be replaced by the following KKT optimality constraints:

$$fx_2^d + gx_3^d \geq \max_{x_3 \in Z} gx_3 + fx_2 \tag{51}$$

$$Dx_2 \leq R - Lx_1 - Ex_3, D\pi \geq f, x_2(D\pi - f), \pi(R - Lx_1 - Ex_3 - Dx_2) = 0 \tag{52}$$

$x_2(D\pi - f), \pi(R - Lx_1 - Ex_3 - Dx_2) = 0$ are the complementarity condition. Therefore, the BLMIPd in (34)–(36) is equivalent to its single-level formation or the *master problem*, which is constructed through enumeration and the KKT optimality method.

Step 2.2. Under a given x_1^* derived from the *master problem*, repeat steps from 1.2 to 1.4.

Step 2.3. If LB = UB, stop and report the best solutions; otherwise, let $l = l + 1$ and create a *master problem augmented* with adding (46)–(49) to the *master problem*.

Step 2.4. Solve the *augmented master problem* and derive x_1^*.

Step 2.5. Repeat steps from 1.2 to 1.4 under a given x_1^* derived from the *augmented master problem.*

Step 2.6. If LB = UB, stop and report the best solutions; otherwise, let $l = l + 1$ and add equations (46)–(49) to the *augmented master problem.*

Step 2.7. Solve the new *augmented master problem* and derive x_1^*.

Step 2.8. Repeat steps 2.5–2.7.

Proposition. *By assuming that $Z = \left\{x_3^1, \ldots, x_3^l\right\}$ is finite, the developed decomposition algorithm converges to the optimal value of BLMIPd.*

Proof. It is sufficient to demonstrate that x_3^* results in LB = UB. Now, assume that, in the current iteration l_1, the value of $\left(x_1^*, x_2^{d^*}, x_3^{d^*}\right)$ is obtained in step 2.1 and the value of z^* is derived in step 2.2 with UB > LB. Furthermore, we assume $\left(x_1^*, x_2^{d^*}, x_3^{d^*}\right)$ and x_3^* were also derived in the prior iteration, l_0. Therefore, since

$\mathrm{UB}-\mathrm{LB}>0$, the augmented master problem contains a new set of variables and constraints associated with is construct (steps 2.3, 2.4). However, as those added variables and constraints are similar to those of iteration l_0, the augmentation does not change the master problem. Consequently, the value of master problem LB in iteration l_0 is he same as in iteration l_1+1.

Now, we mathematically show that $\mathrm{LB}\geq\mathrm{UB}$ in iteration l_1+1:

$$\begin{aligned}\mathrm{LB}&=ax_1^*+bx_2^{d^*}+cx_3^{d^*}=ax_1^*+\min\{bx_2^d+cx_3^d:(34),(35),(45)\text{–}(49),x_1=x_1^*\}\\&\geq ax_1^*+\min\{bx_2^d+cx_3^d:Dx_2^d+Ex_3^d\leq R-Lx_1^*,fx_2^d+gx_3^d\geq fx_2^{l_1+1}+gx_3^{l_1+1},Dx_2^{l_1+1}\leq R\\&\quad-Lx_1^*-Ex_3^{l_1+1},D^t\pi^{l_1+1}\geq f^t,x_2^{l_1+1}\left(D^t\pi^{l_1+1}-f^t\right),\pi^{l_1+1}\left(R-Lx_1^*-Ex_3^{l_1+1}-Dx_2^{l_1+1}\right),\\&\quad x_3^d\in Z_+^{n_d},x_2^d,x_2^{l_1+1}\in R_+^{n_c},\pi^{l_1+1}\in R_+^{n_1}\}\geq ax_1^*+\min\{bx_2^d+cx_3^d:Dx_2^d+Ex_3^d\leq R-Lx_1^*,\\&\quad fx_2^d+gx_3^d\geq\varphi_1(x_1^*),x_3^d\in Z_+^{n_d},x_2^d\in R_+^{n_c}\}=ax_1^*+\varphi_2(x_1^*).\end{aligned}$$

By considering that $x_3^{l_1+1}$ is an optimal value for $\varphi_1(x_1^*)$ and the KKT condition constraints ensure that $fx_2^{l_1+1}+gx_3^{l_1+1}=\varphi_1(x_1^*)$, the second inequality is established. Therefore, we easily obtain in step 2.3 that $\mathrm{LB}\geq\mathrm{UB}$, which terminates the proof. □

5. Numerical study and insights

In this section, we first solve a small-sized illustrative example to validate our proposed bi-level PDSP model. Based on this example, we present and discuss the model's application and the optimal strategy of each actor. We finally evaluate the performance of the developed duplication-based decomposition algorithm by solving several test problems.

