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DECISION SUPPORT SYSTEM FOR MANAGING MULTI-ECHELON SUPPLY CHAIN NETWORKS AGAINST DISRUPTIONS USING ADAPTIVE FRACTIONAL ORDER CONTROL ALGORITHM

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Abstract. Managing highly evolving supply chains can be challenging, especially when vulnerable to disruptions and risks. This paper deals with a supply chain system's dynamical analysis and efficient management strategy using a four-stage hyperchaotic Lorenz–Stenflo equation under disruptive events. Nonlinear behaviors are intensely investigated by eigenvalue and bifurcation analysis to identify supply chain risks. Then phase portraits are presented to illustrate the bullwhip effect negatively influencing the performance of various stages of multi-echelon supply chains. Resilient supply chains have been developed along with dynamic identification by realizing an adaptive fractional-order controller. An efficient control algorithm can optimize the management system while reducing potential risks by employing control theory in a decision support system. Performance criteria have been exploited to validate the control methodology. Using digital management algorithms, decision-makers might effectively cope with chaos suppression and synchronization problems, ensuring productivity and sustainability. Finally, the novel decision-making strategy can offer new insights into effectively managing digital supply chain networks against market volatility.

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1. Introduction

Supply chain management deals with complex dynamic activities involving communication, coordination, and collaboration between multiple constituents to manage the flow of goods and services efficiently. Sometimes, the supply chain system exhibits a chaotic behavior that represents nonlinear characteristics due to dynamic interactions of subsystem dynamics, such as manufacturers, retailers, distributors, and end customers, which will form multiple-stage network models [1]. Most leading enterprises are concerned about their inability to predict future business performance. They have worried about their supply chains' strategic resilience strategy and sustainability planning in the face of unrelenting market challenges. Today, business industries with such rigid supply chains cannot keep pace with the current market changes. In unpredictable markets, the management

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strategy of each constituent in the supply chains should fully and promptly meet the customers' demand, achieving high profitability and ensuring maximum customer satisfaction with efficiency and reliability [2].

Recently, chaos theory has been a topic of considerable interest among researchers. In the deterministic chaos of seemingly showing irregular or agitated dynamical behaviors, it is possibly predictable in a short period but volatile in a long period [2]. Supply chain optimization is highly challenging because of the various risk factors impacting enterprise decision-making processes. The unexpected disruptions are mainly caused by uncertainties in demand, supply, delivery, and forecasting [3, 4]. Some disturbances as uncontrollable events include natural disasters, strikes, trade barriers, global pandemics, and wars. The bullwhip effect is widely recognized in operational management, leading to significant supply chain inefficiencies [5]. Distorted or untimely information from one stage in the network to the others might result in extreme inefficiencies of supply chain activities: excess or shortage of inventory, inaccurate forecasts, transportation delays, poor customer service, and missed factory shipments. The pressure of the shift towards leaner supply chains forces practitioners to focus on identifying the causes and mitigating bullwhip behavior [6]. Supply chain management software aims to coordinate the flow of products or information most efficiently and cost-effectively. This study will therefore address the following research questions against a volatile market:

– How to gain insight from the dynamical properties of multi-echelon chaotic supply chains?
– How to implement an active controller to mitigate supply chain risks posed by unexpected disruptions?
– How to improve productivity and increase supply chain profits using control theory?

The supply chain risk or vulnerability arises from system uncertainties, in which many events cannot be foreseen in reality [7]. System uncertainty is mainly due to a lack of knowledge of the system parameters, modeling errors, and external disturbances. The complexity and uncertainty will increase the supply chain risks. The network risk is the probability that an inbound supply problem will disrupt a business. Supply chain risks can also result in quality, liability, and reputational issues. The network risks should be effectively identified, analyzed, and controlled for efficient decision-making.

In the context of risk analysis, the applicability of control theory has been investigated in supply chain management [8], in which active control systems can reduce uncertainty or volatility. Today's supply-chain fluctuations require systemic solutions using counterintuitive actions. No articles have been published realizing fractional-order control algorithms to improve dynamic performance in multi-echelon supply chain management. This paper proposes an adaptive fractional-order PID control scheme to accommodate all kinds of fluctuations and chaotic behaviors of four-echelon supply chain networks against market volatility. Simultaneously, supervisory control is implemented to deal with perturbations when the dynamic behaviors exceed some bounds. Business enterprises that understand a volatile market's dynamics can take decision-making actions that guarantee competitive advantage and better supply chain performance than competitors.

To summarize, this paper presents a dynamic model of a four-stage supply chain system to optimize its network operations under uncertainty. This study proposes a comprehensive strategy that exploits the eigenvalue characterization and bifurcation theory to gain more insight into the dynamic properties of the supply chain network. Chaotic behaviors have been systematically investigated through a stability analysis. Then, a novel adaptive fractional-order control algorithm realized chaotic suppression and synchronization. The stability of closed-loop supply chain systems has been demonstrated using the Lyapunov theory. Two types of performance criteria, including integral of squared error (ISE) and integrated of time-weighted-squared-error (ITSE), have been exploited to evaluate control system performance. There is a way to design a new supply chain management system that allows companies to take advantage of the evolving market, giving them a competitive advantage. In the face of uncertainty, this study proposes that supply chains must quickly react and adapt to any changes and disruptions.

The rest of this paper is organized as follows. The literature review is presented in Section 2. Section 3 deals with a multi-stage supply chain model with parameter perturbation. Dynamical behaviors and the bullwhip effect are also shown. Section 4 presents adaptive fractional order control synthesis with supervisory control.

In Section 5, numerical simulations are carried out to demonstrate the effectiveness of the proposed approach. Finally, conclusions are made in Section 6.

2. Literature review

2.1. Supply chain disruption risks

Many supply chains are becoming increasingly complex and fragile under highly uncertain markets. Complicated behaviors of a supply chain system stem from several disruption factors that policymakers have to consider seriously for efficient decision-making. In an increasingly globalized world, disruptions may be caused by unforeseen external factors such as supplier bankruptcy, a political dispute, war, a pandemic, a natural disaster, or strikes. An interruption in any link along the networks can profoundly affect the rest of the supply chains [9, 10]. Supply chain managers might face a new difficulty due to the rippling effects of disruptions [3, 11], as opposed to the parametrical aberrations in the bullwhip effect, which are susceptible to structural disturbances in the supply chains. Over the past 20 years, much work has been done on inventory management, production constraints, and demand changes [3, 12, 13]. Academic-practitioner would pay more attention to supply chain disruptions as natural or man-made disasters increase. For example, Cuong *et al.* [14] investigated the impact of the COVID-19 pandemic on maritime supply chain performance using nonlinear analysis methods, including equilibrium analysis, stability evaluation, and time series investigation. The dynamical analysis indicates that the supply chain system shows highly nonlinear behavior due to system uncertainty and interruption.

