On a timetabling problem in the health care system

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Abstract

This paper proposes a mathematical goal program for the design of timetables for radiologists. The goal program converts the tedious monthly tasks of the head of the radiology department of a leading hospital to a simple goal optimization problem that abides to the regulations of the Ministry of Health and avoids conflicting issues that may arise among coworkers. The optimization problem which is designed for the tactical level can also be used at the strategic level (i.e., account for a long time horizon) to plan for longer term constraints such vacations, medical and study leaves, recruitment, retirement, etc. Despite its large size, the problem is herein solved using an off-the-shelf solver (CPLEX). Empirical tests on the design of timetables for the case study prove the efficiency of the obtained schedule and highlights the time gain and utility of the developed model. They reflect the practical aspects of timetabling and radiologists' availability. Specifically, not only does the model and its solution reduce the effort of the Department head in this design stage, but it also promotes social peace among the technicians and a sense of fairness / unbiasedness. In addition, the designed model can be used at the operational level as a rescheduling tool by those technicians wishing to trade their shifts, and as a sensitivity analysis tool by managers wishing to study the effect of some phenomena such as absenteeism, increasing or decreasing the workforce, and extending work hours on the welfare of patients.

Keywords: Assignment, scheduling, integer programming, timetabling, goal programming.

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1. Introduction

Health care facilities are facing an ever-increasing demand. This demand is due to the improved access to health care and to its better quality of service. This, in turn, resulted in longevity of the local population, a low infantile mortality rate, and a high fertility rate. This is further compounded by the migration of qualified manpower; a phenomenon that further augments the size of the population and its needs for health services. Health care institutions often resort to optimization techniques that schedule the scare resources in the short run and plans their capacities in the long run [1].

Despite the good planning and the design of a proactive action plan to face this predicted increase of demand, health care facilities are in many instances strained for human resources. To face this demand, they need to better manage their limited resources; including physicians [2], operating rooms [3], nurses and technicians. These specific human resources are fundamental to any facility because they directly impact the welfare and well being of patients [4]. Yet, they have non-fixed time schedules and are in scare demand not only in Kuwait but also in the Gulf region and the Western world. Designing 'good' timetables that fit the needs of a health care facility and that meet the aspirations and preferences of their human resources while abiding to budgetary and regulatory constraints is challenging. Implementing the timetables requires the full cooperation of the staff. This depends on their perception of equity and fairness and on their personal preferences.

Staff timetabling assigns a number of employees to a number of shifts such that the resulting schedule satisfies some constraints (regulations, policies, workload, multiple locations, seniority, aptitude, etc.). This problem is generally very complex because of the intricate and sometimes conflicting nature of its multiple constraints. When alternative schedules can be built and budgeting is of no concern, a schedule that maximizes the satisfaction of the staff is sought [5]. In health care, such a scenario is a luxury. Most timetabling focuses on satisfying demand without 'burning' medical staff; i.e., while ensuring they get a weekly day off [6, 7, 8]. Many health care facilities over ride that right and resort to overtime as a means to satisfy demand and ensure patients' wellbeing. With tightening budgets, they, however, must be cognizant of the cost of overtime while equitably spreading it among staff. Ensuring that the problem produces a minimal cost timetable that meets all constraints further amplifies the problem's difficulty. The Radiology department is no exception.

Most health care institutions and their specific departments including the Radiology department work around the clock. They require different levels of qualified staff during different shifts of weekdays and weekends. This requires building equitable balanced timetables. This is important not only because timetables organize the lives of the staff but also and most importantly because they affect the quality of care delivered to patients.

Staff scheduling is not particular to health care facilities; it appears in manufacturing factories, call centers, transportation hubs, and emergency services. Staff scheduling covers a specific period of time (a day, a week, a month, a year, etc.) called a planning horizon. A schedule must satisfy the constraints of the workplace such as staff work load, availability, days off, etc. These constraints may vary during the planning horizon according to demand, circumstances of the health care facility or of the staff; e.g., retirement [9], maternity or study leave, etc.

Health care facilities are costly to governments. Optimizing the use of their resources reduces this cost while it improves the quality of care delivered and the overall satisfaction of health care professionals and patients alike. During the pandemic, one of these resources is the radiology department, which has been detrimental in detecting COVID-19 patients and the percent of infection of their lungs. In general, Radiology is a detrimental diagnostic and monitoring tool of the spread of certain diseases. Kuwait is no exception. It is facing an increased demand for Radiology departments caused both by an increase and an aging of the population. This occurs at a time when resources are becoming scarce; marked by a shortage of qualified technicians and radiologists, unavailability of immigrant expatriates, and shrinking financial means caused by government budget tightening. In this research, we focus on timetabling the technicians of a Radiology department of a leading hospital in Kuwait. Every month, the Department's Head, who is a leading radiologist, spends 60% of her time designing a timetable of the technicians. This is a time-consuming task of an invaluable resource that should be explored in developing the department, planning training, and attending patients. To avoid this non-judicious use of her time, she requested that her monthly task be automated.

There are generally four different levels of timetabling.

- The strategic level spans an extended period of time, generally about 6 months or a year. It dimensions the workforce and the budget needed for a threshold level of service. It accounts for vacation requirements of the staff, seasonal variability of demand, capacity of the health care unit, and its translation into demand. (See for example [10].)

