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SUSTAINABLE INVENTORY PREDICTION WITH RANDOM DEFECT AND REWORK USING BAT ALGORITHM

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Abstract. The sustainable EPQ models that have been proposed in the inventory literature are insufficient to address the practical scenario of defects in manufacturing and subsequent rework for remedial actions. In this article, sustainable inventory model with rework for the faulty products has been studied. Promotional activities are the key factors that significantly affect the market demand for an item. The impacts of random defects and combining economic and environmental elements on the economic order quantity with price and promotional effort dependent demand have been addressed. Numerical illustrations along with sensitivity analysis are presented to reveal the relevancy as well as computational tractability of the proposed investigation. For the profit optimization, a mixed integer problem has been formulated and analyzed by using Bat meta-heuristic optimization algorithm.

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1. Introduction

In any production system material inventories take place at many stages. The required level of inventory depends on the degree of service according to the demand of customers. The financial power of a company, which includes liquidity, solvency, profitability, operational efficiency and its expanding business potential, is significantly affected by overstocking or under stocking of inventories. Effective storage management techniques help companies to retain self-sufficiency and make them independent from vendor reliance. Any kind of inventory can be represented as having an economic value on the asset side of a company's balance sheet or management books. Many professionals utilize the EOQ and EPQ models as decision-making tools for inventory control and management. To maintain an ideal inventory level, every firm should use inventory management strategies. Although it would be ideal if every product will be 100% error free, it is almost never attainable in actual manufacturing. As a result, the EPQ models have to take the expense of flaws and rework into account.

The absence of an optimal inventory policy for imperfect quality items causes loss of profit. Appliance issues, process degradation and a number of other factors that occurred throughout the production might be one of the precise causes of the defective goods. These management system flaws are significant and should be addressed by providing top-notch customer service. In today's cut-throat competition in business, we cannot just confine ourselves permanently to rework these defects but to eliminate them from the production system. Salameh and

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Jaber [1] extended the EOQ model where imperfect items are made available at a reduced price. Jamal *et al.* [2] proposed an inventory model that focuses on the prediction of the minimum cost with the optimal lot size in a single-stage structure. Sarkar *et al.* [3] continued with their detailed study in a multi-stage manufacturing environment under the corresponding operational practice and Cárdenas-Barron [4, 5] modified this study by incorporating planned backorders. Chung [6] and Khan *et al.* [7] extended the work done by Cárdenas-Barron [5]. Pal *et al.* [8] suggested an EOQ model that was proficient in taking stochastic demand in a defective production system. Wee *et al.* [9] developed an inventory model including rework procedure for a single-stage production unit by considering intended backlogs. Kumar and Goswami [10] investigated an imperfect production inventory system with stochastic demand. The defective rate and process shifting time are considered as fuzzy variables. The numerical illustrations are obtained by the fuzzy simulation-based particle swarm optimization (PSO) technique. Kim *et al.* [11] developed an integrated inventory model with imperfect production by using an effective process of calculating the number of defectives. The defective products are sent to the supplier during the next lot-delivery duration. Salamah [12] proposed an inventory model for an imperfect manufacturing process by considering the synchronous and asynchronous flexible rework rates. Dey *et al.* [13] examined cost-effective smart autonomation policy for a hybrid manufacturing-remanufacturing system. Marchi *et al.* [14] studied the effects of greenhouse gas in an imperfect production system. Sarkar *et al.* [15] studied lot-sizing strategies for a multi-item imperfect production system with random defect rate. Dey *et al.* [16] created a one-stage smart cleaner manufacturing model with an autonomous strategy to manage work-in-progress inventory. Tayyab *et al.* [17] suggested a scheduled backorder strategy to overcome projected shortages and raise the system's service level.

Efficiency and the effect of operations on social, financial and environmental decisions are frequently used to evaluate sustainability. Effective sustainable goods guarantee energy-efficient manufacturing processes by utilizing water use, chemical inputs and other renewable non-polluting energy resources to their fullest potential. Sustainable inventory management focuses on the reduction of the adverse environmental and communal influences on a manufacturing system without disturbing the profit earned by the organization.

A few studies have enriched the literature of inventory management with sustainable development in the past few years. Inman [18] put forth several key aspects related to research guidelines in the area of ecologically sensitive inventory management. Barbosa-Póvoa [19] presented a summary of sustainability in the supply chain level as a rising period. Bouchery *et al.* [20] developed an inventory model for an inherent sustainable lot size problem. Bonnet and Jaber [21] came up with an idea related to some costs regarding sustainability and recommended a liable economic order quantity model from a cost optimization viewpoint. Battini *et al.* [22] presented a structure based on the direct accounting methodology to establish a sustainable EOQ model by adding some carbon emission costs and suggested a modified ordering policy. A deterministic inventory problem that considers carbon emissions was dealt by Hammami *et al.* [23]. Kazemi *et al.* [24] procreated an EOQ model taking into account the ecological concerns with the defective products. Lin and Sarker [25] developed a pull system inventory model for imperfect quality goods by incorporating a carbon tax policy with quantity discount. Taleizadeh *et al.* [26] discussed partial backordering in the EPQ model by including different carbon emission costs. Tiwari *et al.* [27] developed a sustainable inventory model for deteriorating items by using carbon emission costs. A multi-trade credit policy and selling price dependent demand was used to develop the model. Mishra *et al.* [28] investigated a carbon tax and cap-based production model that incoperated investment in green technology initiatives to manage carbon emission rate with shortages. Priyamvada *et al.* [29] developed a sustainable production quantity model for deteriorating items. The defective proportion in the production lot was assumed to follow a probability distribution function. The deterioration rate is controlled by making an investment in the preservation equipment. Gautam *et al.* [30] studied a production inventory model where demand is based on pricing and the level of green technology. Under the effect of three pollution control strategies, Yadav *et al.* [31] studied a sustainable production model with variable pollution costs.

Businesses have acknowledged pricing strategy as a useful tool for resolving the crucial problem of optimal price over the last few decades. Many businesses seek to optimize their selling price in order to attract more market demand. To optimize the profits of companies facing price-sensitive and predictable demand over the

TABLE 1. A brief of studies relevant to proposed study.

Authors(s)	Price based demand	Random defect rate	Promotional effort	Imperfect production system	Flexible/ Multi-stage production system	Inspection	Sustainability	Meta-heuristic
Cárdenas-Barrón [5]				✓				
Sarkar *et al.* [15]		✓		✓				
Dey *et al.* [16]	✓	✓		✓		✓		
Tayyab *et al.* [17]		✓		✓	✓	✓	✓	
Mishra *et al.* [28]							✓	
Priyamvada *et al.* [29]		✓		✓		✓	✓	
Hatibaruah and Saha [36]				✓				
Chang *et al.* [39]				✓	✓			
Yadav *et al.* [31]				✓	✓		✓	
This Paper	✓	✓	✓	✓			✓	✓

planning period, Kim and Lee [32] proposed fixed-capacity as well as variable-capacity single-item pricing and ordering policies. Several researchers used pricing policies with coordination mechanisms under various assumptions [33,34]. Sahoo *et al.* [35] considered the selling price dependent demand in an inventory problem in which items deteriorate linearly. Hatibaruah and Saha [36] discussed a production inventory system for goods by considering how demand was affected by time, price, and frequency of advertising. In the marketing and operations research literature, sales effort or other different factors have been seen as essential strategies that have received a lot of attention. Cárdenas-Barrón and Sana [37] studied an EOQ inventory model where demand is dependent on the marketing efforts. Kim and Chung [38] investigated the most effective capacity allocation strategy for a freight firm with regional sales offices.