A framework was created by Ali *et al.* [10] to recognize, examine, and evaluate the causes and effects of supply chain disruptions. Based on market scenarios and risks, four types of disruption factors are identified and investigated in a real-world setting: natural, man-made, system accidents, and financials. Additionally, Dolgui and Ivanov [11] presented a detailed analysis of state-of-the-art and projected outcomes of ripple effects on supply chain systems. Among their suggested approaches, their work includes simulation, game theory, control theory, data-driven analytics, network complexity, reliability theory, and empirical results. Olivares-Aguila and ElMaraghy [15] proposed a system dynamics framework for observing supply chain behavior and evaluating disruptions. They examined how disruptions impacted supply chain service levels, costs, profits, and inventory levels. Based on the outcomes of many business scenarios, it was found that interruptions at downstream levels are more likely than those at upstream levels to have adverse impacts on supply chain performance. Last, Wang *et al.* [1] developed a measuring technique for the supply chain network structure and employed the design structure matrix model to determine its complexity. The construction of supply chain networks under interruption risks is theoretically supported by analysis of the internal link between complexity value, network size, and connection probability. Moreover, digital twins and data-driven frameworks based on machine learning and metaheuristics optimization have been introduced to enhance the performance of supply chains subject to unpredictable disruptions [59, 60]. As listed in Table 1, several approaches are presented for managing supply chain disruptions and risks. No studies have yet offered a comprehensive method dealing with nonlinear analysis and control theory to estimate disruption risks and optimize supply chain systems. Despite a growing volume of literature dealing with dynamical analysis in supply chain management, most findings are pretty fragmented and lack an analytical framework that utilizes the nonlinear approach to analyze supply chain disruptions and risks. Accordingly, this research provides a comprehensive methodology employing equilibrium analysis, bifurcation evaluation, and phase portrait to evaluate disruption risks in the supply chains. A decision support system based on adaptive fractional control theory is also realized for managing multi-echelon supply chain networks against disturbances.

2.2. Dynamical analysis and active decision-making strategy

A business contingency plan should be quickly put into action at the control stage in the reactive mode to help enterprises recover from an unexpected event. Only competent managers use such a proactive strategy to overcome the obstacle. A platform that facilitates collaboration and supply chain visibility solutions is

Table 1. Summary of analyzing supply chain disruptions and risks.

Authors	Method of analysis	Supply chain applications	Main contribution
Wang *et al.* [1]	Barabasi-Albert and Watts-Strogatz networks	None identified	Exploring enterprise risks, operational robustness and flexibility, completeness of market information, and network topology.
Li and Liu [2]	Equilibrium decisions analysis, fuzzy uncertainties	Two-echelon green supply chain models	Investigating green supply chain game models with governmental interventions and risks under fuzzy uncertainties.
Ali *et al.* [10]	Delphi method and the fuzzy analytic hierarchy process	Ready-made garment supply chain	Developing a framework to identify, analyze, and assess supply chain disruption factors. Defining top six disruption drivers.
Naim *et al.* [12]	Control block diagram manipulation with analogical reasoning	Production-distribution system	Identifying the bullwhip effect with an appropriate source model and establishing a correct candidate solution.
Olivares-Aguila and ElMaraghy [15]	Mathematical modeling, system dynamics analysis, and simulation	Four-echelons supply chains	Employing a system dynamics paradigm to assess the effects of disturbances. Upstream levels are more susceptible than downstream levels to disruptions.
Modgil *et al.* [16]	Artificial Intelligence (AI)	Supply chain resilience during COVID-19	Employing AI-based supply chains to realize resilience in its structure and network
Dolgui *et al.* [17]	Optimal control models, qualitative methods of performance analysis	Production, supply chain, and Industry 4.0 systems	Explaining control engineering models in terms of industrial engineering and production management.
Spiegler *et al.* [18]	Mathematical analysis, linearization method, an approximation method	Production distribution system	Developing simplified linear representations of complex supply chain models.
Corsini *et al.* [59]	Artificial Neural Network and Particle Swam Optimization	Two-echelon supply chain	Introducing a data-driven framework for dynamically selecting the optimal replenishment strategy in the supply chain.
Badakhshan and Ball [60]	Digital twin integrating Machine Learning	Inventory and cash throughout the supply chain	Investigating the potential of an SC digital twin framework to help decision-makers manage inventory and cash across the SC network under disruptions

necessary for the execution of such recovery plans to evaluate the disruption's impact on the supply chain and the costs of rerouting material flows. Based on the system dynamics framework, many management policies and control methods have been demonstrated in the supply chain literature, including linear control theory [19], nonlinear approach [20,21], and analysis of the randomness, disturbances, and fluctuations [22,23]. The authors have also emphasized the optimal control algorithms to overcome supply chain challenges and issues, such as optimal production planning [24, 25], optimal control applications to Industry 4.0 and cloud manufacturing [26], the integrated production-marketing decisions [27,28]. The control theory has been utilized as a decision-support tool to explore complex behaviors for multi-echelon supply chain networks in uncertainty [29,30]. When dynamical behaviors are appropriately investigated, an efficient management strategy can be implemented to achieve sustainable competitive advantage [31]. Since most systems are inherently nonlinear, one of the standard analytical methods is a linear approximation because it's a quick and straightforward method. Although this approximation scheme is not readily applicable to reproducing complex nonlinear systems, linear system theories are very diverse and widely applied in many fields of supply chains for simplicity [12,20]. In addition, supply chain management can be seen as a complex system in which a dynamic relationship between different components is interrelated through the transfer functions in control system theory [13].

While linear control theory is well established in general, there is little known about specifically nonlinear control theory. The nonlinear control theory is complicated and requires comprehensive knowledge of algorithms, analytical tools, and design techniques. A few studies focus on exploring complex dynamical behaviors using nonlinear system theory, contrasting with simpler linear systems [21, 26, 32–35]. Moreover, typical analysis algorithms are built for a ripple effect in the supply chains or short-term supply chain scheduling in the smart factory Industry 4.0 [36]. For nonlinear systems, well-known analysis tools, such as eigenvalues, bifurcation analysis, phase portraits, and time history investigation, have been utilized in complex dynamical analysis [37–41]. The principles of stability and controllability are used to study several decision configurations that can eliminate or mitigate undesirable behaviors. In robust nonlinear control systems, many exciting challenges and issues against uncertainty exist.

Strategic risk mitigation and efficient management strategy are fundamental tasks for guaranteeing optimal operations of the supply chain networks. The manufacturing system operations' dynamic performance directly affects the supply chain's overall efficiency and profitability, and this process mismanagement can disturb the whole supply chain operations [42]. Therefore, with feedback mechanisms and sufficient analytical tools, the control theory has been employed by many articles to identify how decision policies can generate specific optimal behaviors in the supply chain system [43]. Moreover, supply chain management is recognized as a chaotic system accompanied by bullwhip effects impacting multiple stages of the system networks. It requires a comprehensive strategy to regulate system performance and incorporate resiliency into the network core. In this study, market conditions constantly change with volatility, becoming more unstable or chaotic. Recently, some researchers have investigated the supply chain system's suppression and synchronization of chaotic behaviors. Specifically, the adaptive super-twisting sliding mode control algorithm is proposed to manage a three-stage supply chain system [44]. The adaptive sliding mode control is intended to deal with parameter uncertainties, modeling errors, and external disturbances in nonlinear supply chain systems.