- The tactical level spans a shorter period of time, generally a month or five weeks. Using the number of technicians obtained at the strategic level, it generates a schedule for each of available technician. For each technician who is not on-leave, the schedule specifies the days off, the shift, day of the week and a provisional radiology room to which s/he is assigned. Similarly, for each radiology room, it specifies the provisional technician(s) assigned to the room for each shift and day.

- The off-line operational level spans one week. It is generally either a confirmation or an alteration of the provisional staff assigned at the tactical level to each of the rooms. It rarely changes the shifts of the staff but may accommodate swaps between staff.

- The on-line operational level spans one day. It accounts for last minute changes of circumstances such as absenteeism, and miscellaneous events (eg. accidents, emergencies, etc.). This type of planning has been particularly effective during the COVID-19 early days (cf. [11]).

In this paper, we study the tactical timetabling of radiologists at a leading hospital referred to hereafter as H. We search for a feasible assignment that satisfies Kuwait's labor laws and the Ministry of Health rules and regulations. To do so, we first collected data pertinent data and defined the constraints of the problem. We then modeled the problem as an integer linear goal program (GP), and solved it using an off-the-shelf solver for the length of the planning horizon (4-5 weeks). Finally, we compare our results to the timetable generated manually by the Department Head. The results indicate the superiority of the generated timetables and the possibility of extending it for a full year period. The generated solutions eliminated the overtime and the lack of weekly days off.

Section 2 reviews pertinent literature on timetabling of nurses and radiologists. Section 3 defines the problem, and models it as a GP. Section 4 presents the results of the proposed model, highlights its importance as a decision support system at the tactical level, and explains how it can be extended to the strategic and operational level. Section 5 presents the computational results, which assess the efficiency of the methods in terms of solution quality. Finally, Chapter 6 summarizes the findings and presents future extensions.

2. Literature Review

Staff roistering has been extensively investigated in the literature. This interest emanates not only from the financial implications of hiring and paying staff but also and most importantly from its effect on the quality of life of the staff and of the patients alike. The numerous surveys (e.g., [12]) on the subject reflect this importance. The survey of [13] focuses on timetabling of nurses, categorizing them by skills, level of planning, length of the timetabling horizon, constraints, objectives, key performance indices, etc. The reviews of Lang et al. [14] and Griffiths et al. [15] offer a systematic evaluation and summary on nurse staffing and their interaction with management of clinical risk, quality and safety; points to new staffing methodology; and highlights the importance of equity of nursing workload. The surveys of [16], [1] and Abdalkareem et al. [17] cover scheduling of all types of personnel of a hospital (or health care unit), propose alternative classifications, identify research gaps. On the other hand, the literature review of Afshar-Nadjafi [18] accounts for the different sets of skills and levels of proficiency. Finally, Gür and Eren [19] detail the applications of GP to scheduling and planning in service systems.

Table 1 lists studies that investigated factors impacting the performance of health care professionals. For example, Abd-El-Aziz et al. [20] study the association between nurse's satisfaction with their shift schedule and the patient satisfaction in Nasr City Hospital in Cairo. Stec et al. [21] present a systematic review of the current literature on radiologist's fatigue and interpretation errors. They infer that radiologists' fatigue is omnipresent, and affects their interpretative accuracy. Khalili et al. [8] consider ergonomic factors that affect the health of nurses when scheduling their shifts in Shahid Labbafinejad Hospital in Iran and point to nurses' fatigue on the key performance indices.

Table 1: Factors impacting the wellbeing of health care staff

Reference	Staff	Well being	Shortage	Demand	Work load	Staffing	Patient outcome	Interpretive accuracy	Solution approach
[22]	R			X	X	X	X	X	Statistical analysis
Ernst et al. [12]				X	X	X	X	X	Review and discussion
[23]	R				X				Statistical analysis
[24]	R	Radiation			X				Statistical analysis
[25]	L			X		X			Statistical analysis
[26]	R	Cancer& Eye Effect							Statistical analysis
[27]	R				X				Statistical analysis
[28]	R					X			Statistical analysis
[29]	R				X				Quantitative methods
[30]	R				X	X			Statistical analysis
[31]	R	Wellness	X	X	X			X	Reviews and discussion
[32]	RT					X			Statistical analysis
[33]	R							X	Reviews and discussion
Ali et al. [1]									Reviews and discussion
[10]	R			X		X			Forecasting
[34]	R	Wellness							Engagement strategies
[35]	R				X			X	Statistical analysis
[21]	R	Fatigue							Reviews and discussion
[20]	N				X		X		Statistical analysis
[36]	N		X	X		X			Reviews and discussion
[37]	R	Fatigue							Reviews and discussion
[38]	R	Burnout							Statistical analysis
[39]	R				X	X			Review & Statistical analysis
[15]	N								Review and discussion
[40]	N	Fatigue				X			Statistical analysis
Abdalkareem et al. [17]	H								Review and discussion
[41]	R						X		Inductive approach
[42]	R	Mortality							Statistical analysis
[43]	N	Fatigue			X				Multilevel linear regression analysis
[41]	R						X		Framework
[44]	H	Sleep&Emotion							Statistical analysis
[45]	HS				X				Forecasting
[46]	R	Burnout							Multimodal

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Reference	Staff	Well being	Shortage	Demand	Work load	Staffing	Patient outcome	Interpretive accuracy	Solution approach
Afshar-Nadjafi [18]	H								Reviews and discussion
[47]	R	Burnout							Demonstration
[48]	P	Radiation							Multi-level linear regression
[49]	N						X	X	Reviews and discussion
[50]	ICUN						X	X	Reviews and discussion