In this study, we propose inventory models with a rework process along with a special feature of sustainability. In the inventory literature, it is seen that there is not enough effort being done to meet sustainability for an imperfect production system. The reworking of damaged products may be one of the causes of the carbon emissions. This gap inspired us to create an inventory model that incorporates expenses associated with carbon emissions and rework for faulty products. This research is a step toward expanding the production-inventory model studied by Taleizadeh *et al.* [26] with shortages. Incorporating random malfunctions, rework and lost sales into order quantity models are the main goals of this study. The suggested sustainable economic production quantity (SEPQ) model incorporates sale and rework difficulties in sustainable inventory management and deals with a realistic scenario of production inventory systems. The outputs of the suggested model may be advantageous to several businesses who have production bottlenecks owing to faulty products and are affected by the environment.

Table 1 outlines the important previous studies related with our proposed model. It is observed that the price sensitive demand is not included in most of the inventory studies with imperfect production. Also, in the existing literature, the promotional efforts have not been used frequently to study EPQ models. The carbon control policies have received less attention among the inventory practitioners in spite of requirement of sustainable inventory systems embedded with industries. In order to fill this research gap, we develop an SEPQ model for imperfect production where defect rate follows a known probability distribution and includes the features of sales team initiatives and price sensitive demand. To make the model sustainable, various carbon emission costs are considered. A metaheuristic, namely, Bat algorithm is employed to solve the concerned mixed integer inventory optimization problem.

The continuing introduction part is one of seven sections and rest contents of article are structured as follows. In Section 2, assumptions and notations are mentioned to describe the inventory model. In Section 3, two SEPQ models with rework and without/with lost sale have been developed. Section 4 examines a few special cases related to nature of defect rate. The fundamental idea of Bat algorithm (BA), a soft computing approach to establish the optimal pricing, inventory cycle and sales team activities, is discussed in Section 5. By utilising a suitable example, Section 6 presents the numerical results obtained using the analytical technique and Bat algorithm. Sensitivity analysis and managerial implications are also discussed in Section 6. Finally, Section 7 is devoted to the conclusions and future work on the suggested model.

2. Model description

In this study, we develop two models to consider the rework process and lost sale concepts. In the first model, we shall consider the rework process with a random defect rate. The defective products are assumed to follow a known probability distribution. The expected value is taken to formulate the inventory model. Different carbon emission costs are also considered. In the second model, we have incorporated the concept of lost sale by considering the goodwill cost. Both the models deals with sustainability by including carbon emission costs (CEC) where demand is based on sales team initiatives and selling prices. The assumptions and notations used to formulate the mathematical models for the concerned inventory problem are provided below.

Assumptions

Both the models are formulated based on the following assumptions:

(1) A single item is produced during the whole cycle.
(2) The rework process is employed to remove the defects of the faulty items produced during the production time.
(3) The rate of demand $D(s,\sigma)$ for the customers is dependent on price and sales effort. $D(s,\sigma)$ is a decreasing function of selling price. The symbols D and $D(s,\sigma)$ are used interchangeably in this article.
(4) Shortages are not permitted in the first model. The second model allows complete lost sale.
(5) The rate of production is finite. For the validation of the model, the production must be faster than the demand.
(6) The rework rate and the production rate are equal.
(7) Scrap items are not permissible within a cycle.

Notations

P	Production rate
C_p	Cost of production of a unit item
C_s	Setup cost per setup
C_g	The cost associated with goodwill loss
C_i	Cost of inventory holding for a unit item
C_l	Lost sale cost of a unit item ($C_l = (s - C_p + C_g)$)
C	Manufacturing cost per unit item
κ	Obsolescence rate
α	A scale parameter

λ	Space required for each unit item
m_0	Mass of an obsolescent item
C_{ei}	CEC for holding inventory
C_{eo}	CEC for inventory obsolescence
C_{ep}	CEC for each unit production
d_r	Random variable denoting the defective items
$E(d_r)$	Expected random value denoting the defective items in each cycle
Q	Production size
I	The maximum level of inventory
$\bar{I}$	Average inventory level
f	Cycle length fraction with positive inventory level
ꞵ	Total profit function
TC	Total cost function

Decision variables

T	Inventory period length (Continuous decision variables)
s	Price of a unit item (Continuous decision variables)
σ	Initiatives of the sales team (Discrete decision variables)

3. Mathematical model

The proposed model is formulated using a sustainable approach, where demand is based on marketing initiatives and sales price. This study is also concentrated on reducing waste *via* rework,which is produced as a result of imperfect production process. We have developed two models for imperfect production, the first one without shortages and other one with shortages.

3.1. Model A: SEPQ model with the rework process

By considering the carbon emission cost, we develop a SEPQ model by considering the rework process without lost sale. During the production, a fraction of total produced units (Q) may be defective for which rework will be done. The inventory level of an inventory cycle is depicted in Figure 1. During the time T_1, the items are produced at a rate of P and Q units are produced within time T_1. The effective production rate is $P[1-E(d_r)]$ due to the imperfect production process. After the time T_1 , production stops and defectives are reworked for the time duration T_2. Only demand occurs after the time point T_1+T_2 and inventory level will be zero at the end of this cycle.

From the △UVW, it is easy to obtain

$$\tan\theta_1 = P[1-E(d_r)] - D = \frac{I_1}{T_1} \tag{1}$$

$$\text{and } T_1 = \frac{Q}{P}\cdot \tag{2}$$

Similarly, in △KWX, we notice

$$\tan\theta_2 = P - D = \frac{I_2}{T_2} \tag{3}$$

$$\text{and } T_2 = \frac{E(d_r)Q}{P}\cdot \tag{4}$$

From the above equations (1)–(4), we have

$$I_1 = \frac{Q}{P}[P[1-E(d_r)] - D] \tag{5}$$

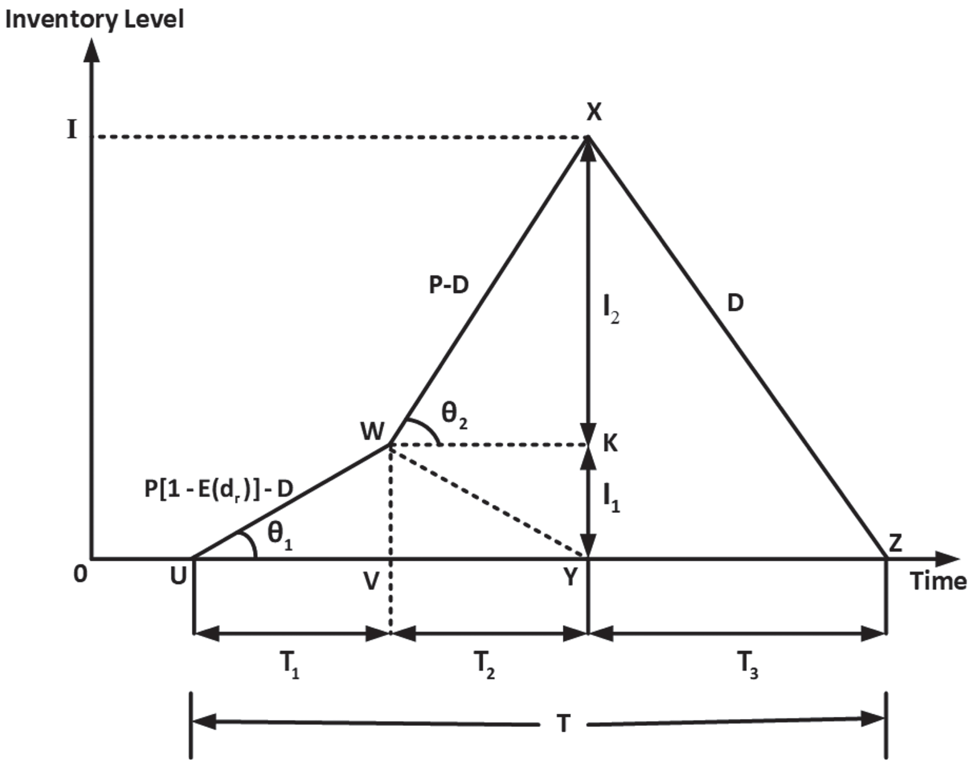


FIGURE 1. An inventory cycle for SEPQ model with rework process.