Chaos synchronization is studied using unidirectional error feedback to alleviate the bullwhip effect [45]. Furthermore, a feedback controller was presented based on the adaptive integral sliding mode controller to synchronize two identical chaotic systems and to estimate the unknown parameters [46–50]. So far, many approaches have been presented to cope with supply chain challenges using different control algorithms. However, conventional control algorithms might not work as expected for complex nonlinear systems. The transient performance is significantly degraded regarding time domain specifications, such as longer rising time, slower setting time, and a more considerable percentage overshoot.

To achieve better dynamic performance, fractional-order controllers have been employed for chaotic synchronization [51]. With the development of mathematical theory, it is noted that actual dynamical systems are better characterized by a non-integer order model based on fractional order calculus. In addition, adaptive control schemes can be realized to handle a broad range of parameter variations. The controller can modify itself according to the parameter changes to achieve better system performance [52]. The fractional-order control synthesis with adaptation mechanisms can efficiently cope with hyper-chaotic complex systems. Yang *et al.* [53] have successfully realized adaptive fractional-order PID (AFOPID) control to manipulate wind energy conversion systems. The experimental results have shown that tracking performance is greatly improved compared with conventional PID control counterparts. Ma and Li [54] investigated the effects of a fractional controller on the supply chain financial system. This paper proposes an adaptive fractional order PID control scheme to accommodate all kinds of fluctuations and chaotic behaviors of four-echelon supply chain networks against market volatility. Simultaneously, supervisory control is implemented to deal with perturbations when the dynamic behaviors exceed some bounds. The proposed control algorithm enables business enterprises to balance opportunities to drive growth with profit against downside risks of any disruptive events that might occur in the unpredictable market.

3. Supply chain model and dynamical analysis

3.1. Multi-echelon supply chain model

The business market is always too complex to analyze entirely in reality. In this regard, chaotic and other nonlinear behaviors have been investigated in various fields since Lorenz presented a chaotic attractor on weather prediction. A chaotic system would behave so randomly and sensitively that even the slightest change in the system input causes an unpredictable or different response. Chaos theory and its related phenomena can be employed to describe actual business activities. In recent years, considerable interest has been in applying chaotic phenomena to financial management studies. Complex dynamical behaviors with chaotic phenomena have also been observed in real supply chain systems. Many researchers in the literature have employed two or three-dimensional models to describe the supply chain system [44,45,47]. However, the actual supply chain behaviors are more complex than those in the two or three echelons' cases. The dynamic models with flexible structures and closer to reality can help the network optimization get closer to matching manufacturing output to actual demand with customer satisfaction. This research proposes the beer distribution model based on the four-echelon structure for dealing with supply chain dynamics and control synthesis. As shown in Figure 1, the four-level supply chain model includes key components: end customers, retailers, distributors, and manufacturers.

In the supply chain system, orders propagate from customers to the factory while products are shipped to the customer. Endogenous and exogenous factors in the networks always have a cumulative effect on both flows. The dynamical model describes the time evolutions of four state variables representing the volume demanded of each system stage. At time instant i, the model includes the product demand at a retailer x_i, the quantity of product the distributor can supply y_i, the quantity produced by the manufacturer z_i, and customer consumption w_i. The detailed notations with the descriptions are listed in Table 2.

The supply chain models are used to explore the dynamic interactions among components of the system networks [55,56]. First, the number of goods received by the customer in the current period w_i is linearly coupled with the product demand at a retailer in the previous period x_{i-1}, and it is influenced by total inventory at the end of the previous period mw_{i-1}, in which m is the safety stock coefficient of customer and retailer. Then the dynamic behavior between the end customer and retailer is described as follows:

$$w_i = -(x_{i-1} + mw_{i-1}) \tag{1}$$

where the negative sign in front of the expression indicates the consumption of goods for the last stage of the supply chain. As described in equation (1), the retailer plays a crucial role in the supply chain network as it is responsible for understanding customers' demands and placing orders with the distributor. Next, the system behavior between customers and retailers is described in equation (2). Product demand at a retailer in the current period, x_i, is linearly coupled with how much product was received by the customer in the previous period dw_{i-1}, product demand at a retailer in the previous period mx_{i-1}, and also the quantity of the product that are closed order in the previous period but the distributor not yet delivered to the retailer due to transport delay, ay_{i-1}. As a result, the integrated dynamical relationship is described as follows:

$$x_i = ay_{i-1} - mx_{i-1} + dw_{i-1} \tag{2}$$

where a is the transport risk coefficient, and d is the contingency reserve coefficient. Those system parameters should be appropriately chosen to ensure that the current retailer volume matches customers' needs in the same period.

As illustrated in Figure 1, the distributors are the bridge between the manufacturer and retailer, and the dependency among them is nonlinear. The distributor order is influenced by both upstream and downstream, and considering the effect caused by the producer and retailer is necessary before placing the order with the manufacturer. The quantity of product that the distributor can supply in the current period, y_i, is not linear due to the coupling effect from product demand at the retailer x_{i-1}, and product produced at the manufacturer in

TABLE 2. Key state variables and model parameters for supply chain model.

Notation	Description
$i, i-1$	Time period (subscript)
x_i, x_{i-1}	Product demand at a retailer in the current and previous period
y_i, y_{i-1}	Quantity of product that the distributor can supply in the current and previous period
z_i, z_{i-1}	Product produced at a manufacturer in the current and previous period
w_i, w_{i-1}	Product received by the customer in the current and previous period
a	Transport risk coefficient
b	Quality risk coefficient
c	Distortion coefficient
d	Contingency reserve coefficient
m	Safety stock coefficient of customer and retailer

the previous period z_{i-1}. The distributor always considers product demand at the retailer that may be distorted as, cx_{i-1}. Then this combined consequence is written as follows:

$$y_i = x_{i-1}(c - z_{i-1}) \tag{3}$$

where c is the distortion coefficient. Finally, the manufacturer is the heart of the supply chain system. Since this stage is located at the bottom of the information flow, it is severely influenced by the bullwhip effect. Bullwhip effects refer to the magnification of market demand fluctuations and system instability. So, the ordering quantity received by the manufacturer is not the same as the requested order at any level due to the bullwhip effect from the upstream stages. The product quantity produced at the factory in the current period, z_i, is not linear due to the combined impact of the retailer and distributor in the previous period $x_{i-1}y_{i-1}$. The rejected products also influence it at the factory in the previous period, bz_{i-1}, which needs to be compensated by products in the current period. This behavior is modeled by

$$z_i = x_{i-1}y_{i-1} - bz_{i-1} \tag{4}$$

where b is a quality risk coefficient. It represents defect detection at the customer or any inspection process in the supply chains. This may be caused by problems in the production process, damage during transportation, or other factors from the inventory conditions.