HP =hospital personnel
HS =health services
ICU =Intensive care unit nurses
L =laboratory staff
N =Nurses
R =Radiologists
RT =Radiation therapy
P =Physicians

Because of the abundance of the literature on the subject of health care timetabling, we focus on those most pertinent to us. M'Hallah and Alkhabbaz [51] develop a binary integer program that minimizes the number of outsourced nurses in Kuwaiti health care facilities. They consider a panoply of constraints including those related to a feasible schedule, to the Ministry of Health's restrictions, Ministry of Labor's requirements, and to practical considerations. For instance, they include constraints relative to gender, religion, country of origin, heterogeneity of staff, and qualifications. Because nurses can float among different health care units and wards of hospitals, the model minimizes the number of floaters. In fact, managers of health care units resent resorting to floaters, which generally are not familiar with the traditional work set up of the unit. The authors solve the problem exactly for both a health care unit and a ward of a hospital for extended cycle lengths. They show that the health care unit should build timetables of four-week cycles while the ward should adopt a six-week cycle for its timetables. Their analysis of the obtained timetables highlights the unbalanced nature of the excess availability of nurses across the shifts of the timetables.

Adoly et al. [52] study the case of scheduling nurses in Egyptians hospitals, which have a low nurse to patients' ratio. They propose a multi-commodity-network flow (MCF) model to minimize the overall hospital cost and maximize nurses' preferences. Khalili et al. [8] develop a multi-objective model that accounts for nurses' work fatigue among other objectives. They employ a comprehensive standard decisional method to solve a reduced-size problem in the hospital, and metaheuristics to solve the real life large problem. Specifically, they use a Particle Swarm Optimization and a Non-dominated Sorting Genetic Algorithm. Wong et al. [53] tackle the scheduling problem of emergency department's staff using a two-stage heuristic approach. The first stage builds an initial schedule that satisfies all hard constraints (shift assignment). The second stage is a sequential local search that improves the initial schedules by accounting for nurses' preferences as soft constraints. Burke et al. [54] consider a real case study of a Dutch hospital. They assign four types of shifts to 16 nurses. They propose a hybrid multi-objective model that combines integer programming and variable neighborhood search. An Integer program (IP) solves the sub-problem, which includes both hard and soft constraints. Then a variable neighborhood search (VNS) follows to improve the IP's resulting solutions. Zhong et al. [55] present a two-stage heuristic algorithm for nurse scheduling. The first stage generates a feasible two-week schedule, where a feasible schedule satisfies the capacity, demand, work and rest regulations. The second stage makes the schedule as fair and flexible as possible by allocating the night and weekend shifts as evenly as possible while striving to maximize the satisfaction of nurse preferences. The authors apply their approach to a US hospital to highlight the cost savings it may achieve.

Tassopoulos et al. [56] design a two-phase variable neighborhood search algorithm. To create feasible and efficient nurse rosters, their metaheuristic explores distant neighborhoods of the current incumbent solution and moves the focal point of the search if and only if when it identifies a neighboring solution that improves the incumbent. Their nurse rostering problem assigns shifts to nurses in accordance with any given set of constraints. They illustrate the performance of their metaheuristic for the nurse rostering problem instances considered in the International Nurse Rostering Competition (INRC-2010). Their proposed algorithm obtains better objective function values. Similarly, Luo et al. [57] apply a two-stage approach: The first stage calculates the number of staff via a queuing model. The second stage designs schedules using the number of staff determined in stage one via a mixed integer program. They apply their approach to scheduling nurses in a blood center of a large hospital in China. They infer that their approach reduces patient's waiting times while it balances work and rest schedules of nurses. Cappanera et al. [58] apply a two-phase approach to design timetables that increase physicians' satisfaction and workload equity. Cappanera et al. [59] consider timetabling emergency medical doctors while focusing on workload balancing in terms of night and weekend shifts for the strategic level and morning and afternoon shift for the tactical level. For this purpose, they apply a two-phase approach, each based on integer programming.

Table 2 presents other manuscripts dwelling on the timetabling of health care professionals. It specifies the type of personnel along with the solution technique. The latest trend considers more than one objective to the timetabling problems. Obviously, this is only pertinent when the objectives are conflicting in nature. What constitutes a 'good'timetable from a radiologist perspective may be perceived as a bad one from a management point of view or may be infeasible from a legislative side. For instance, radiologists are exposed to high doses of radiation; thus, require regular time off to protect their health. Many expatriate technicians would rather not have days off during the year and trade them for vacation days to extend their annual leave.