$$\text{and } I_2 = T_2(P - D) = \frac{E(d_r)Q}{P}(P - D). \tag{6}$$

The maximum inventory level I is obtained as the addition of I_1 and I_2

$$I = I_1 + I_2 = Q\left[1 - \frac{D}{P}(1 + E(d_r))\right]. \tag{7}$$

From the $\triangle$UVW, $\triangle$KWX and $\triangle$XYZ, we see

$$T_1 = \frac{I_1}{P[1 - E(d_r)] - D}, \; T_2 = \frac{I_2}{P - D} \; \text{ and } \; T_3 = \frac{Q[1 - \frac{D}{P}(1 + E(d_r))]}{D}. \tag{8}$$

Since Q items are consumed during the period T, we have

$$Q = DT \tag{9}$$

where, $T = T_1 + T_2 + T_2$.

The average lot-size is calculated by estimating the area of the inventory curve.

$$\bar{I} = \frac{1}{T}[\text{Area of } \triangle\text{UVW} + \text{Area of } \triangle\text{VYW} + \text{Area of } \triangle\text{XYW} + \text{Area of } \triangle\text{XYZ}]. \tag{10}$$

After some manipulations, we get

$$\bar{I} = \frac{Q^2}{2TD}\left[1 - \frac{D}{P}\left\{1 + E(d_r) + (E(d_r))^2\right\}\right]. \tag{11}$$

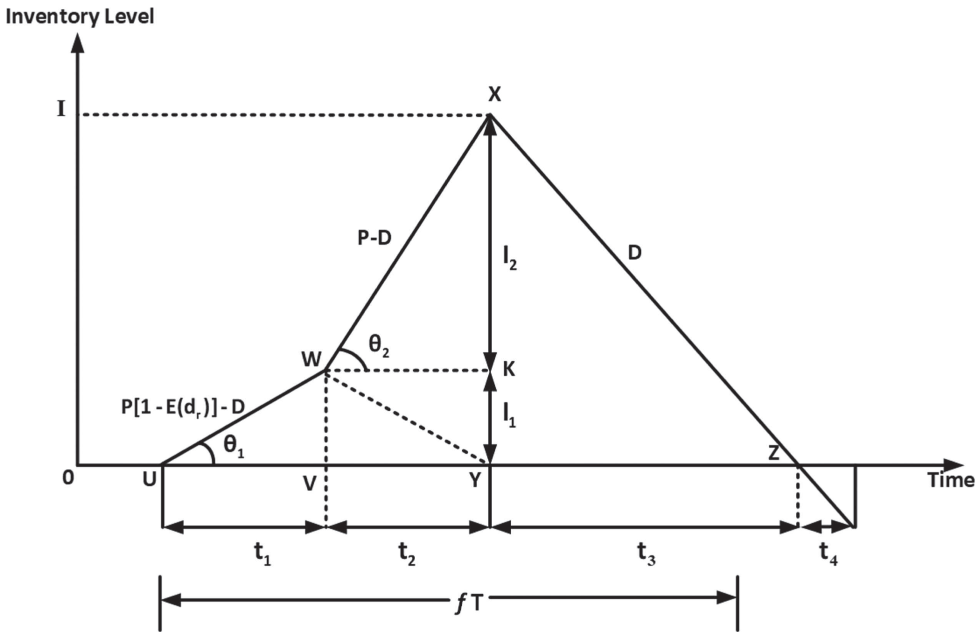


FIGURE 2. SEPQ model with rework process and lost sale.

By including environmental costs due to carbon emission, the average profit function for an inventory cycle is

$$\begin{aligned}\beta_{\text{SEPQ\_Basic}}(T,s,\sigma) &= sD - CD[1+E(d_r)] - C_pD - C_{ep}D - \frac{C_s}{T} - C_i\bar{I} - \lambda C_{ei}\bar{I} - C_{eo}\kappa m_0\bar{I}\\ &= sD - CD[1+E(d_r)] - C_pD - C_{ep}D - \frac{C_s}{T} - \frac{\omega DT}{2}\left[1-\frac{D}{P}\{1+E(d_r)+(E(d_r))^2\}\right]\end{aligned} \tag{12}$$

where $\omega = C_i + \lambda C_{ei} + \kappa m_0 C_{eo}$.

As we know that initiative of the sales team is a discrete variable. The profit optimization problem is a mixed integer problem. Analytically, it is not easy to find optimal values of decision variables. So, we shall use metaheuristic approach to find optimal values. The concavity of profit function is examined by evaluating the second order derivatives w.r.t. two variables T & s. The required derivatives are provided in Appendix A.

3.2. Model B: SEPQ model with the rework process and lost sale

This model is developed to analyze the rework process under the scenario of lost sales. When an item is not in stock, selling opportunities for that product will be missed. The customers do not restrict to wait until the product becomes available. Thus, the lost sale of that item occurs. The cost associated with the lost sale is assessed as the cost related to the goodwill loss. The loss of trade opportunities and unavailability of the desired brand product is also considered as the lost sale. Some environmental parameters are used to determine the profit function in addition to goodwill cost.

The production lot size, in this case, is given by

$$Q = fT. \tag{13}$$

From the △UVW, △KWX and △XYZ depicted in Figure 2, the following expressions can be obtained

$$t_1 = \frac{fTD}{P}, \quad t_2 = \frac{fTDE(d_r)}{P}, \quad t_3 = fT\left[1 - \frac{D}{P} - \frac{DE(d_r)}{P}\right]. \tag{14}$$

After some algebraic manipulation, the average inventory can be attained, similar to model A.

$$\bar{I} = \frac{Tf^2D}{2}\left[1 - \frac{D}{P}\left\{1 + E(d_r) + (E(d_r))^2\right\}\right]. \tag{15}$$

The average profit function, in this case, will be

$$\begin{aligned}\beta_{\text{SEPQ-Lost}}(T, s, \sigma) &= (s - C_p + C_g)Df - CDf(1 + E(d_r)) - C_{ep}Df - \frac{c_s}{T} - \omega\bar{I}\\ &= C_lDf - CDf(1 + E(d_r)) - C_{ep}Df - C_gD - \frac{c_s}{T}\\ &\quad - \frac{\omega f^2TD}{2}\left[1 - \frac{D}{P}\left[1 + E(d_r) + (E(d_r))^2\right]\right].\end{aligned} \tag{16}$$

To show concavity of the average profit function ($\beta_{\text{SEPQ-Lost}}$) all the required analysis is explained in Appendix B.