5.1. Numerical illustration

We consider a typical decentralized supply chain, comprising of a manufacturer and a distributor. It is assumed that, in one working day, the production company receives five different orders from five different customers, located in different geographic areas of a city. There are three multi-purpose machines with different processing costs. Additionally, we assume that each job consists of three operations. It is also assumed that the distributor owns eight nonhomogeneous vehicles including various cost coefficients and loading capacity. The algorithm and solving approach are implemented in CPLEX environment *via* its APIs and executed in Win 10, with 2.2 GHz and 8 GB DDR II.

The data on travel distance between customer i and $j(t_{ijk})$ are reported in Table 2. Table 3 represents the capacity (q_k), fixed cost (fc_k), and variable transportation cost of each vehicle (vc_k) per hour. The machine processing time (pt_{jsm}) and process-machine matrix (β_{jsm}) are given in Tables 4 and 5, respectively. The processing cost of machine per minute, the inventory holding cost of each job per hour and the size of orders are respectively $pc_m=[100,200,400]\times 1000\,\mathrm{IRR}$, $h_j=[2000,3000,4000,5000,6000]\times 1000\,\mathrm{IRR}$, and $l_j=[1,1.5,1.75,2,3]$ in aggregate units.

The optimal values of the upper- and lower-level decision variables and the resulting optimal costs for the manufacturer and the distributor are shown in Tables 6, 7, and 8, respectively. Additionally, Figure 2 shows the operation assignment to each machine (optimal decisions of the manufacturer). Furthermore, the distribution scheduling and routes followed by the vehicles (optimal decisions of the distributor) are shown in Figure 3.

As previously explained, each decision-maker is affected by the decision of the other actor and aims to reduce its incurred costs. The manufacturer targets minimizing the machine processing and inventory holding costs. To reduce the inventory holding cost, the manufacturer needs fast pickups immediately after production, which results in an increased use of vehicles. However, the distributor aims to reduce the total number of vehicles used, which is in conflict with the fast pickup concept.

According to the Figures 2 and 3, the production and distribution configurations are designed to deliver finished products immediately after production with the minimum number of vehicles. More specifically, pickup

Table 2. Transportation time between customers.

Customer j	0	1	2	3	4	5
0	0	88	71	45	31	21
1	88	0	41	75	93	50
2	71	41	0	38	90	73
3	45	75	38	0	29	15
4	31	93	90	29	0	88
5	21	50	73	15	88	0

Table 3. Vehicle data.

Vehicle type k	Capacity (aggregate unit)	Fixed cost (1000 IRR)	Variable cost (1000 IRR)
1	2	500	800
2	3	600	1000
3	5	800	1300
4	6	900	1400
5	9	1300	1700
6	14	1700	2300
7	19	2200	2700
8	22	2800	3200

Table 4. Processing times.

Machine	Job	1			2			3			4			5		
	Operation	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1		7	6	6	7	6	6	6	6	6	7	6	8	6	7	6
2		10	9	7	8	8	10	9	8	9	10	9	11	9	10	7
3		12	12	11	12	10	12	12	11	11	12	10	12	10	12	8

Table 5. Process-machine matrix.

Machine	Job	1			2			3			4			5		
	Operation	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1		1	0	0	0	1	0	0	0	1	1	0	0	0	0	1
2		1	0	1	1	1	0	0	1	1	0	0	1	1	0	1
3		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1

Table 6. Optimal decisions for production scheduling.

S_{jk}	S_{11}	S_{12}	S_{13}	S_{21}	S_{22}	S_{23}	S_{31}	S_{32}	S_{33}	S_{41}	S_{42}	S_{43}	S_{51}	S_{52}	S_{53}
Value	7	29	56	0	14	51	63	75	83	0	41	64	8	17	77
CC_{jk}	CC_{11}	CC_{12}	CC_{13}	CC_{21}	CC_{22}	CC_{23}	CC_{31}	CC_{32}	CC_{33}	CC_{41}	CC_{42}	CC_{43}	CC_{51}	CC_{52}	CC_{53}
Value	14	41	63	8	20	63	75	83	89	7	51	75	17	29	83

TABLE 7. Optimal decisions for distribution scheduling and routing.

PV_{jk}	Value	TT_{jk}	Value	SS_k	Value	E_k	Value
PV_{12}	63	TT_{12}	175	SS_1	75	E_1	137
PV_{22}	63	TT_{22}	134	SS_2	63	E_2	263
PV_{33}	89	TT_{33}	134	SS_3	89	E_3	170
PV_{41}	75	TT_{41}	106				
PV_{53}	89	TT_{53}	149				

TABLE 8. Resulting optimal costs in the system.