Table 2: On scheduling of health care staff

Reference	Staff	Approach
[60]	N	Fuzzy Goal Programming
[61]	N	Metaheuristic (Genetic algorithm (GA) with an immigrant scheme)
[62]	N	Heuristics(a Particle Swarm Optimization)
[63]	N	Heuristic (Simulated Annealing)
[64]	R	Sequential Optimization
[65]	N	Meta-Heuristic algorithms(Jaya algorithm)
[66]	N	Heuristic, Metaheuristic, and MIP solver
[67]	N	Metaheuristic (Hydrologic Cycle)
[68]	N	Heuristic and Metaheuristic
[69]	MS	Heuristic
[70]	N	Heuristic (Generic variable-fixing) and MIP
[71]	N	Metaheuristic(Plant Propagation Algorithm)
[72]	N	Meta-heuristics(Keshtel algorithm, genetic algorithm, tabu search)
[73]	N	Metaheuristic (differential evolution (DE) algorithm)
[8]	N	Metaheuristic
[74]	N	Metaheuristic (Tabu Search)
[75]	N	Goal Programming
[76]	N	Heuristic (Column generation)
[77]	N	Rescheduling (Penalization Scheme)
[78]	N	Metaheuristic Ant Colony Optimization
[79]	N	Z-number method
[80]	N	Hybrid Genetic Algorithm

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Reference	Staff	Approach
[78]	N	Heuristic (Ant Colony)
[81]	P	Metaheuristic (Variable neighborhood search (VNS)) and Dynamic programming
[41]	R	Inductive approach to formulate staffing requirement
[82]	MS	Metaheuristic (Bat algorithms)
[83]	N	Hybrid heuristic
[84]	N	GP
[85]	N	GP
[86]	N	GP
[87]	N	GP

N	=Nurse
R	=Radiologist
HP	Hospital personnel
MS	Medical staff

Özcan et al. [85] study the features of radiology technicians, considering government regulations, hospital conditions, and staff requests, by application area. They propose a fair and balanced personnel schedule for radiology technicians where the objective is to maximize the satisfaction of the technicians while abiding to the laws regulating the sector in Turkey. They focus on a small scale hospital of Ankara with eight technicians. They model the problem of timetabling the technicians into four daily shifts (day, afternoon, night, and rest) as a goal program. Simon et al. [88] propose a successful modification of the radiology technology to reduce the radiologists' burnout. Exploring technological and computer science advances my decrease the length of non-value added tasks of radiologists and enhance patient care.

Because of the multitude of objectives or because of over-constrained nature of the problem, a recent trend in the literature is opting for goal programming GP. However, most of these use approximate approaches rather than exact methods to solve the resulting GP. Gür and Eren [19] gave an extensive review on GP for service systems putting in evidence its effectiveness as a planning tool. Ariyani et al. [84] address the timetabling of the nurses caring for the COVID19 inpatients' ward at Universitas Sebelas Maret Hospital. They obtain the optimal schedule using a GP that minimizes the deviation of the schedule from hospital regulations. However, the number of their nurses is limited and their regulations differ from those of the radiology department at hand. Similarly, Rerkjirattikal et al. [75] use GP to solve the nurse scheduling problem for the operating theatre of a private hospital in Thailand over a 28-day span. Their model maximizes nurses' preferences on shifts and day-off. It minimizes three goals: unbalanced workload, non-preferred shift, and non-preferred day off. Al-Hinai et al. [86] study the nurse scheduling problem for the emergency department at a public Hospital in Oman to minimize the shortage of nurses while balancing the distribution of their workload. They formulate the problem as a GP but only solve a small scale problem using the commercial solver LINGO. They point to the need of resorting to approximate techniques for larger problems. To solve a similar nurse scheduling problem, Hakim et al. [87] model it as a GP but use a nonlinear optimization model whose objective function is the workload distribution. In all the above cases, solving GP **exactly** was never possible for a real life sized instance with more than a hundred technicians.

This paper follows the trend of the literature. It addresses the tactical timetabling level of radiologists of AlAmiri hospital. Yet, it does not opt for a heuristic approach because heuristics have neither guaranteed average performance nor worst case performance. Thus, it is not reasonable to produce timetables that are on average good. Therefore, an exact method is needed. In this case, the problem is modeled as an integer goal program and solved optimally using CPLEX. This approach is also motivated by the fact that runtime is not pertinent given that the Head of the Department spends more than half of her productive time generating these timetables.

3. Problem Definition

The timetabling problem under consideration applies to the Radiology Department of Al-Amiri hospital. This specific department has a set $I = \{1, \dots, n\}$ of n radiology technologists. These radiologists are either outsourced or in-house. Even though both outsourced and in-house technicians have the same workload, they may have different vacation days and rules. In addition, they are not eligible for certain types of leave (such as maternity or study leave).

For every day of the week, each technician should be at rest or working a shift in a room $r \in R$ of the Department. The rooms are not physically located in the same building. They are located in

- the Sabah Al-Ahmad center for heart diseases,
- the emergency room of Al-Amiri hospital,
- the radiology center of Al-Amiri, or
- one of three health care clinics: Hamed Saqer, Al-Abdul Hadi, or Al-Nifisi clinic.

The rooms are not polyvalent. For instance, the Al-Amiri radiology center has few casualty rooms, two Barium rooms, two computerized tomography rooms, one computerized tomography-cardiac room specialized in heart diseases, one endoscopic retrograde cholangiopancreas-tomography room, twelve portable X-ray equipment units, four operating rooms, one angiogram room, one mammogram and ultra sound breast room, two magnetic resonance imaging rooms, a radiology information system room, and one bone mineral density room. Let $n_r = |R|$ denote the total number of radiology rooms (including Department's reception area, which requires qualified radiologists to interact with arriving patients and with medical staff and doctors of both the hospital and the clinics). The rooms offer n_p specialties, with specialty $p,\ p = 1, \dots, n_p$, offered in a room $r,\ r \in R_p$, where $R_p \subseteq R$, and $\cup_{p=1}^{n_p} R_p = R$.