Remarks

(i) When demand is constant.
The derivatives of average profit function of model A with respect to T are

$$\frac{\mathrm{d}\beta}{\mathrm{d}T} = \frac{C_s}{T^2} - \frac{\omega D}{2}\left[1 - \frac{D}{P}\left[1 + E(d_r) + (E(d_r))^2\right]\right] \tag{17}$$

$$\text{and} \quad \frac{\mathrm{d}^2\beta}{\mathrm{d}T^2} = \frac{-C_s}{T^3} < 0\cdot \tag{18}$$

Using $\frac{\mathrm{d}\beta}{\mathrm{d}T} = 0$, the optimal value of T is obtained by

$$T^* = \sqrt{\frac{2C_s}{D\omega\delta}} \tag{19}$$

where, $\delta = [1 - \frac{D}{P}(1 + E(d_r) + (E(d_r))^2)]$.
The equation (18) indicates that the profit function is strictly concave. Thus at T^*, the total optimal profit is

$$\beta_{\text{SEPQ-Basic}}(T^*) = sD - CD[1 + E(d_r)] - C_pD - C_{ep}D - \left[2C_s\omega D\left[1 - \frac{D}{P}\left[1 + E(d_r) + (E(d_r))^2\right]\right]\right]^{\frac{1}{2}} \tag{20}$$

Similarly, for model $B, T^* = \sqrt{\dfrac{2C_s}{\omega f^2 D\delta}}\cdot$ (21)

By using (16), the total maximum profit function is

$$\beta_{\text{SEPQ-Lost}}(T^*) = f\left(C_lD - CD(1 + E(d_r)) - C_{ep}D - \sqrt{2\omega C_sD\delta}\right) - C_gD. \tag{22}$$

(ii) In the case of $f = 1$, model B will convert into model A. Hence model A (when $f = 1$) is a particular case of model B.

4. Special cases

The different distributions of defect process will provide different profit functions for both the models. We will consider the following cases.

Case A: Rectangular distribution with range $[\alpha, \beta], 0 < \alpha < \beta$

The expected value of defect rate

$$E(d_r) = \frac{\alpha+\beta}{2}\cdot \tag{23}$$

The maximum profit in this case is

$$\begin{aligned}\beta_{\text{SEPQ\_Basic}}(T,s,\sigma) &= sD - CD\left[1+\frac{\alpha+\beta}{2}\right] - C_pD - C_{ep}D - \frac{C_s}{T}\\ &\quad - \frac{\omega DT}{2}\left[1-\frac{D}{P}\left[1+\frac{\alpha+\beta}{2}+\left(\frac{\alpha+\beta}{2}\right)^2\right]\right]\end{aligned} \tag{24}$$

and

$$\begin{aligned}\beta_{\text{SEPQ-Lost}}(T,s,\sigma) &= C_lDf - CDf[1+\frac{\alpha+\beta}{2}] - C_gD - C_{ep}Df - \frac{C_s}{T}\\ &\quad - \frac{\omega f^2DT}{2}\left[1-\frac{D}{P}\left[1+\frac{\alpha+\beta}{2}+\left(\frac{\alpha+\beta}{2}\right)^2\right]\right]\cdot\end{aligned} \tag{25}$$

Case B: Triangular distribution with range $[\alpha, \beta, \gamma]$

In this case

$$E(d_r) = \frac{\alpha+\beta+\gamma}{3}\cdot \tag{26}$$

The maximum profit in this case is

$$\begin{aligned}\beta_{\text{SEPQ\_Basic}}(T,s,\sigma) &= sD - CD\left[1+\frac{\alpha+\beta+\gamma}{3}\right] - C_pD - C_{ep}D - \frac{C_s}{T}\\ &\quad - \frac{\omega DT}{2}\left[1-\frac{D}{P}\left[1+\frac{\alpha+\beta+\gamma}{3}+\left(\frac{\alpha+\beta+\gamma}{3}\right)^2\right]\right]\end{aligned} \tag{27}$$

and

$$\begin{aligned}\beta_{\text{SEPQ-Lost}}(T,s,\sigma) &= C_lDf - CDf\left[1+\frac{\alpha+\beta+\gamma}{3}\right] - C_gD - C_{ep}Df - \frac{C_s}{T}\\ &\quad - \frac{\omega f^2DT}{2}\left[1-\frac{D}{P}\left[1+\frac{\alpha+\beta+\gamma}{3}+\left(\frac{\alpha+\beta+\gamma}{3}\right)^2\right]\right]\cdot\end{aligned} \tag{28}$$

Case C: Beta distribution with parameters $[\alpha, \beta], 0 < \alpha < \beta < 1$

In this case

$$E(d_r) = \frac{\alpha}{\alpha+\beta}\cdot \tag{29}$$

Hence

$$\beta_{\text{SEPQ\_Basic}}(T, s, \sigma) = sD - CD\left[1 + \frac{\alpha}{\alpha+\beta}\right] - C_p D - C_{ep} D - \frac{C_s}{T} - \frac{\omega DT}{2}\left[1 - \frac{D}{P}\left[1 + \frac{\alpha}{\alpha+\beta} + \left(\frac{\alpha}{\alpha+\beta}\right)^2\right]\right] \tag{30}$$

and

$$\beta_{\text{SEPQ-Lost}}(T, s, \sigma) = C_l Df - CDf\left[1 + \frac{\alpha}{\alpha+\beta}\right] - C_g D - C_{ep} Df - \frac{C_s}{T} - \frac{\omega f^2 DT}{2}\left[1 - \frac{D}{P}\left[1 + \frac{\alpha}{\alpha+\beta} + \left(\frac{\alpha}{\alpha+\beta}\right)^2\right]\right]. \tag{31}$$

5. Bat algorithm

Bat algorithm (BA) is a nature-inspired optimization technique, which is based on the echolocation behaviors of micro-bats. The idea of this algorithm was first conceived by Yang [40]. The main idea of the algorithm was based on the moment of bats towards the prey, avoiding the obstacles. The microbats use a type of sonar, called echolocation, to find the target. These bats emit a thunderous sound pulse and capture the echo from the surrounding objects.

In the realm of optimization and computational intelligence, BA is the first algorithm of its kind since it incorporates frequency tuning. It is observed by Yang [40] that in some optimization problems, the BA is more suitable than the PSO. Moreover, Khan and Sahai [41] have shown the advantages of BA over PSO and genetic algorithm. Sadeghi *et al.* [42] utilized a hybrid BA, with calibrated parameters in redundancy allocation problem. Srivastava and Sahana [43] used BA to tackle network congestion and solved the transport network design problem. Shehab *et al.* [44] analyzed several Bat metaheuristic variants and discussed their applications in engineering problems. The pseudo-code for the BA is given in Figure 3.

The solution update equations are given as:

$$F_i = F_{\min} + (F_{\max} - F_{\min})\nu \tag{32}$$

$$V_i^{k+1} = V_i^k + (B_i^k - B^*)F_i \tag{33}$$

$$B_i^{k+1} = B_i^k + V_i^{k+1} \tag{34}$$

where

ν	:	A random number drawn uniformly between 0 and 1
F_i	:	Emission frequency for i^{th} bat ($i \in \{1, 2, ..., n_{\text{B}}\}$, where $n_{\text{B}} =$ no. of bats)
$F_{\min}, F_{\max}$	:	Lower & upper bound for the frequency
B_i^k	:	The position of i^{th} bat at k^{th} iteration
B^*	:	Current best bat position
V_i^k	:	The velocity of i^{th} bat at k^{th} iteration.

A local solution around the chosen best solution can be generated by using the concept of a random walk. The new bat position is then given by

$$B_{\text{new}} = B_{\text{old}} + \vartheta r_k l^k \tag{35}$$

Initialization Step:

Define objective function þ$(\boldsymbol{X})$, $X \in (x_1, x_2, \ldots, x_m)^T$.
Set the parameter values of $\boldsymbol{F_{min}}$, $\boldsymbol{F_{max}}$, $\boldsymbol{\vartheta}$, $\boldsymbol{n_B}$, $\boldsymbol{MIter}$, $\boldsymbol{\phi}$ and $\boldsymbol{\eta}$.
Initialize bat population B_i and velocity V_i,
Define pulse frequency F_i for each B_i
Initialize pulse rate pr_i^0 and loudness l_i for each B_i.
Set k=1.