Manufacturer cost $\times 10^3$	Distributor cost $\times 10^3$
35 000 IRR	356 800 IRR

times, arrangements of production, transportation and routes are significantly scheduled to improve the whole efficiency of the system in a win-win game. In fact, as the manufacturer considers the optimal reaction of the distributor, it provides a suitable production configuration, in which the interests of both parties are considered. More importantly, through completing the jobs within the close times, the manufacturer provides an interactive pickup times scheme, in which the batch delivery method can be employed by the distributor to reduce its delivery costs, and also prevent the accumulation of the finished goods in the stock. The distributor who reacts to the optimal decision of the manufacturer, needs three routes and delivers finished goods immediately after production, except for job 5, which is held in stock. As seen in Tables 6 and 7, orders 1 and 2 are completed in time 63 and delivered in a batch immediately after production. Moreover, order 5 is completed in time 83 and order 3 is completed in time 89 and both are delivered in a batch in time 89. Finally, the production of order 4 is finished in time 75 and is delivered after its production in time 75.

5.2. Performance of the decomposition algorithm

To study the efficiency of the proposed algorithm, fifteen extra test problems with different dimensions are generated and optimally solved *via* CPLEX using the duplication-based decomposition algorithm. The obtained results are reported in Table 9. Iteration number stands for those iterations in which the process of the algorithm is terminated or, in other words, the values of the lower and upper bounds are equal. In the generated test problems, the information data are as follows. The process-machine matrix, processing time, processing cost of machine, and inventory holding cost are randomly drawn from $\beta_{jsm} \in \{0, 1\}$, $pt_{jsm} \in [6, 12]$, $pc_m \in [100, 800] \times 1000\,\mathrm{IRR}$, and $h_j \in [2000, 10\,000] \times 1000\,\mathrm{IRR}$, respectively. The information data about travel distance between customers i and j, capacity, fixed cost, and variable transportation cost of vehicles are randomly generated between $t_{ijk} \in [10, 100]$, $q_k \in [2, 45]$, $fc_k \in [500, 3600] \times 1000\,\mathrm{IRR}$ and $vc_k \in [800, 4500] \times 1000\,\mathrm{IRR}$. Finally, the size of orders in terms of the aggregate unit is randomly drawn between $l_j \in [1, 10]$.

As per Table 9, the upper and lower bounds of the algorithm converge together in a few iterations. In all test problems, without exception, the gap between the lower bound and upper bounds decreases for a reasonable time. When the scale of the problem increases, the speed of the convergence decreases and the number of iterations needed to reach the predefined epsilon increases smoothly.

6. Comparison between decentralized and centralized decision-making

Finally, we investigate the value of decentralization by comparing the results of the presented bi-level PDSP model with those of the centralized approach, in a single objective structure. The centralized PDSP model aims

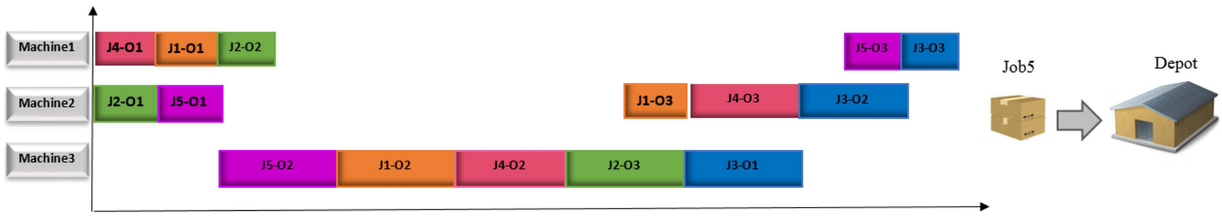

Figure 2. Schematic results of the manufacturer's optimal decisions for production scheduling.

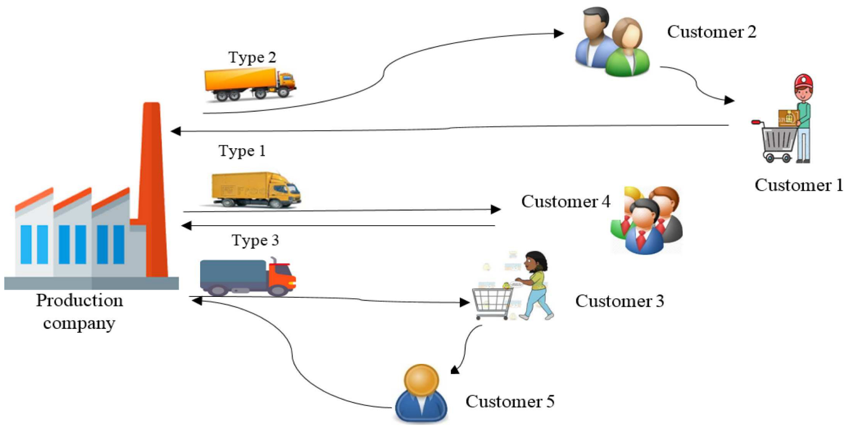

Figure 3. Schematic results of the distributor's optimal decisions for distribution scheduling.