The Department's head designs a schedule for the n technicians of the Department for a planning horizon of n_w weeks. This is a lengthy process that builds upon her expertise and on her knowledge of the compatibility, complementarity, competence, and preference of the technicians. The timetable indicates who is assigned to every radiology room $r,\ r \in R$, during each shift $s,\ s \in S = 1, 2, 3$, of each day $d,\ d \in D = \{1, \ldots, 7\}$, of week $w,\ w \in W = \{1, \ldots, n_w\}$, of the planning horizon whose length is $n_w = |W|$ weeks. The Department follows a three-shift pattern. Shift $s = 1$ (morning shift) is a seven-hour shift that extends from 7:00 A.M. to 2:00 P.M. Shift $s = 2$ (long-day shift) is a thirteen-hour shift that starts at 7:00 P.M. and ends at 8:00 P.M. Shift $s = 3$ (night shift) is an eleven-hour shift that spans from 8:00 P.M. to 7:00 A.M.

During a weekday $d,\ d \in \{1, \ldots, 5\}$, the length of a work shift $s,\ s \in S$, corresponds to the number of work hours h_{sd} rewarded to the technician; that is, $h_{1d} = 7,\ h_{2d} = 13$, and $h_{3d} = 11$. However days 6 and 7 of a week correspond to the weekend. During these two days of the weekend, the number of hours rewarded is amplified in order to make these shifts attractive. Specifically, $h_{16} = 7,\ h_{26} = 13,\ h_{36} = 11, h_{17} = 7,\ h_{27} = 13$, and $h_{37} = 11$.

Technicians are characterized by seniority level, ability of work in a specific room, training, availability status, and gender. Gender is important because pregnant women radiologists can only be assigned to the Department's reception, and are expected to be on-leave once they deliver. The above characteristics of the technicians are defined by the following parameters:

- $l_i = 1$ if technician $i,\ i \in I$, is a senior radiologist and 0 otherwise;
- $f_i = 1$ if technician $i,\ i \in I$, is female and 0 otherwise;
- $a_{ir} = 1$ if technician $i,\ i \in I$, is (or will become) qualified for the specialty offered by room $r \in r_p$ and 0 otherwise;
- $t_{ipw} = 1$ if technician $i,\ i \in I$, is in-training in specialty $p,\ p = 1, \ldots, n_p$, during week $w,\ w \in W$;
- $v_{idw} = 1$ if technician $i,\ i \in I$, is on leave during day $d,\ d \in D$, of week $w,\ w \in W$.

Finally, the primary concern of the Head of the Radiology Department is to generate a feasible timetable; i.e., a timetable that satisfies the following constraints.

C_1 Every radiology room $r,\ r \in R$, must have at least $n_{sdwr} \in \mathbb{N}$ technicians needed during shift $s,\ s \in S$, of day $d,\ d \in D$, of week $w,\ w \in W$.

C_2 There must be a senior technician in each radiology room.

C_3 The weekly workload of a technician must be at least 42 hours, but can't exceed 48 hours.

C_4 A night shift is necessarily followed by more than 24 hours of rest.

C_5 Female technologists can't be assigned to a night shift.

C_6 Every technologist has at least one day off per week.

C_7 A technician is assigned to a room $r,\ r \in R$, if and only if s/he is qualified to work in it.

C_8 A technician who is in training in a specialty $p,\ p = 1, \ldots, n_p$ during a week $w,\ w \in W$, must be assigned to a room $r \in R_p$; that is, to a room that offers specialty p.

During our interview of the Head of the Radiology Department, she mentioned additional constraints:

- Every technician must work at least 12 hours of afternoon, night, and weekend shifts.
- Male technicians should work at least two night shifts per month.
- Each technologist must work a long day at least twice a month.

However, these constraints are the result of her personal experience and of the constraints mentioned above. For instance, suppose that a technologist works only morning shifts. To satisfy the minimal 42-hour workload, s/he must work six days (i.e., $\frac{42}{7} = 6$, where 7 denotes the number of workload hours accredited to a morning shift); that is, s/he must have a morning shift during one of the weekend days. Alternatively, to avoid working during the weekend, this technologist must work on average (i.e., $\frac{42}{5} = 8.4$ hours per weekday where 5 is the number of weekdays). That is, s/he must work an additional $1.4 * 5 = 7$ hours per week. This requires that s/he works at least two afternoon, night or weekend shifts per week. (A long day brings only an additional 6 h per shift while a night shift brings only 4 h per shift).

Similarly, the constraint that male technicians should work at least two night shifts per month emanates from the workload requirements. Let n_m

denote the number of male technologists. Then, the number of night shifts that must be filled during a planning period is

$$\frac{1}{n_m} \sum_{w=1}^{n_w} \sum_{r=1}^{n_r} n_{swr}.$$

Using the data of the Radiology Department, this number is two; thus, the constraint.

The last constraint translates what happens when male technicians are assigned night shifts. It is possible that a male technician is assigned four night shifts during a week w. In such a case, in day 1 of week $w+1$, he must be off-duty. Thus, he can't be assigned any night shift for week $w+1$, and must subsequently work at least one long day shift. It follows that during any 4-week planning horizon, a male technologist must work at least two long day shifts.

These constraints are not essential to the model but follow as a result to Constraints $C_1 - C_8$. They reduce the search space by inducing certain patterns. They are used when the problem is solved manually.