Main Loop:

While $k < MIter$
  For $i = 1$ to n_B
   Update bat population and velocities using eqs. (33) and (34)
    If $(rand > pr_i)$
    Choose a solution among the best solutions.
    Obtain a local solution B_{new} around the selected solution using eq. (35)
    End If
    If (rand $<$ l_i and þ(B_{new}) $<$ þ(B^*))
      Accept the solution
      Reduce l_i and increase pr_i using eqs. (36) and (37).
    End If
  End For
  Rank the best solutions and select current best solution $\boldsymbol{B^*}$.
  $k = k + 1$
End while

FIGURE 3. Pseudo code for BA.

where

r_k: A random number chosen from the standard normal distribution $N(0, 1)$.
l^k: Average loudness of bats at iteration k.
ϑ: A scale parameter to avoid overshooting or undershooting.

The loudness and pulse emission rate are updated by using the formulae:

$$l_i^{k+1} = \phi l_i^k \tag{36}$$

$$pr_i^{k+1} = pr_i^0[1 - \exp(-\eta k)]. \tag{37}$$

Here pr_i^0 is the initial pulse emission rate and it is generally selected randomly between 0 and 1. Also, ϕ and η are constants such that $0 < \phi < 1$ and $\eta > 0$.

6. NUMERICAL AND SENSITIVITY ANALYSIS

The cement industry is one of the emerging sectors in recent years. The existing production system technology for cement clinker is harmful to the ecological system. It consumes a lot of energy and natural resources and emits pollutants. The desired optimal profit may be evaluated by using the proposed models, which include

TABLE 2. Numerical results by using BA.

Model	Illustration	Optimal policy				Run time (s)
		T^* (Month)	s^* (\$)	σ^*	β^* (\$)	
A	1	0.5111	-	-	1004.80	0.0396
B	2	0.6888	-	-	788.83	0.0423
A	3	0.5891	51.30	22	953.27	0.0574
B	4	0.7307	51.73	19	740.06	1.0341

the rework and carbon emission costs. Here we are presenting some numerical results for the model to see the behavior of profit function in each case considered in the paper.

To show the industrial advantage of the proposed inventory model, this section contains an industrial example along with four different scenarios. The parameter values are taken from the inventory literature. This numerical data is undertaken to authenticate this research. To study the nature of profit function regarding different situations, we will consider some estimated parameters that are taken by inspection. For computational purposes, the following default parameters are considered.

$P = 45$ units/month, $s = \$60$ per unit, $C_p = \$7$ per unit, $C_s = \$50$ per setup, $C_i = \$30$ per unit, $C_g = \$3$ per unit, $\sigma = 1$ ton per unit, $C = \$5$ per unit, $\kappa = 10\%$, $\lambda = 1.30$ cube-meter per unit, $C_{ei} = \$0.55$ per cube-meter, $E(d_r) = 0.05$, $C_{eo} = \$10$ per ton, $C_{ep} = \$0.30$ per unit, $m_0 = 1$ ton per unit.

The following illustrations are considered for the numerical simulation of the proposed models.

I. Illustration 1 (I-1): Constant Demand without Lost Sale.
$D = 25$ units/month, $f = 1$, $s = \$60$ per unit.

II. Illustration 2 (I-2): Constant Demand with Lost Sale.
$D = 25$ units/month, $f = 0.80$, $s = \$60$ per unit.

III. Illustration 3 (I-3): Price and Promotion Sensitive Demand without Lost Sale.
$D = a - bs + \alpha\left(1 - \frac{1}{\sigma+1}\right)$, $f = 1$. Where, $a = 56$, $b = 0.7$ and $\alpha = 10$.

IV. Illustration 4 (I-4): Price and Promotion Sensitive Demand with Lost Sale.
$D = a - bs + \alpha\left(1 - \frac{1}{\sigma+1}\right)$, $f = 0.8$. Where, $a = 56$, $b = 0.7$ and $\alpha = 10$.

Illustrations 1 and 2 only assume constant demand. A closed form solution can be obtained by using equations (24) and (25). The optimal decision obtained as:

For illustration 1, $T^* = 0.5511$ and corresponding $\beta_{\text{SEPQ\_Basic}}(T^*) = \$1004.80$.

For illustration 2, $T^* = 0.6888$ and corresponding $\beta_{\text{SEPQ-Lost}}(T^*) = \$788.83$.

6.1. BA implementation

To optimize the profit function, BA is implemented by using MATLAB (R2021b). The initial loudness is selected randomly between 0 and 1. After performing parameter tuning experiments, the following initial parameter values are taken to employ Bat metaheuristic:

$\phi = 0.87$, $\chi = 0.42$, $\nu = 0.011$, $F_{\min} = 0$, Maximum no. of bats (population size) $n_{\text{B}} = 50$, Maximum number of iterations MIter $= 100$.

Our study tests for the applicability of Bat metaheuristic for the proposed model. Table 2 shows the numerical outcomes obtained by implementing BA to the proposed profit maximization model. The following observations are obtained:

– The optimal solution of the optimization problem obtained by BA is similar to that obtained by the classical method for Illustrations 1 and 2. This fact ensures us about the applicability of the BA for the other illustrations where demand function is price sensitive and the corresponding optimization problem is a mixed integer programming problem.