Table 9. Performance of the developed duplication-based decomposition algorithm.

Problems	Orders	Operations	Machines	Vehicles	Variables	CPU time	Iteration number	LB $\times 10^3$	UB $\times 10^3$
P_1	2	2	2	6	478	00:05:47:23	1	5700	5700
P_2	2	3	3	6	914	00:10:55:59	1	2800	2800
P_3	3	3	2	6	1126	00:14:48:20	2	9500	9500
P_4	3	3	3	10	1720	00:26:37:00	2	20 900	20 900
P_5	4	3	3	10	2506	00:31:06:58	3	27 800	27 800
P_6	5	3	3	10	3440	01:11:04:29	4	35 000	35 000
P_7	7	3	4	12	7380	01:20:26:40	4	48 050	48 050
P_8	8	3	4	12	9158	01:26:00:00	5	44 350	44 350
P_9	9	3	5	12	13 054	01:31:00:24	5	51 730	51 730
P_{10}	10	3	3	15	11 940	01:44:56:12	6	62 000	62 000
P_{11}	12	3	2	15	13 075	01:53:15:39	8	65 786	65 786
P_{12}	12	3	3	15	19 486	01:55:57:43	8	67 340	67 340
P_{13}	15	3	4	18	30 742	02:11:10:01	9	78 453	78 453
P_{14}	20	3	5	22	63 986	02:29:04:09	10	96 333	96 333
P_{15}	25	4	7	25	193 999	02:30:00:00*	10	143 453	145 021

Notes. (*)Algorithm stopped and reported the solution after 2 hours and half.

Table 10. Comparison between decentralized and centralized production–distribution models.

Problems	Orders	Operations	Machines	Vehicles	Variables	Decentralized decision-making cost $\times 10^3$			Centralized decision-making costs $\times 10^3$			Comparison		
						Manufacturer	Distributor	Total cost	Manufacturer	Distributor	Total cost	$\Delta 1$	$\Delta 2$	$\Delta 3$
P_1	2	2	2	6	478	5700	7653	13 353	7100	4039	11 139	0.25	0.89	0.20
P_2	2	3	3	6	914	6110	8190	14 300	8486	4500	12 986	0.39	0.82	0.10
P_3	3	3	2	6	1126	9399	11 018	20 417	12 076	6170	18 246	0.28	0.79	0.12
P_4	3	3	3	10	1720	11 198	14 931	26 129	15 061	8821	23 882	0.34	0.69	0.09
P_5	4	3	3	10	2506	16 610	20 018	36 628	20 294	13 430	33 724	0.22	0.49	0.09
P_6	5	3	3	10	3440	31 994	35 080	67 074	37 500	28 800	63 300	0.17	0.22	0.06
P_7	7	3	4	12	7380	49 976	56 880	106 856	62 550	40 201	102 751	0.25	0.41	0.04
P_8	8	3	4	12	9158	58 348	68 766	127 114	72 505	52 334	124 839	0.24	0.31	0.02
P_9	9	3	5	12	13 054	64 788	70 200	134 988	71 120	57 882	129 002	0.10	0.21	0.05
P_{10}	10	3	3	15	11 940	74 268	80 198	154 466	84 378	65 666	150 044	0.14	0.22	0.03
P_{11}	12	3	2	15	13 075	89 145	97 580	186 725	100 377	80 870	181 247	0.13	0.21	0.03
P_{12}	12	3	3	15	19 486	108 355	104 476	212 831	120 477	81 479	201 956	0.11	0.28	0.05
P_{13}	15	3	4	18	30 742	111 564	109 931	221 495	122 551*	97 801*	220 352*	0.10	0.12	0.01
P_{14}	20	3	5	22	63 986	128 754	126 029	254 783	131 911*	122 444*	254 355*	0.02	0.03	0.00
P_{15}	25	4	7	25	193 999	146 652	156 067	302 719	152 331*	143 100*	295 431*	0.04	0.09	0.02

Notes. (*)CPLEX cannot reach the optimal solution after 2 hours and half. So, the best feasible solutions are reported.

to minimize the sum of the manufacturer and distributor costs subject to production and distribution constraints. Specifically, in the centralized PDSP model, the manufacturer, and distributor aim to reduce total costs in an integrated way, including machine processing costs and inventory holding costs $(\sum_{j=1}^{\rho}\sum_{s=1}^{\phi_j}\sum_{m=1}^{\omega}(pc_m \times pt_{jsm} \times Z_{jsm}) + \sum_{j=1}^{\rho} h_j(\sum_{k=1}^{\upsilon}\mathrm{PV}_{jk} - C_j))$ and fixed and variable vehicle usage costs $\sum_{k=1}^{\upsilon}(fc_k W_k + (E_k - \mathrm{SS}_k)vc_k)$. In this context, fifteen test problems are solved with both decentralized and centralized approaches. The obtained results are shown in Table 10. The comparison between the two problems setting Δ^is is reported in the three last columns, which are calculated as follows:

$$\Delta 1 = \frac{\text{decentralized manufacturer cost} - \text{centralized manufacturer cost}}{\text{decentralized manufacturer cost}}$$
$$\Delta 2 = \frac{\text{centralized distributor cost} - \text{decentralized distributor cost}}{\text{centralized distributor cost}}$$
$$\Delta 3 = \frac{\text{centralized total cost} - \text{decentralized total cost}}{\text{centralized total cost}}.$$