4. Mathematical Program

The interview of the Head of Radiology Department revealed that it is difficult to build a feasible timetable that satisfies all constraints. This section models the problem as an integer program and explains how it is best suited to solve it as a goal program. This technologists' timetabling problem can be modeled as a binary integer satisfaction program with two classes of binary decision variables:

- $x_{idswr} = 1$ if technician $i,\ i \in I$, is assigned to room $r,\ r \in R$, during shift $s,\ s \in S$, of day $d,\ d \in D$, of week $w,\ w \in W$, and 0 otherwise.

- $y_{idw} = 1$ if technician $i,\ i \in I$, has day $d,\ d \in D$, during week $w,\ w \in W$, and 0 otherwise.

Using the above binary variables, the constraints of the problem are modeled as follows.

$$\sum_{i=1}^{n} a_{ir}(x_{id1wr} + x_{id2wr}) \geq n_{1dwr} \quad d \in D, w \in W, r \in R : n_{1dwr} > 1 \quad (1)$$

$$\sum_{i=1}^{n} a_{ir} x_{id2wr} \geq n_{2dwr} \quad d \in D, w \in W, r \in R : n_{2dwr} > 1 \quad (2)$$

$$\sum_{i=1}^{n} a_{ir}(1 - f_i) x_{id3wr} \geq n_{3dwr} \quad d \in D, w \in W, r \in R : n_{3dwr} > 1 \quad (3)$$

$$\sum_{i=1}^{n} a_{ir}(x_{id1wr} + x_{id2wr}) = 0 \quad d \in D, w \in W, r \in R : n_{1dwr} = 0 \quad (4)$$

$$\sum_{i=1}^{n} a_{ir} x_{id2wr} = 0 \quad d \in D, w \in W, r \in R : n_{2dwr} = 0 \quad (5)$$

$$\sum_{i=1}^{n} a_{ir}(1 - f_i) x_{id3wr} = 0 \quad d \in D, w \in W, r \in R : n_{3dwr} = 0 \quad (6)$$

$$\sum_{i=1}^{n} l_i(x_{id1wr} + x_{id2wr}) \geq 1 \quad d \in D, w \in W, r \in R : n_{sdwr} > 1 \quad (7)$$

$$\sum_{i=1}^{n} l_i x_{id2wr} \geq 1 \quad d \in D, w \in W, r \in R : n_{sdwr} > 1 \quad (8)$$

$$\sum_{i=1}^{n} l_i(1 - f_i) x_{id3wr} \geq 1 \quad d \in D, w \in W, r \in R : n_{sdwr} > 1 \quad (9)$$

$$\sum_{d=1}^{7} \sum_{s=1}^{3} h_{sd} \sum_{r=1}^{n_r} x_{idswr} \geq 42 \quad i \in I \quad (10)$$

$$\sum_{d=1}^{7} \sum_{s=1}^{3} h_{sd} \sum_{r=1}^{n_r} x_{idswr} \leq 48 \quad i \in I \quad (11)$$

$$\sum_{r=1}^{n_r} x_{id3wr} \leq y_{i(d+1)w} \quad i \in I, w \in W, d \in D : d \neq 7 \quad (12)$$

$$\sum_{r=1}^{n_r} x_{i73wr} \leq y_{i1(w+1)} \quad i \in I, w \in W \quad (13)$$

$$\sum_{i=1}^{n} f_i x_{id3wr} = 0 \quad d \in D, w \in W, r \in R \quad (14)$$

$$\sum_{r=1}^{n_r} x_{id1wr} + x_{id2wr} + x_{id3wr} \leq 1 - y_{idw} \quad i \in I, w \in W, d \in D, d \neq 1 \quad (15)$$

$$\sum_{d=1}^{7} y_{idw} \geq 1 \quad i \in I, w \in W, \quad (16)$$

$$
\begin{aligned}
x_{idswr} &\leq a_{ir} && i \in I, r \in R, && (17)\\
\sum_{s=1}^{3} \sum_{r \in R_p} x_{idswr} &= 1 - y_{idw} && i \in I : t_{ipw} = 1, r \in R_p, p = 1, \ldots, n_p && (18)\\
x_{idswr} &\in \{0, 1\}, && i \in I, d \in D, s \in S, w \in W, r \in R, && (19)\\
y_{idwr} &\in \{0, 1\}, && i \in I, d \in D, w \in W, r \in R, && (20)
\end{aligned}
$$

Equations (1)-(6) ensure that each room must be assigned at least n_{sdwr} qualified technologists when $n_{sdwr} > 0$ and exactly 0 radiologist otherwise. Equation (1) adresses the case where $s = 1$. It states that the number of technicians physically present are those assigned to the morning shift to a long day. Equation (2) focuses on $s = 2$ and corresponds to those qualified technologists assigned a long day. Equation (3) ensures that there are enough male qualified technologists assigned a night shift for each open room. In fact, a room is open for use if $n_{sdwr} > 0$. When a room $r \in R$ is closed during a shift $s \in S$ of a day $d \in D$ of a week $w \in W$, its $n_{swdr} = 0$ and no technician should be assigned to that room during that shift. Subsequently, Equations (4) - (6) detail the case of closed radiology rooms for shifts 1, 2, 3; thus mirroring Equations (1)-(3) for open radiology rooms. Equations (7)-(9) ensure that each room is assigned at least one senior technologist, for each of the three shifts. Equation (7) addresses the case where $s = 1$, and where the senior technologist might be assigned either a morning or a long day shift. Equation (8) considers $s = 2$. Equation (9) ensures that there are enough male qualified technologists assigned a long day. Equations (10) and (11) guarantee that every radiologist is assigned a weekly workload that varies between 42 and 48 h, as imposed by the labor law. The 48 h workload corresponds to radiologist working one long day and five morning shifts and having one day off. In fact, $48 = 1 * 13 + 5 * 7$. Equations (12) and (13) stipulate that every radiologist who is assigned a night shift on a day d must have the next day off. When $d \leq 6$, the next day $d+1$ is part of the same week; thus, Equation (12). On the other hand, when $d = 7$, the next day is $d = 1$ of nest week; thus, Equation (13). In either case, the left hand side of the constraint is 1 if the radiologist works a night shift; forcing the right hand side to be 1 and the person to be off duty. On the other hand, when the left hand side of the constraint is 0 (signaling that the radiologist is not assigned a night shift), the constraint is redundant. Equation (14) ensures that no female technologist is assigned a night shift all along the duration of the planning horizon. Alternatively, this equation could have been replaced by forcing $x_{i3dwr} = 0$ for $i \in I$ and $f_i = 1$ or simply removing them from the model. Equations (15) and