As described in Table 2 and Figure 1, the state variables are defined by x, y, z, and w, and the model parameters are written by a, b, c, and $d(\in \Re)$. It is well known that the number of goods in each stage in the current period is closely related to the previous inventory and governed by the remaining products of other stages in the supply chains. Consequently, if the volume of product in one stage fluctuates, the variability will widely influence the other stages.

Four-echelon integrated supply chains given in equations (1)–(4) are described in discrete terms of the system in each time period. The continuous systems are proposed using the approximation method with proper time intervals for convenient representation to examine the long-term chaotic behaviors. For example, the difference equation over time is given as follows: $\dot{x}_{i-1} \approx x_i - x_{i-1}$ by assuming unity time difference. Similarly, the continuous approximations are considered for other state variables over proper time intervals: $\dot{y}_{i-1}, \dot{z}_{i-1}$, and $\dot{w}_{i-1}$. Then the continuous-time descriptions of four first-order differential equations are rewritten by

$$\begin{aligned}
\dot{x} &= ay - (m+1)x + dw \\
\dot{y} &= cx - xz - y \\
\dot{z} &= xy - (b+1)z \\
\dot{w} &= -x - (m+1)w.
\end{aligned} \tag{5}$$

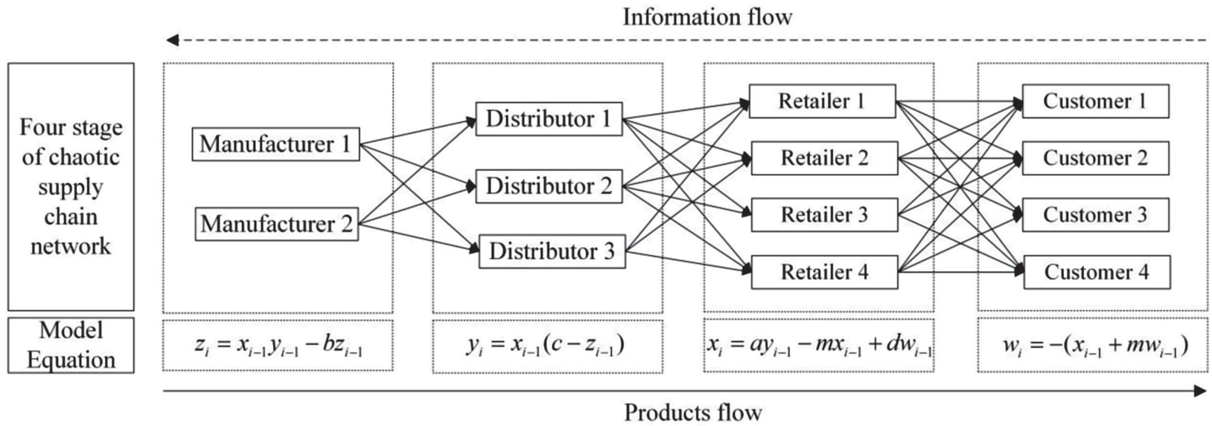


FIGURE 1. Beer distribution model of the multi-level supply chain network.

By letting $a = m + 1$ and $r = b + 1$, the dynamical models in equation (5) lead to the hyperchaotic Lorenz–Stenflo equation [53,57]. From the deterministic chaos perspective, this dynamical model is susceptible to initial conditions and exhibits various nonlinear behaviors depending upon specific parameter values. Also, it will demonstrate complex supply chain characteristics, where a parametric change at each level may lead to chaos at other levels. The operating status of the supply chain in the current period heavily depends on the inventory or pending delivered products in the previous period (initial conditions). The key parameters have been considered in the dynamical model to deal with the uncertainties: distortion coefficient, safety stock of manufacturer and customer, retailer delivery efficiency, and quality risk coefficient. To illustrate the supply chain behaviors, the system parameters are selected as follows. The distributor estimating the information received from the retailer is set to $c = 26$ (units), and this coefficient will be considered before placing the order with the distributor. The safety stock, a, is set to 10% ($m = 0.1$) of the actual production volume to avoid market fluctuations at customer and retailer. There is always a quality risk back up to 5% ($b = 0.05$). Finally, to provide urgent customer requests or to stabilize the market, the contingency reserve coefficient is set to $d = 1.5$. All parameter values are selected based on typical scenarios in supply chain management.

3.2. Dynamical analysis of chaotic supply chain system

Dynamical analysis and control optimization will be based on a mathematical supply chain network model. The dynamic properties of the supply chain system, describing changes in variables over time, are explored to gain more significant insights into nonlinear phenomena, such as periodicity, stability with bifurcation, and chaotic behaviors. Specifically, the parameter values are selected as follows: $a = m + 1 = 1.1$, $r = b + 1 = 1.05$, $c = 26$, and $d = 1.5$. Each level in the supply chain is intended to gain advantages, whether in a cooperative or a competitive relationship with others. An equilibrium status will determine the local behaviors of the supply chain network. The eigenvalue analysis of equilibrium points (E_i) can help us better understand the dynamical system's local qualitative behaviors. They can be defined as follows:

$$E_1 = (\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) = (0, 0, 0, 0)$$

$$E_1 = (\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) = \left(a\sqrt{\frac{\left[c - \left(1 + \frac{d}{a^2}\right)\right]r}{a^2 + d}}, \left(a + \frac{d}{a}\right)\sqrt{\frac{c - \left(1 + \frac{d}{a^2}\right)r}{a^2 + d}}, c - \left(1 + \frac{d}{a^2}\right), -\sqrt{\frac{c - \left(1 + \frac{d}{a^2}\right)r}{a^2 + d}} \right)$$

Table 3. Local dynamical behaviors of the supply chain system.

Equilibrium point	Eigenvalues	Stability
E_0	$\lambda_1 = -0.9000, \lambda_2 = -1.1000, \lambda_3 = -6.3981, \lambda_4 = 4.2981$	Unstable
E_1	$\lambda_1 = -0.2204 + 3.3624i, \lambda_2 = -0.2204 - 3.3624i, \lambda_3 = -3.2820, \lambda_4 = 0.9772$	Unstable
E_2	$\lambda_1 = 3.1947, \lambda_2 = -3.3342 + 0.9900i,\ \lambda_3 = -3.3342 - 0.9900i,\ \lambda_4 = -1.2262$	Unstable

$$E_2 = (\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) = \left(-a\sqrt{\frac{\left[c - \left(1 + \frac{d}{a^2}\right)\right] r}{a^2 + d}}, -\left(a + \frac{d}{a}\right)\sqrt{\frac{c - \left(1 + \frac{d}{a^2}\right) r}{a^2 + d}}, c - \left(1 + \frac{d}{a^2}\right), \sqrt{\frac{c - \left(1 + \frac{d}{a^2}\right) r}{a^2 + d}} \right). \tag{6}$$