According to Table 10, for all test problems, the total cost of the decentralized system is higher than that of the centralized system, which is expected. Less intuitive is the fact that the cost of the manufacturer under the centralized setting is significantly higher than the cost of the distributor, whereas there is no significant difference between the two costs in the decentralized approach. Therefore, although the decentralized approach leads to a higher total cost, it provides a suitable balance between the cost of the manufacturer and the distributor. Adopting centralized decision-making is beneficial for the distributor, while the manufacturer incurs losses. In most practical industrial supply chains, the manufacturer has more power than the distributor. In this case, decentralized decision-making will be preferred by the manufacturer. Furthermore, as seen in Figure 4, when the problem size increases as what happens in real-world cases, the difference between the total cost of two systems is reduced since the proposed decentralized model provides the manufacturer with more options (such as a wide variety of multi-purpose machines) to reduce the production costs and consequently the transportation cost is reduced due to a bigger homogeneous fleet size which allows the distributor to merge more cargos and benefits from the economies of scale.

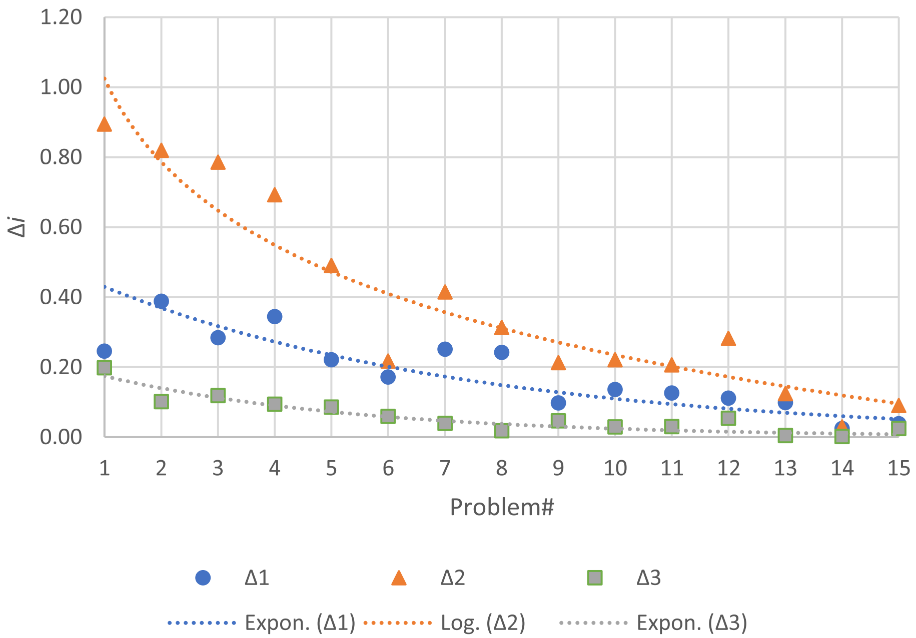

Figure 4. Comparison between the centralized and decentralized settings with different problem sizes.

7. Conclusion

This paper proposes a bi-level mixed-integer nonlinear model for competitive production scheduling and vehicle routing decisions in an FJS environment that widely exists in MTO businesses. The model finds a trade-off between the opposite interests of two decision-makers at two different levels of decision-making (a manufacturer and a distributor who compete on minimizing their incurred costs). At the first level, the manufacturer decides on the production scheduling of received orders and tends to the fast pickup of finished products due to incurring inventory holding costs while meeting the limits of the distributor. At the second level, the distributor handles finished product transportation and decides about distribution scheduling but does not prefer quick pickups because of the increasing delivery cost. In the proposed bi-level model, both the first and second levels are formulated as MIP problems. Solving those bi-level models with an MIP lower-level problem is too difficult. We developed an exact decomposition algorithm and used it to solve the proposed model with different sizes, which reflects the high capability of the algorithm.

Theoretically, our model performs as a generic framework that significantly can be applied to deal with other bi-level production–distribution problems with simpler production configurations (*e.g.* open-shop, flow-shop, etc.) and delivery options (*e.g.*, transshipment, direct shipment, etc.). From a solution perspective, we solved an NP-hard BLMIP model with an MIP lower-level problem by adopting an exact method. From a managerial perspective, we provided a win-win situation for practitioners in a leader–follower game, where the total costs of the manufacturer and distributor are minimized. Finally, we investigated the benefits of decentralized decision-making by comparing the results of decentralization with those of the centralized structure.