(16) guarantee that every radiologist gets at least one day-off. The former determines whether the radiologist is working on a day d or has the day-off while ensuring that a radiologist $i \in I$ is not assigned more than one shift per day d. When i has the day d off, $y_{idw} = 1$ and the right hand side becomes zero. This in turn forces the left hand side to be zero too. On the other hand, when i does not have the day d off, $y_{idw} = 0$. This in turn forces the right hand side to be one too. This translates in assigning i to an open room r during one of the three shifts. Equation (17) allows the assignment of a radiologist i to a room r if s/he has received the appropriate training for the specialties of that room. Equation (18) forces the assignment of a radiologist in training for specialty p to be to a room $r \in R_p$ during the week of his/her training. Finally, Equations (19) and (20) define the nature of the two assignment variables.

The problem can be transformed into a goal program, where the goals relate to the satisfaction of the workload specified by MoH. This requires introducing one surplus and two deficiency variables:

- $\epsilon^+_{iw} \geq 0$, denoting the surplus workload (in excess of the 48-hour maximum weekly workload tolerated by MoH) of technologist i during week w; and
- $\epsilon^-_{iw} \geq 0$, denoting the deficiency workload (shortage of the 42-hour maximum weekly workload tolerated by MoH) of technologist i during week w.
- $\psi_{iw} = 0$ if technologist i has at least one day off during week w and 1 otherwise.

The goal program uses the above deficiency variables to meet the weekly workload window imposed by MoH. This will add a goal, modify workload and days off constraints i.e., Equations (EQ3.a), (EQ3.b), and (EQ6.b)), append the model with non-negativity constraints. In fact, the goal program becomes as follows:

$$\min z = \sum_{w=1}^{n_w}\sum_{i=1}^{n} \epsilon_{iw}^{+} + \epsilon_{iw}^{+} + \alpha\psi_{iw} \tag{21}$$

$$s.t.(1)-(9),(12)-(17), \quad (19)-(20) \tag{22}$$

$$\sum_{d=1}^{7}\sum_{s=1}^{3} h_{sd}\sum_{r=1}^{n_r} x_{idswr} \quad \geq 42 - \epsilon_{iw}^{-} \quad i \in I \tag{23}$$

$$\sum_{d=1}^{7}\sum_{s=1}^{3} h_{sd}\sum_{r=1}^{n_r} x_{idswr} \quad \leq 48 + \epsilon_{iw}^{+} \quad i \in I \tag{24}$$

$$\sum_{r=1}^{n_r} x_{id3wr} + \psi_{iw} \quad \leq y_{i(d+1)w} \quad i \in I, w \in W, d \in D : d \neq 7 \tag{25}$$

$$\epsilon_{iw}^{+} \quad \geq 0, \quad i \in I, w \in W \tag{26}$$

$$\epsilon_{iw}^{-} \quad \geq 0, \quad i \in I, w \in W \tag{27}$$

$$y_{idwr} \quad \in \{0,1\}, \quad i \in I, w \in W \tag{28}$$

Equation (21) defines the goal. It has two components: the sum over all technicians over the length of the planning horizon of all deviations from the weekly workloads, and the sum of the compensation of α work hours for days off. In some hospitals, overload is compensated not financially by via future days off that can be planned during a week upon the request of a technician. Equation (23) defines the surplus workload for a technician on a given week while Equation (24) defines the deficiency for the same technician and week. Equation (25) determines whether a technician has a day off or requires compensation. Finally, Equations (26)-(28) declares the nature of the surplus and deficiency variables. These variables are declared as positive reals. However, there exists at least one optimum with integer values for these variables.

5. Computational Results

This section illustrates the usefulness of the mathematical goal program and the adequacy of its solutions to the technologists' timetabling problem for the radiology department of a leading hospital in Kuwait. The goal program is modeled using GAMS 25.0.2 and solved using CPLEX 12.2 on a 64-bit 11th Gen Intel Core i7, 2.80GHz, and 32 GB RAM. CPLEX is allocated a maximum run time of 36,000 seconds. Its branch and bound is run with the default parameters, as the optimization of the performance of the solver is not an objective per say.