More specifically, the equilibrium points of the Lorenz–Stenflo system are calculated by $E_0 = (0, 0, 0, 0)$, $E_1 = (3.27244, 7.32918, 23.76033, -2.97494)$, and $E_2 = (-3.27244, -7.32918, 23.76033, 2.97494)$. Then the Jacobian matrix (J_i) of equation (5) evaluated at the equilibrium point E_i is given by

$$J_0 = \begin{bmatrix} -a & a & 0 & 0 \\ c & -1 & 0 & 0 \\ 0 & 0 & -d & r \\ -1 & 0 & 0 & -a \end{bmatrix} = \begin{bmatrix} -1.1 & 1.1 & 0 & 0 \\ 26 & -1 & 0 & 0 \\ 0 & 0 & -1.5 & 1.05 \\ -1 & 0 & 0 & -1.1 \end{bmatrix}. \tag{7}$$

Similarly, the Jacobian matrices at other equilibrium points are evaluated as

$$J_{1,2} = \begin{bmatrix} -a & a & 0 & 0 \\ c - \bar{x}_3 & -1 & -\bar{x}_1 & 0 \\ \bar{x}_2 & \bar{x}_1 & -d & b \\ -1 & 0 & 0 & -a \end{bmatrix}. \tag{8}$$

As described in Table 3, all equilibrium points are unstable based on the Routh–Hurwitz criterion since at least one eigenvalue (λ_i) has a positive real part of a complex number for each equilibrium point. This result is consistent with the chaotic behaviors shown in Figures 2 and 3.

The supply chain model described in equation (5) represents complex dynamical behaviors that illustrate multi-periodic, hyperchaotic, and chaotic states depending on the eigenvalues with initial conditions. The evolution of state variables characterizes the system behavior at each point. The dynamical system has one chaotic motion and six periodic motions [56]. Figure 2 illustrates the bifurcation diagram of the state variables according to changes in control parameters. The initial conditions are selected as follows: $(x_0, y_0, z_0, w_0) = (0.040, 0.020, 0.035, 0.025)$. The chaotic system sensitively depends on the initial conditions. For the time-series simulation, the time units can be chosen arbitrarily depending on specific applications, *i.e.*, daily, weekly, or monthly. Supply chain cycle time specifies the overall performance and efficiency of supply chain management. To make quick decision making in today's changing business world, short cycles make supply chain management more efficient and agile. When designing a control strategy, it is usually assumed that the model parameters are reasonably well-known. However, this setting on the dynamic model does not work in reality. It is noted that many control problems involve uncertainties in the model due to parameter variations. The bifurcation diagram in Figure 2a is obtained by varying control parameter an in the range $a = [0, 3]$ and keeping others constant (*i.e.*, $r = 1.05$, $c = 26$, and $d = 1.5$). When the value begins with $a = 0$, one identical period-1 limit cycle is observed in the range of $a = [0, 0.4]$. Increasing the value of a, a single period-1 limit cycle changes its shape until the pair of period-2 limit cycles is obtained in the range $a = [0.4, 1]$. Then the period-2 limit cycles tend to be period-3 stable limit cycles in the range $a = [1, 2]$. After observing a pair of period-3 limit cycles, one can expect the system to lead to chaotic behaviors from $a = 2$. The dynamical

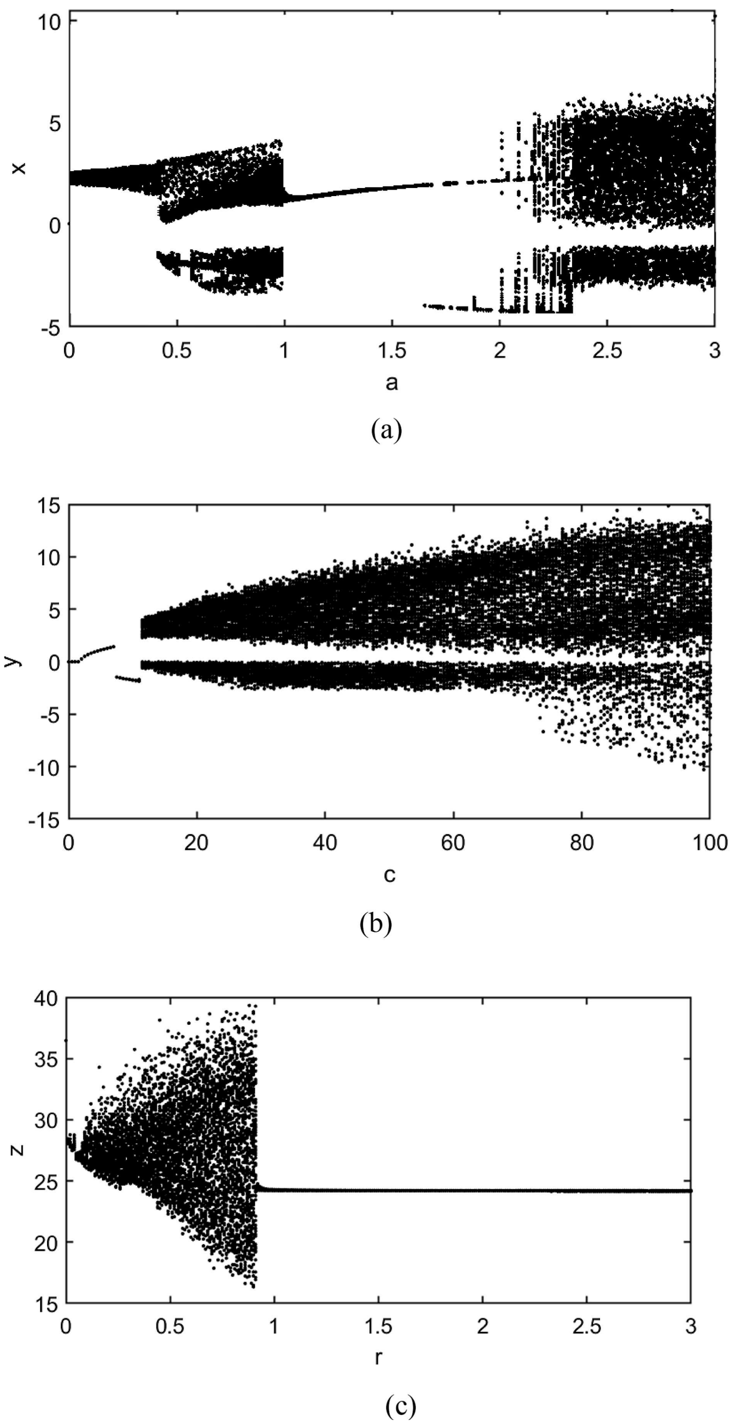


FIGURE 2. Bifurcation diagram of chaotic supply chain system. (a) $a = [0, 3]$, $r = 1.05$, $c = 26$ and $d = 1.5$. (b) $a = 1.1$, $r = 1.05$, $c = [0, 100]$ and $d = 1.5$. (c) $a = 1.1$, $r = [0, 3]$, $c = 26$ and $d = 1.5$.