In future research, one can extend the model to include more cost components on both sides of the producer and/or distributer. It would be interesting to extend the comparison with centralized approaches in which the producer's costs are "penalized" compared to the distributor's. This could allow the construction of a Pareto front and move towards cooperative strategies with possible compensation if one of the actors has to make sacrifices to the detriment of the other. It is also worthwhile to study situations where the processing time in production and/or the traveling time in distribution are uncertain. Moreover, due to the increasing concern of global warming, considering GHG emissions in the production process and/or the pollution routing problem in distribution are other avenues for further research.

Appendix A.

Sub-problem 1

$$\text{Min } \varphi_1^* = \sum_{k=1}^{\upsilon}(fc_k W_k + (E_k - \text{SS}_k)vc_k) \tag{A.1}$$

$$\sum_{k=1}^{\upsilon} U_{jk} = 1 \qquad \forall_j \in P \tag{A.2}$$

$$U_{jk} = \sum_{i=0}^{\rho} Y_{ijk} \qquad \forall_j \in P, \quad \forall_k \in V \tag{A.3}$$

$$\sum_{i=0}^{\rho} Y_{ijk} = \sum_{i=1}^{\rho+1} Y_{jik} \leq 1 \qquad \forall_j \in P, \quad \forall_k \in V \tag{A.4}$$

$$\sum_{j=1}^{\rho} Y_{[0]jk} = \sum_{i=1}^{\rho} Y_{i[\rho+1]k} \leq 1 \qquad \forall_k \in V \tag{A.5}$$

$$\sum_{j=1}^{\rho} U_{jk} l_j \leq q_k \qquad \forall_k \in V \tag{A.6}$$

$$\text{TT}_{0k} = \text{SS}_k = \max_{j \in P} U_{jk} C_j \qquad \forall_k \in V \tag{A.7}$$

$$\text{PV}_{jk} = \text{SS}_k U_{jk} \qquad \forall_j \in P, \quad \forall_k \in V \tag{A.8}$$

$$\text{TT}_{jk} = \sum_{i=0}^{\rho} Y_{ijk}(\text{TT}_{ik} + t_{ijk}) \qquad \forall_j \in P, \quad \forall_k \in V \tag{A.9}$$

$$T_j = \sum_{k=1}^{\upsilon} U_{jk}\text{TT}_{jk} \qquad \forall_j \in P \tag{A.10}$$

$$E_k = \text{SS}_k + \sum_{i=0}^{\rho}\sum_{j=1}^{\rho+1} Y_{ijk} t_{ijk} \qquad \forall_k \in V \tag{A.11}$$

$$W_k = \max_{j \in P} U_{jk} \qquad \forall_k \in V \tag{A.12}$$

$$U_{jk}, Y_{ijk}, W_k \in \{0,1\} \qquad \forall_{i,j} \in P, \quad \forall_k \in V \tag{A.13}$$

$$T_j, \text{TT}_{jk}, \text{SS}_k, \text{PV}_{jk}, E_k \qquad \forall_{i,j} \in P, \quad \forall_k \in V. \tag{A.14}$$

Appendix B.

Sub-problem 2

$$\text{Min } \varphi_2^* = \sum_{j=0}^{\rho} h_j \text{PV}_{jk} \tag{B.1}$$

s.t.

$$\sum_{k=1}^{\upsilon} U_{jk} = 1 \qquad \forall_j \in P \qquad \text{(B.2)}$$

$$U_{jk} = \sum_{i=0}^{\rho} Y_{ijk} \qquad \forall_j \in P, \quad \forall_k \in V \qquad \text{(B.3)}$$

$$\sum_{i=0}^{\rho} Y_{ijk} = \sum_{i=1}^{\rho+1} Y_{jik} \leq 1 \qquad \forall_j \in P, \quad \forall_k \in V \qquad \text{(B.4)}$$

$$\sum_{j=1}^{\rho} Y_{[0]jk} = \sum_{i=1}^{\rho} Y_{i[\rho+1]k} \leq 1 \qquad \forall_k \in V \qquad \text{(B.5)}$$

$$\sum_{j=1}^{\rho} U_{jk} l_j \leq q_k \qquad \forall_k \in V \qquad \text{(B.6)}$$

$$\mathrm{TT}_{0k} = \mathrm{SS}_k = \max_{j \in P} U_{jk} C_j \qquad \forall_k \in V \qquad \text{(B.7)}$$

$$\mathrm{PV}_{jk} = \mathrm{SS}_k U_{jk} \qquad \forall_j \in P, \quad \forall_k \in V \qquad \text{(B.8)}$$