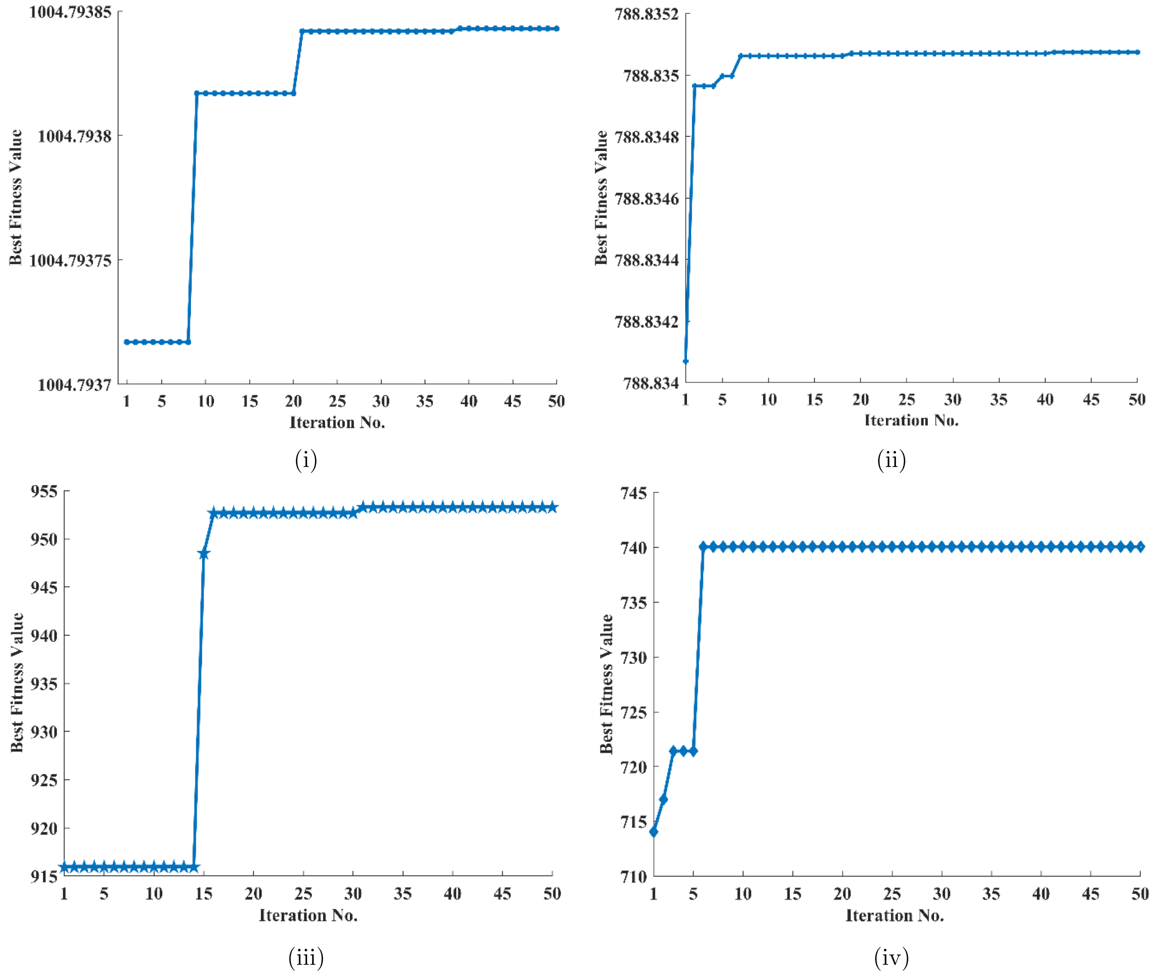
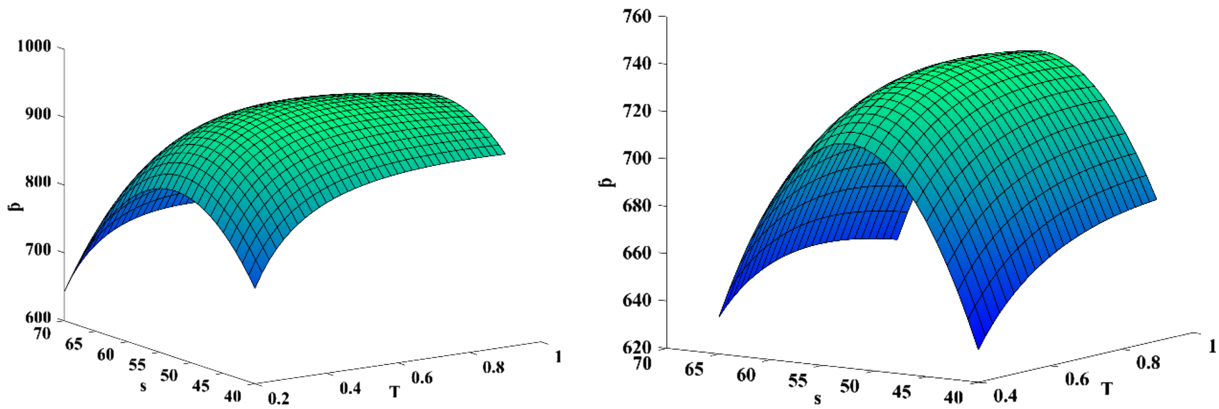


FIGURE 4. The convergence of BA for (i) I-1, (ii) I-2, (ii) I-3, (iii) I-4.

– The optimal policies for Illustrations 3 and 4 are shown in Table 2. The computation run times for all the illustrations are not more than 1 second. Thus, Bat metaheuristic is also computationally inexpensive.
– The convergence of Bat metaheuristic is depicted in Figures 4i–4iv. The convergence patterns indicate that BA converges to the optimal solution within 20–25 iterations. Thus, BA shows good convergence for the proposed optimization problem.
– The concave nature of the average profit function is depicted in Figures 5i and 5ii for illustrations I-3 and I-4, respectively. The average profit function is plotted against inventory cycle time (T) and selling price (s), where the sales team initiative (σ) is fixed at its optimal value obtained by using BA. The concave nature of the average profit function supports the claim of using Bat metaheuristic to our proposed optimization problem.

6.2. Sensitivity analysis

By using the above data of parameters, the graphical representation of profit function has been done in Figures 6i–6vi for varying values of parameters. The following outcomes are obtained.



(i) (ii)

FIGURE 5. The trend in average profit with T and s for (i) I-3, (ii) I-4.

- Figures 6i–6iii show the concave nature of profit function in different situations with respect to T for different values of f, D and P, respectively.
- Figure 6iv displays the variations in total optimal profit by varying the demand rate (D) for different values of f at a constant production rate (P). This variation seems to almost linear.
- Figure 6v exhibits the change in optimal profit function with respect to the production rate (P) at a constant demand (D). It is observed that as the production rate increases, profit goes on decreasing at a minimal rate.
- Figure 6vi depicts the trend of the optimal profit function with respect to the expected defect rate. It is noticed that rework is profitable as the profit is higher if the rework is done.

Now, we prefer the sensitivity of parameters for illustration-(I-1) to explore the changes in cost parameters on the optimal cycle length and total optimal profit.

- The impacts of changes in inventory cost parameters on the total optimal profit (β) and optimal inventory cycle (T) are shown in Table 3. The parameter values are varied by -20% to $+20\%$ from their respective values taken in the data set.
- The cost parameters, *viz.*, setup cost (C_s) and holding cost (C_i) are more sensitive to the optimal inventory cycle length (T) among all the cost parameters.
- The sensitivity analysis of sustainable cost parameters is provided in Table 4. The sustainability cost parameters are microscopic in comparison to setup cost (C_s) and holding cost (C_i). The sensitivity of cost elements, ranging from -60% to $+60\%$ to analyze the effective changes, is displayed in Table 4.
- The sustainable cost parameters are less sensitive to the optimal profit. It is noticed that all the changes in parameters are consistent as we expect in realistic scenarios.
- The other parameters, *viz.*, s, $E(d_r)$, λ, m_0 and κ, involved in the profit function are examined in Table 5, separately.
- The percentage changes demonstrate that the sensitivity of parameter "s" is more than all the remaining parameters.

6.3. Managerial insight

In this study we have developed two EPQ models. The first model covers the EPQ problem that considers rework without shortages. The second model focuses on the EPQ problem with shortages, when it is prudent to