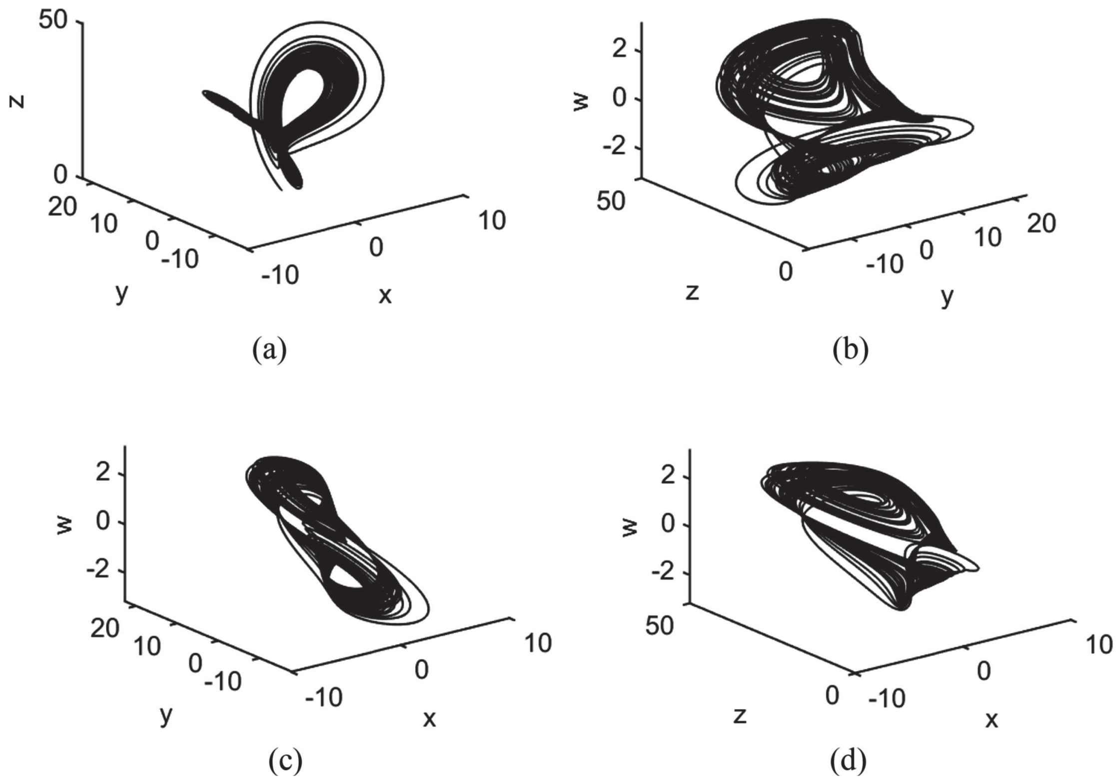


FIGURE 3. Phase portraits of the supply chain system: (a) $x-y-z$, (b) $y-z-w$, (c) $x-y-w$, and (d) $x-z-w$.

behavior demonstrates that the transport risk coefficient (a) significantly influences the dynamic response of shipment at the retailer. An increased transport risk might dramatically affect the dynamic behavior of product quantity sent to the customer, eventually making the system response chaotic. In Figure 2b, the bifurcation diagram is plotted for the control parameter $c = [0, 100]$. The first saddle-node bifurcation (in the direction of increasing c) occurs around $c = 1.27$, and the first period-doubling bifurcation occurs around $c = 13.2$. The distributor stage becomes more and more chaotic by increasing the distortion coefficient. For the lower value of c, the system shows a fixed point solution, and for the higher values of c, the supply chain network becomes more chaotic. As mentioned in Section 2, this phenomenon by the distortion factor is a significant contributor to the bullwhip effect, which always causes volatility in the supply chains. Next, the bifurcation diagram of the factory level is analyzed by considering the quality coefficient (r) as a bifurcation parameter, as depicted in Figure 2c. The chaos phenomena are more severe at the beginning, in the range of $r = [0, 0.91]$. However, those irregular motions tend to be periodic motions near $r = 0.91$, and the responses maintain stable periodic motion in the range of $r = [0.91, 3]$. Since r represents the system quality coefficient, the increase in the safety factor contributes to the stability of the supply chain management. Besides improving operational capacity, governance and risk assessment play a significant role in stabilizing supply chains for enterprises.

The three-dimensional phase plane of the strange attractor of the chaotic system is illustrated in Figure 3. Moreover, the time evolutions of state variables are depicted in Figure 4. From the test results, the chaotic behavior appears at all levels of the system over time along with varying amplitudes and frequencies. Sometimes, these are considered some common undesirable behaviors from a management perspective and should be eliminated to ensure smooth supply chain management (chaotic suppression). To summarize, system behaviors