$$\mathrm{TT}_{jk} = \sum_{i=0}^{\rho} Y_{ijk}(\mathrm{TT}_{ik} + t_{ijk}) \qquad \forall_j \in P, \quad \forall_k \in V \qquad \text{(B.9)}$$

$$T_j = \sum_{k=1}^{\upsilon} U_{jk} \mathrm{TT}_{jk} \qquad \forall_j \in P \qquad \text{(B.10)}$$

$$E_k = \mathrm{SS}_k + \sum_{i=0}^{\rho} \sum_{j=1}^{\rho+1} Y_{ijk} t_{ijk} \qquad \forall_k \in V \qquad \text{(B.11)}$$

$$W_k = \max_{j \in P} U_{jk} \qquad \forall_k \in V \qquad \text{(B.12)}$$

$$\sum_{k=1}^{\upsilon} (fc_k W_k + (E_k - \mathrm{SS}_k) vc_k) \leq \varphi_1^* \qquad \text{(B.13)}$$

$$U_{jk}, Y_{ijk}, W_k \in \{0,1\} \qquad \forall_{i,j} \in P, \quad \forall_k \in V \qquad \text{(B.14)}$$

$$T_j, \mathrm{TT}_{jk}, \mathrm{SS}_k, \mathrm{PV}_{jk}, E_k \qquad \forall_{i,j} \in P, \quad \forall_k \in V. \qquad \text{(B.15)}$$

Appendix C.

Master problem

$$\text{Min } \varnothing^* = \left(\sum_{j=1}^{\rho} \sum_{s=1}^{\phi_j} \sum_{m=1}^{\omega} (pc_m \times pt_{jsm} \times Z_{jsm}) + \sum_{j=1}^{\rho} h_j \left(\sum_{k=1}^{\upsilon} \mathrm{PV}_{jk} - C_j \right) \right) \qquad \text{(C.1)}$$

s.t.

$$\sum_{m=1}^{\omega} Z_{jsm} = 1 \qquad \forall_j \in P, \quad \forall_s \in O_j \qquad \text{(C.2)}$$

$$Z_{jsm} \leq \beta_{jsm} \qquad \forall_j \in P, \quad \forall_s \in O_j, \quad \forall_m \in M \qquad \text{(C.3)}$$

$$Z_{irm} = \sum_{j=1}^{\rho+1} \sum_{s=1}^{\phi_j} X_{irjsm} \qquad \forall_i \in P, \quad \forall_r \in O_i, \quad \forall_m \in M \tag{C.4}$$

$$Z_{jsm} = \sum_{i=0}^{\rho} \sum_{r=1}^{\phi_j} X_{irjsm} \qquad \forall_j \in P, \quad \forall_s \in O_j, \quad \forall_m \in M \tag{C.5}$$

$$S_{js} \geq \max\left\{ \mathrm{CC}_{j(s-1)}, \sum_{i=0}^{\rho} \sum_{r=1}^{\phi_i} \sum_{m=1}^{\omega} \mathrm{CC}_{ir} \times X_{irjsm} \right\} \qquad \forall_j \in P, \quad \forall_s \in O_j \tag{C.6}$$

$$\mathrm{CC}_{js} = S_{js} + \sum_{m=1}^{\omega} pt_{jsm} \times Z_{jsm} \qquad \forall_j \in P, \quad \forall_s \in O_j \tag{C.7}$$

$$C_j = \mathrm{CC}_{j\phi_j} \qquad \forall_j \in P \tag{C.8}$$

$$\sum_{k=1}^{\upsilon} U_{jk}^d = 1 \qquad \forall_j \in P \tag{C.9}$$

$$U_{jk}^d = \sum_{i=0}^{\rho} Y_{ijk}^d \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.10}$$

$$\sum_{i=0}^{\rho} Y_{ijk}^d = \sum_{i=1}^{\rho+1} Y_{jik}^d \leq 1 \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.11}$$

$$\sum_{j=1}^{\rho} Y_{0jk}^d = \sum_{i=1}^{\rho} Y_{i[\rho+1]k}^d \leq 1 \qquad \forall_k \in V \tag{C.12}$$

$$\sum_{j=1}^{\rho} U_{jk}^d l_j \leq q_k \qquad \forall_k \in V \tag{C.13}$$

$$\mathrm{TT}_{0k}^d = \mathrm{SS}_k^d = \operatorname*{Max}_{j \in P} U_{jk}^d C_j \qquad \forall_k \in V \tag{C.14}$$

$$\mathrm{PV}_{jk}^d = \mathrm{SS}_k^d U_{jk}^d \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.15}$$

$$\mathrm{TT}_{jk}^d = \sum_{i=0}^{\rho} Y_{ijk}^d \Big(\mathrm{TT}_{ik}^d + t_{ijk} \Big) \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.16}$$