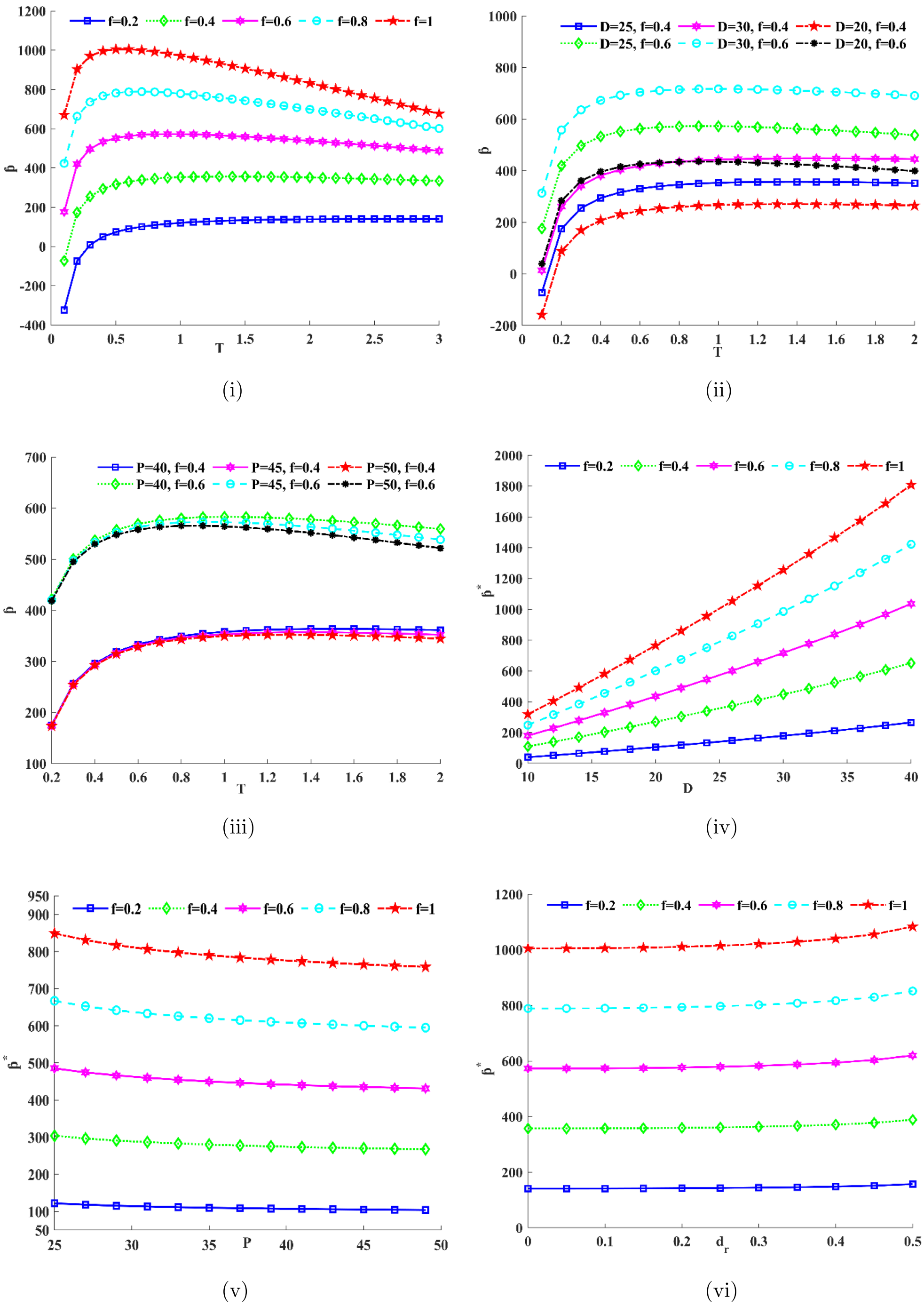


FIGURE 6. (i) Concave nature of the profit function for different values of f. (ii) Concave nature of the profit function for different values of D and f. (iii) Concave nature of the profit function for different values of P and f. (iv) Optimal profit with demand rate at a constant production rate. (v) Optimal profit *vs.* production rate for different values of f. (vi) Optimal profit *vs.* defect rate for different values of f.

TABLE 3. Sensitivity analysis of cost parameters.

Cost parameters (CP)	% Change in CP	T^*	% Change in T^*	$\beta(T^*)$	% Change in β (T^*)
	+20	0.60	+8.87	987.47	−1.72
	+10	0.58	+5.24	995.94	−0.88
C_s	0	0.55	+0	1004.80	0
	−10	0.52	−5.64	1014.10	+0.92
	−20	0.49	−11.08	1024.00	+1.91
	+20	0.50	−9.27	988.37	−1.63
	+10	0.52	−5.64	996.40	−0.83
C_i	0	0.55	0	1004.80	0
	−10	0.58	+5.24	1013.60	+0.87
	−20	0.61	+10.68	1022.90	+1.80
	+20	0.55	0	969.79	−3.84
	+10	0.55	0	987.29	−1.74
C_p	0	0.55	0	1004.80	0
	−10	0.55	0	1022.30	+1.74
	−20	0.55	0	1039.80	+3.48
	+20	0.55	0	978.54	−2.61
	+10	0.55	0	991.67	−1.31
C	0	0.55	0	1004.80	0
	−10	0.55	0	1017.90	+1.30
	−20	0.55	0	1031.00	+2.61

TABLE 4. Sensitivity analysis of sustainable cost parameters.

Sustainable Cost Parameters (CP)	% Change in SCP	T^*	% Change in T^*	β (T^*)	% Change in β (T^*)
	+60	0.5460	−0.92	1003.10	−0.17
	+40	0.5477	−0.62	1003.70	−0.11
	+20	0.5494	−0.31	1004.20	−0.06
C_{eo}	0	0.5511	0	1004.80	0
	−20	0.5528	+0.31	1005.40	+0.06
	−40	0.5546	+0.63	1005.90	+0.11
	−60	0.5564	+0.96	1006.50	+0.17
	+60	0.5511	0	1000.30	−0.45
	+40	0.5511	0	1001.80	−0.3
	+20	0.5511	0	1003.30	−0.15
C_{ep}	0	0.5511	0	1004.80	0
	−20	0.5511	0	1006.30	+0.15
	−40	0.5511	0	1007.80	+0.3
	−60	0.5511	0	1009.30	+0.45
	+60	0.5474	−0.67	1003.60	−0.12
	+40	0.5786	−0.45	1004.00	−0.08
	+20	0.5499	−0.22	1004.40	−0.04
C_{ei}	0	0.5511	0	1004.80	0
	−20	0.5523	+0.22	1005.20	+0.04
	−40	0.5536	+0.45	1005.60	+0.08
	−60	0.5549	+0.69	1006.00	+0.012

TABLE 5. Sensitivity analysis of inventory parameters.

Inventory Parameters (IP)	% Change in IP	T^*	% Change in T^*	β (T^*)	% Change in β (T^*)
	+60	0.5511	0	1904.80	+89.57
	+40	0.5511	0	1604.80	+5971
	+20	0.5511	0	1304.80	+29.85
s	0	0.5511	0	1004.80	0
	−20	0.5511	0	704.79	−29.85
	−40	0.5511	0	404.79	−59.71
	−60	0.5511	0	104.70	−89.57
	+60	0.5640	+2.34	1005.20	+0.04
	+40	0.5595	+1.52	1005.00	+0.02
	+20	0.5552	+0.74	1004.90	+0.01
$E(d_r)$	0	0.5511	0	1004.80	0
	−20	0.5471	−0.72	1004.70	−0.01
	−40	0.5433	−1.41	1004.69	−0.012
	−60	0.5396	−2.08	1004.68	−0.011
	+60	0.5474	−0.67	1003.60	−0.12
	+40	0.5486	−0.45	1004.00	−0.08
	+20	0.5499	−0.22	1004.40	−0.04
λ	0	0.5511	0	1004.80	0
	−20	0.5523	+0.22	1005.20	+0.04
	−40	0.5536	+0.45	1005.60	+0.08
	−60	0.5549	+0.69	1006.00	+0.12
	+60	0.5460	−0.92	1003.10	−0.17
	+40	0.5477	−0.62	1003.70	−0.11
	+20	0.5494	−0.31	1004.20	−0.06
m_0	0	0.5511	0	1004.80	0
	−20	0.5528	+0.31	1005.40	+0.06
	−40	0.5546	+0.63	1005.90	+0.11
	−60	0.5564	+0.96	1006.50	+0.17
	+60	0.5460	−0.92	1003.10	−0.17
	+40	0.5477	−0.62	1003.70	−0.11
	+20	0.5494	−0.31	1003.70	−0.06
κ	0	0.5511	0	1004.80	0
	−20	0.5528	+0.31	1005.40	+0.06
	−40	0.5546	+0.63	1005.90	+0.11
	−60	0.5564	+0.96	1006.50	+0.17

invest in a reworking process. The following are key benefits of investigation done on the sustainable inventory model:

- A firm may conduct major findings by employing product marketing, which boosts the company's brand image and earnings. Based on our inventory model, company managers can focus on the effective promotion. A reduced cost per unit effort of sales team activities can motivate the business organizers to make attempts to enhance the consumer demand.
- The rework of faulty goods is required for the manufacturing organizations to take benefits in terms of waste reduction, increased turnover, and market goodwill.
- The proposed model portrays the effect of defects using a probabilistic perspective which may be helpful for the effective management strategy.