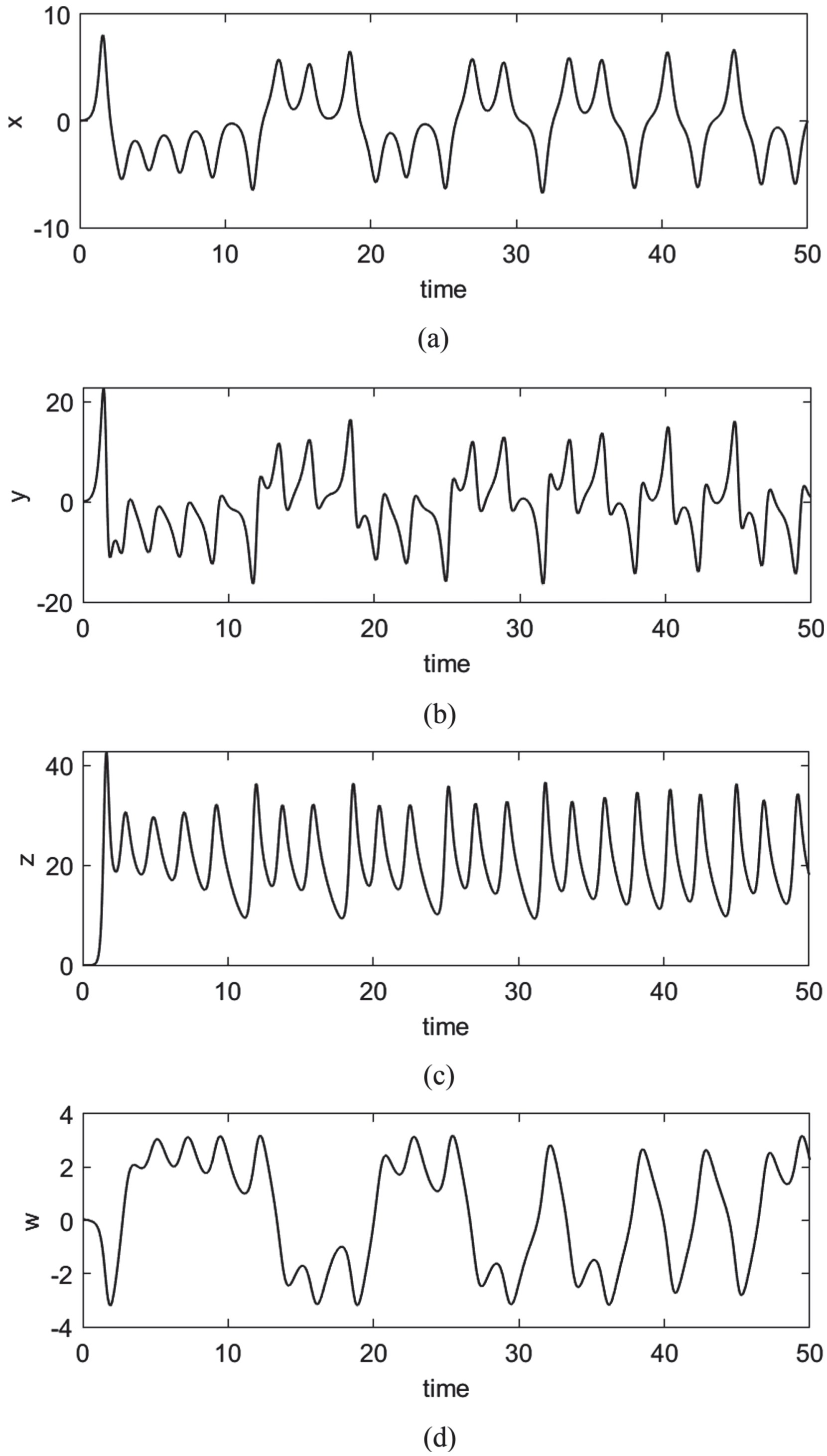


FIGURE 4. Time histories of state variables: (a) $x(t)$, (b) $y(t)$, (c) $z(t)$, and (d) $w(t)$.

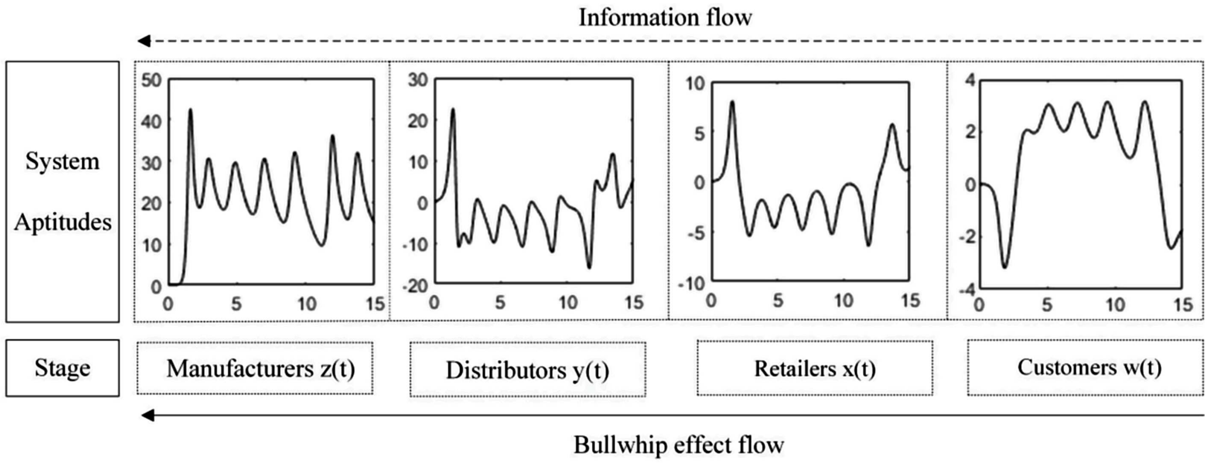


FIGURE 5. Progressive amplification of fluctuations in flow (demand) in the supply chains.

can be described using three dynamic features: a bifurcation diagram, phase portraits, and time history of state variables. All characteristics show chaos uniformity across all supply chain system levels, and limit cycles are obtained with varying control parameters. For supply chain resilience, chaos and limit cycle regions should be managed by active control.

In addition, the amplitude levels of four state variables in the time series are presented in Figure 5. From chaotic motion with trivial magnitude at the customer, fluctuation, and oscillation gradually increase across upstream and then show maximum amplitude at the manufacturer stage. Sometimes, chaos negatively affects supply chain management, such as bullwhip effects. This effect disrupts the smooth material flow of the supply chain networks, keeps production costs high, and finally yields supply chain inefficiencies. Moreover, any tiny change in demand significantly deviates from the predicted results with time delay. Eventually, the factors that cause this effect might lead businesses to have poor inventory forecasting, in which the company runs the risk of overstock, poor customer experience, and inaccurate budgeting abilities. Decision support systems should manage this effect for business operations managers. Although dynamic analysis has been explored on the hyperchaotic Lorenz–Stenflo system, this approach can be beneficial in analyzing other chaotic systems. Next, the active controller can be designed to manage supply chain disruptions and optimize overall business operations.

4. Active management system

According to Ivanov, Tsipoulanidis, and Schönberger [7], management risk arises from uncertainties in an unpredictable market and should be identified, analyzed, and controlled for efficient decision-making. There are two sources of uncertainties in the supply chain system: variations of system parameters and uncertainty from external disturbances. The supply chain uncertainties are originated from demand, design, production, and delivery, as well as time delays. Thus, the risk is a consequence of the external and internal disturbances that affect a supply chain network, such as disasters, pandemics, war, etc. In reality, active control algorithms can eliminate or synchronize chaotic supply chain systems. The feedback control actions can help the decision-makers to get the right business direction for supply chain management under uncertainty. Few papers deal with the synchronization and suppression of supply chains in a system dynamics theory [44, 45, 47]. In this paper, risk management and supply chain optimization are achieved by a novel fractional-order control theory with an adaptive mechanism.

In the study, system uncertainties due to disturbances and parametric variations can be intentionally introduced in designing a control system so that those uncertainties can be tolerated to relate dynamic models to reality. First, the dynamics of the supply chain system in equation (5) are rewritten to incorporate parameter variations and control input as follows:

$$\begin{aligned}\dot{x} &= (a+\Delta a)y - (m+\Delta m+1)x + (d+\Delta d)w + \delta_1 + u_1\\ \dot{y} &= (c+\Delta c)x - xz - y + \delta_2 + u_2\\ \dot{z} &= xy - (b+1+\Delta b)z + \delta_3 + u_3\\ \dot{w} &= -x - (m+1+\Delta m)w + \delta_4 + u_4.\end{aligned} \tag{9}$$

In turn, the state-space representation (9) can be described in the vector-matrix form by

$$\dot{v} = Av + f(v) + \Delta v + \delta + u \tag{10}$$

where $v = [x, y, z, w]^T \in \Re^4$ is the vector of state variables, and $u = [u_1, u_2, u_3, u_4]^T \in \Re^4$ is the control input vector. The system matrix A with fixed nominal parameter is given by

$$A = \begin{bmatrix} -m-1 & a & 0 & d \\ c & -1 & 0 & 0 \\ 0 & 0 & -b-1 & 0 \\ -1 & 0 & 0 & -m-1 \end{bmatrix}. \tag{11}$$