$$T_j^d = \sum_{k=1}^{\upsilon} U_{jk}^d \, \mathrm{TT}_{jk}^d \qquad \forall_j \in P \tag{C.17}$$

$$E_k^d = \mathrm{SS}_k^d + \sum_{i=0}^{\rho} \sum_{j=1}^{\rho+1} Y_{ijk}^d t_{ijk} \qquad \forall_j \in P \tag{C.18}$$

$$W_k^d = \max_{j \in P} U_{jk}^d \qquad \forall_k \in V \tag{C.19}$$

$$\sum_{k=1}^{\upsilon} \Big(fc_k W_k^d + vc_k \Big(E_k^d - \mathrm{SS}_k^d \Big) \Big) \leq \sum_{k=1}^{\upsilon} (fc_k W_k + (E_k - \mathrm{SS}_k) vc_k) \qquad \forall_k \in V \tag{C.20}$$

$$\mathrm{TT}_{0k} = \mathrm{SS}_k = \operatorname*{Max}_{j \in P} U_{jk} C_j \qquad \forall_k \in V \tag{C.21}$$

$$\mathrm{TT}_{jk} = \sum_{i=0}^{\rho} Y_{ijk} (\mathrm{TT}_{ik} + t_{ijk}) \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.22}$$

$$T_j = \sum_{k=1}^{\upsilon} U_{jk} \mathrm{TT}_{jk} \qquad \forall_j \in P \tag{C.23}$$

$$E_k = \mathrm{SS}_k + \sum_{i=0}^{\rho} \sum_{j=1}^{\rho+1} Y_{ijk} t_{ijk} \qquad \forall_k \in V \tag{C.24}$$

$$\mathrm{PV}_{jk} = \mathrm{SS}_k U_{jk} \qquad \forall_j \in P, \quad \forall_k \in V \tag{C.25}$$

$$-u1_{lk} + u2_{lk} - u5_{lk} - \sum_{j=1}^{\rho} (U_{ljk} \times u6_{ljk}) \geq vc_k \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.26}$$

$$u5_{lk} \geq -vc_k \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.27}$$

$$u7_l - u8_l \geq 0 \qquad \forall_l \in L \tag{C.28}$$

$$u3_{lkj} - \sum_{i=1}^{\rho} Y_{ljik} \times u3_{lki} - u4_{lj} \times U_{ljk} \geq 0 \qquad \forall_l \in L, \quad \forall_k \in V, \quad \forall_j \in P \tag{C.29}$$

$$u1_{lk} - \sum_{i=1}^{\rho} (Y_{l0ik} \times u3_{lki}) \geq 0 \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.30}$$

$$\left(-u1_{lk} + u2_{lk} - u5_{lk} - \sum_{j=1}^{\rho} (U_{ljk} \times u6_{ljk}) - vc_k \right) \times \mathrm{SS}_{lk} = 0 \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.31}$$

$$(u5_{lk} + vc_k) \times E_{lk} = 0 \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.32}$$

$$\left(u3_{lkj} - \sum_{i=1}^{\rho} Y_{ljik} \times u3_{lki} - u4_{lj} \times U_{ljk} \right) \times \mathrm{TT}_{ljk} = 0 \qquad \forall_l \in L, \quad \forall_k \in V, \quad \forall_j \in P \tag{C.33}$$

$$u1_{lk} - \sum_{i=1}^{\rho} \left(Y_{l[0]ik} \times u3_{lki} \right) \times \mathrm{TT}_{l[0]k} = 0 \qquad \forall_l \in L, \quad \forall_k \in V \tag{C.34}$$

$$Z_{jsm}, X_{irjsm}, U_{jk}^d, Y_{ijk}^d, W_k^d \in \{0,1\} \qquad \forall_{i,j} \in P, \quad \forall_{r,s} \in O_j, \quad \forall_m \in M, \quad \forall_k \in V \tag{C.35}$$

$$\begin{aligned} &S_{js}, \mathrm{CC}_{js}, C_j, T_j, \mathrm{TT}_{jk}, Y_{jik}, \mathrm{SS}_k, \mathrm{PV}_{jk}, E_k, \\ &T_j^d, \mathrm{TT}_{jk}^d, \mathrm{SS}_k^d, \mathrm{PV}_{jk}^d, E_k^d, U_{ljk}, Y_{ljik}, \mathrm{SS}_{lk}, E_{lk}, U_{ljk}, \\ &u1_{lk}, u2_{lk}, u3_{lkj}, u4_{lj}, u5_{lk}, u6_{ljk}, u7_l, u8_l \geq 0 \qquad \forall_{i,j} \in P, \quad \forall_{r,s} \in O_j, \quad \forall_m \in M, \quad \forall_k \in V. \end{aligned} \tag{C.36}$$

Data Availability Statement. The authors confirm that the data are available upon reasonable request.

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