- Our study suggests to produce an admissible production quantity in order to reduce carbon emission and wastage.
- Bat metaheuristic is suggested to solve the mixed integer problem of the proposed model for which traditional approach seems to pose computational complexity. BA offered the optimum solution of the concerned problem in less computation time.

7. Conclusions

Two sustainable economic production quantity (SEPQ) models in the present article depict various demand scenarios in a production system and may be helpful to the manufactures to enhance the profit. In order to account for imperfect items in a manufacturing system, our study included a rework process in the SEPQ models. The assembly of price-sensitive demand, sales team actions,and sustainability consideration are the main features of the suggested production-inventory model. Two sustainable EPQ models developed with rework without shortage and rework with the lost sale, involves uncertain sustainability subsumed by the cost of stock emissions. A proportion of defective items with rework in the production process *via* a suitable probabilistic distribution incorporated make the model more realistic and beneficial to the manufacturers. The endeavors of the sales teams and promotional efforts may enhance the consumer demand in an oligopolistic marketing environment.

The suggested models may be helpful for the industries where manufacturing system requires the reworking of the flawed goods to appropriately preserve the profitability. The proposed inventory model seems to be more realistic and suited for various industrial production systems, because of the stochastic nature of imperfect defect rate. Initiatives to build and maintain environmental protection should pay close attention to achieve sustainability. The suggested optimization approach has used successfully to solve the challenging inventory problem in estimating the environmental cost parameters.

There is a scope to evolve the sustainability strategy to enhance the applicability of the proposed study. In future, a case study may be done along with the proposed model to enhance the applicability of our study in real business world. To cope up with a more realistic scenario, one may employ the stochastic nature of demand and fuzzy cost parameters. The condition of an equal production rate and rework rate can be relaxed in future research endeavor.

Appendix A. Proof of concavity by using hessian matrix in model A

The following notations are used to simplify the algebraic manipulations:

$$k_1' = C_g - C_p, k_2' = 1 + E(d_r), k_3' = \left[1 - \frac{D}{P}\left[1 + E(d_r) + (E(d_r))^2\right]\right], k_4' = Ck_1 + C_{ep}, k_5' = \sqrt{\frac{2C_s}{\omega}}. \tag{A.1}$$

For the optimum values of β, $\frac{\partial β}{\partial T} = \frac{\partial β}{\partial s} = 0$

$$\frac{\partial β}{\partial T} = \frac{C_s}{T^2} - \frac{\omega D}{2}\left[1 - \frac{D}{P}k_3'\right] = 0 \tag{A.2}$$

$$\Rightarrow T^* = \sqrt{\frac{2C_s}{\omega D\left(1 - \frac{D}{P}k_3'\right)}} = \frac{k_5'}{\sqrt{\left(1 - \frac{D}{P}k_3'\right)D}} \tag{A.3}$$

$$\frac{\partial β}{\partial s} = 0$$

$$\Rightarrow \left[(s - k_4')D'\sqrt{\left(1 - \frac{D}{P}k_3'\right)D} + \left(\frac{\omega DD'k_3'}{P} - \frac{\omega D'}{2}\right)k_5' + D\sqrt{\left(1 - \frac{D}{P}k_3'\right)D}\right] = 0. \tag{A.4}$$

It is not feasible to solve the equation (A.4) explicitly to find the optimal value of "s" that maximizes the average profit. However numerical approaches or soft computing approaches can be used to solve this.

By using second order derivatives, we find the optimal values of "T" and "s" for which the Hessian matrix is formulated as follows:

$$\left|H\left(\text{ß}_{\text{SEPQ-Basic}}\right)\right| = \begin{vmatrix} \frac{\partial^2 \text{ß}}{\partial T^2} & \frac{\partial^2 \text{ß}}{\partial T \partial s} \\ \frac{\partial^2 \text{ß}}{\partial s \partial T} & \frac{\partial^2 \text{ß}}{\partial s^2} \end{vmatrix}. \tag{A.5}$$

We need to calculate all eigen values of hessian matrix. To show the concavity of the profit function, these eigenvalues must be negative. The second order partial derivatives are obtained as follows:

$$\frac{\partial^2 \text{ß}}{\partial T^2} = -\frac{C_s}{T^3} \tag{A.6}$$

$$\frac{\partial^2 \text{ß}}{\partial s^2} = \left[\left(s - k_4' + \frac{\omega T}{2}\right)D'' - \frac{\omega k_3' T}{P}\left(D'^2 + DD''\right) + 2D'\right] \tag{A.7}$$

$$\frac{\partial^2 \text{ß}}{\partial T \partial s} = \left[\frac{\omega k_3' DD'}{P} - \frac{\omega D'}{2}\right]. \tag{A.8}$$

Appendix B. Proof of concavity by using hessian matrix in model B

For optimum values of ß, $\frac{\partial \text{ß}}{\partial T} = \frac{\partial \text{ß}}{\partial s} = 0$ hold. We are using following notations to simplify the algebraic manipulations:

$$k_1'' = C_g - C_p, k_2'' = 1 + E(d_r), k_3'' = \left[1 - \frac{D}{P}\left[1 + E(d_r) + (E(d_r))^2\right]\right], k_4'' = Ck_1 + C_{ep}, k_5'' = \sqrt{\frac{2C_s}{\omega f^2}} \tag{B.1}$$

$$\frac{\partial \text{ß}}{\partial T} = \frac{C_s}{T^2} - \frac{\omega D}{2}\left[1 - \frac{D}{P}k_3''\right] = 0 \tag{B.2}$$

$$\Rightarrow T^* = \sqrt{\frac{2C_s}{\omega f^2 D\left(1 - \frac{D}{P}k_3''\right)}} = \frac{k_5''}{\sqrt{\left(1 - \frac{D}{P}k_3''\right)D}} \tag{B.3}$$

$$\frac{\partial \text{ß}}{\partial s} = 0$$

$$\Rightarrow f\left[(s - k_4'')D'\sqrt{\left(1 - \frac{D}{P}k_3''\right)D} + \left(\frac{\omega f DD' k_3''}{P} - \frac{\omega f D'}{2}\right)k_5'' + D\sqrt{\left(1 - \frac{D}{P}k_3''\right)D}\right] = 0. \tag{B.4}$$

It is not feasible to solve the equation (B.4) explicitly to find the optimal value of "s" that maximizes the average profit. However numerical approaches or soft computing approaches can be used to solve this.

The second order partial derivatives are calculated as follows:

$$\frac{\partial^2 \text{ß}}{\partial T^2} = -\frac{C_s}{T^3} \tag{B.5}$$

$$\frac{\partial^2 \text{ß}}{\partial s^2} = f\left[\left(s - k_4'' + \frac{\omega f T}{2}\right)D'' - \frac{\omega f k_3'' T}{P}\left(D'^2 + DD''\right) + 2D'\right] \tag{B.6}$$

$$= f\left[\left(s - k_4'' + \frac{\omega f k_5''}{2\sqrt{\left(1 - \frac{D}{P}k_3''\right)D}}\right)D'' - \frac{\omega f k_5'' k_3''}{P\sqrt{\left(1 - \frac{D}{P}k_3''\right)D}}\left(D'^2 + DD''\right) + 2D'\right] \tag{B.7}$$

$$\frac{\partial^2 \text{ß}}{\partial T \partial s} = f^2\left[\frac{\omega k_3'' DD'}{P} - \frac{\omega D'}{2}\right]. \tag{B.8}$$

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