Table 3: Number of technicians per department

Specialty p	$\lvert R_p \rvert$	Label
1	4	CAS
2		Inpatient
3	2	Barium X-ray
4	2	CT
5		CT SAC
6		ERCP
7	12	Portable
8	4	Operating Theatre
9		Angiography
10		Mammography and breast US
11		Relieve (F)
12		Relieve (M)
13	2	MRI
14	1	Reception
15	1	RIS & PACS
16	1	QC
17	1	Edelya HC
18	1	Yarmouk HC
19	1	Nifissi HC
20	2	BMD
21		Dasman HC

Table 3 presents the data of the radiology Department, where columns 1-3 give the specialty p, $p = 1, \ldots, n_p$ and $n_p = 20$ specialties, the number $|R_p|$ of rooms, and the label of the specialty. Furthermore, the Radiology Department has 114 technicians: 61 males and 53 females. It is tested on the data of 2018. The choice is motivated by the last pre-pandemic available data. during the month of October 2018, only 57 males corresponding to $i = 1{-}57$ and 52 females corresponding to $58{-}109$ were on duty. Technicians $110 - 114$ were on-leave for study and maternity leaves.

The goal program obtained a first feasible schedule of the 109 radiologists for the five-week period, within 2145.84 seconds of runtime, a 733.44 Mb tree and a 454 hours of overtime. This schedule is further improved with the best scheduled obtained from a tree size of 3348.85 MB, after 32742.22 seconds. The schedule has neither workload deficiency nor over time. In addition, it allocates to every technician at least a day off per week. Obviously, those having more than one night shift per week may end up with more than one day off. The generated schedule has been validated by the head of the Radiology department, who is an expert with more than 12 years of experience of technologists' timetabling design. Its results are unbiased and are fairer than those obtained manually by the Department head. They incorporate all her constraints and concerns. She can also use it as a decision support system for her timetables to avoid clashes with technologists or claims of favouritism. This validated model can be made more comprehensive; i.e., covering other setups, clinics, departments and classes of technicians.

Suppose now that the goal program is to be used to generate a schedule for a longer span of time. In fact, the goal program can be applied for strategic planning. It can determine the number of radiologists needed to smoothly run the Department. Suppose that no radiologist is requesting a vacation, and a 10-week schedule that satisfies all the constraints is to be generated. The goal program may only find a feasible solution if the Department has at least 147 technicians. Having fewer technicians will result in an infeasible solution; indicating an over-constrained problem. It is the night shift constraints for male radiologists that are the source of infeasibility. The fact that a night shift must be followed by a whole day off limits the search space especially that female radiologists are not concerned by night shifts. Within the preset 10-hour run time limit for CPLEX, the best overtime for a ten-week period is 490 hours; which is on average $\frac{1}{3}$ hour per technician. Obviously, this average is not very meaningful as it turns out that some technicians may have 6 to 8 hours of overtime while others have a weekly workload of 42 to 48 hours.

The goal program can be run for a 12-month period imposing 4 weeks of continuous vacation times for expatriates and a 5-week vacation time for Kuwaiti radiologists over the planning horizon. Testing the goal program with different numbers of technicians reveals that such schedules are possible if the number of technicians is at least 150. When technicians are equally qualified or polyvalent, it is possible to allow them to choose their yearly work schedule ahead of time as a function of the available vacation times.

The proportion of male to female radiologists is another important factor that should be planned at the strategic level. Having a disproportionate ratio may lead to the non-satisfaction of the night shifts, large over times for male radiologists caused by the non-satisfaction of the day off following a night shift.

The results provided computational evidence that it is possible to design timetables of technologists for a month and for a year. When run for long periods of time, it takes into account the vacations, days off, requests and special circumstances of technologists (pregnancy, sickness, study leave, reduced workload, etc.). The model can be extended to take into account staff's preferences. Finally, it can be at the online operational level to account for last minute changes of a schedule.

The availability of the goal program constitutes a decision tool that can be used to determine the needs of the Department in the long term, not only in terms of number of radiologists but also in terms of their diversity. During the pandemic, radiology departments played a crucial role in determining the level of progress of the disease. Their good operation was only possible because most of them had reserve surplus capacity. This allowed them to handle staff absenteeism while avoiding staff exhaustion. In fact, the strategic level should build in slack capacity to enable more flexible operational planning and robustness when dealing with unexpected events such as pregnancies of female radiologists, study leaves, etc.

6. Conclusion

This research designs a mathematical model for radiologists' timetabling. It focuses on scheduling shifts and off-days of radiologists such that the resulting timetable balances the workload and optimizes the over and under deficiencies of workloads, as mandated by the Ministry of Health. Generating this schedule manually is both time-consuming and complicated whereas using the automatically generated timetables ensures a fairer schedule and enhances the wellbeing of the radiologists; thus, satisfying demand and avoiding favouritism. This research presented the problem's constraints, formulated it as a goal integer program, and solved it for the technology

Department of a leading Kuwaiti hospital. Solving the model provided computational evidence of the simplicity of generating the schedules at the tactical level while reducing overtime hours across all weeks. The timetables can be generated for the strategic level to determine the optimal number of technicians needed and the potential hiring needs. In addition, they can be applied at both the off-line and online operational levels to remedy last minute changes in patients and availability of staff. In summary, the proposed model is a decision support tool that can save a large percentage of hours of the Department's Head. Despite its specificity, the model can be applied to any Department and hospital. The model can also account for outsourcing and floaters. Advanced integer programming techniques such as column generation and branch and price can be applied to speed the resolution of this problem. Alternatively, constructive heuristics can be used to generate solutions that can be fed to the goal program and further speed it.

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