In addition, $f(v)$ is a vector of given functions generally defined in a nonlinear vector field, $f(v) = [0, -xz, xy, 0]^T$. The perturbed term Δv represents the parametric uncertainty applied to the four states of the system, and it is given by

$$\Delta = \begin{bmatrix} -\Delta m & \Delta a & 0 & \Delta d \\ \Delta c & 0 & 0 & 0 \\ 0 & 0 & -\Delta b & 0 \\ 0 & 0 & 0 & -\Delta m \end{bmatrix}. \tag{12}$$

The external disturbance vector is described by $\delta = [\delta_1, \delta_2, \delta_3, \delta_4]^T$. Three positive upper bounds ($\in \Re^+$) are proposed for stability proof on the given functions: $Av^u(\cdot), v^u(\cdot)$ and $\Delta v^u(\cdot)$ are bounded by $\|Av(\cdot)\| \leq Av^u(\cdot)$, $\|f(v)\| \leq v^u(\cdot)$, and $\|\Delta v(\cdot)\| \leq \Delta v^u(\cdot)$, respectively, where $\|(\cdot)\|$ denotes vector norm with appropriate dimensions. The disturbance vector is bounded by a positive constant β, satisfying $|\delta| \leq \beta$. Let $e = [e_1, e_2, e_3, e_4]^T \in \Re^4$ be the error vector between the desired values $v_d = [x_d, y_d, z_d, w_d]^T \in \Re^4$ and the actual output v. The state error vector is rewritten as follows:

$$e = v - v_d = [(x - x_d), (y - y_d), (z - z_d), (w - w_d)]^T. \tag{13}$$

Then, the closed-loop error dynamics are given by

$$\dot{e} = A(e + v_d) + \Delta v(e + v_d) + f(e + v_d) + \delta + u. \tag{14}$$

The control algorithms are designed to ensure that the error dynamics are asymptotically stable for all initial conditions, *i.e.*, $\lim_{t\to\infty}\|e_i(t)\| = 0$ for all $e_i(0) \in \Re$. The active control action in equation (14) is given by

$$u = u_s + u_{\text{AFOPID}} \tag{15}$$

where $u_{\text{AFOPID}}(\in \Re^4)$ is the vector of the adaptive fractional PID controller (AFOPID). This controller acts as the main controller to reduce the management risk. In addition, $u_s(\in \Re^4)$ is an extra supervisory control vector [58] which will be activated when the system state exceeds some bounds. It refers to a high level of overall monitoring of individual controllers to allow operation integration. When the state variables exceed

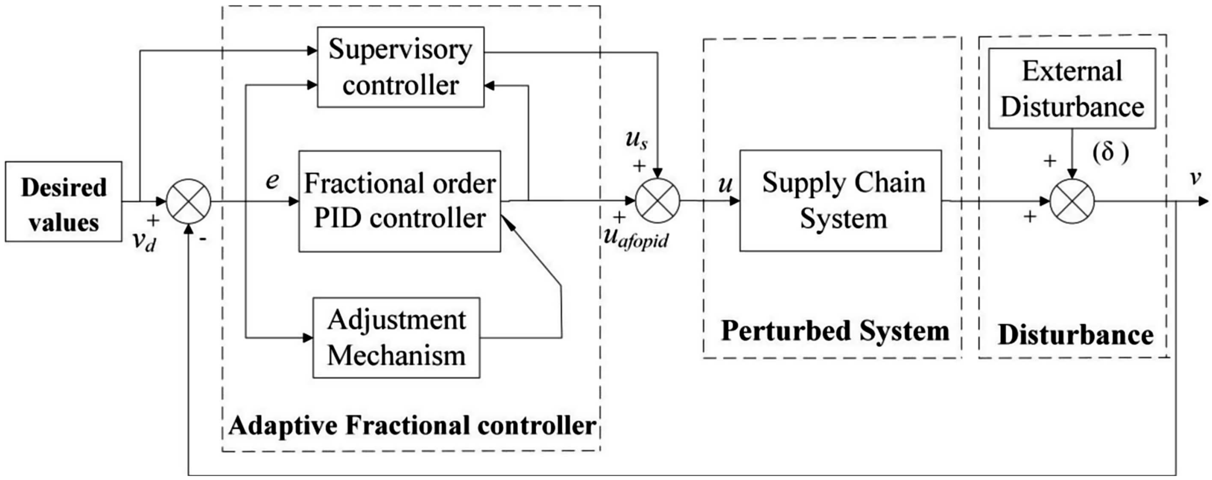


FIGURE 6. Block diagram of the complete control system.

certain thresholds under uncertainty, causing severe overshoots or deteriorating the transient responses, the supervisory strategy will help the complete control system eliminate these undesired phenomena, ensuring closed-loop stability and performance. The schematic diagram of the supply chain system with the active controller is illustrated in Figure 6. This two-stage hybrid method makes realizing a robust algorithm tailored to each design objective possible.

4.1. Supervisory control

System uncertainties and external disturbances matter for business fluctuations. The supply chain might exhibit high variability and unpredictability, and it is challenging to regulate these risks by conventional control algorithms. The supervisory management strategy is set up to eliminate the impacts of these phenomena. For synthesizing nonlinear control systems, stability is the primary concern, in which stability concepts are provided by Lyapunov theory. To ensure stability under the supervisory controller, the Lyapunov function is considered as follows:

$$V_e(e) = \frac{1}{2} e^T P e \tag{16}$$

where P is a positive definite symmetric matrix. For the given matrix Q, there exists a unique positive definite symmetric matrix P which is the solution of the following Lyapunov equation:

$$A^T P + PA = -Q. \tag{17}$$

Taking the time derivative of V_e along with the closed-loop error system (14) leads to the following inequality:

$$\begin{aligned} \dot{V}_e &= \frac{1}{2}\left(e^T A^T P e + e^T P A e\right) + \frac{1}{2} e^T P[A v_d + \Delta v(e + v_d) + f(e + v_d) + \delta + u_{\text{AFOPID}} + u_s] \\ &= -\frac{1}{2} e^T Q e + \frac{1}{2} e^T P[A v_d + \Delta v(e + v_d) + f(e + v_d) + \delta + u_{\text{AFOPID}} + u_s] \\ &\leq -\frac{1}{2} e^T Q e + \frac{1}{2}\left|e^T P\right|[|A v_d| + |\Delta v(e + v_d)| + |f(e + v_d)| + |\delta| + |u_{\text{AFOPID}}|] + \frac{1}{2} e^T P u_s. \end{aligned} \tag{18}$$

From equation (18), the supervisory controller is selected as

$$u_s = -\text{sgn}\left(e^T\right)\left(A v^u(e + v_d) + v^u(e + v_d) + \Delta v^u(e + v_d) + \beta + |u_{\text{AFOPID}}|\right) \tag{19}$$

where the sign function is defined by

$$\operatorname{sgn}(e^T) = \begin{cases} +1, & e^T > 0 \\ -1, & e^T < 0. \end{cases} \tag{20}$$

4.2. Adaptive fractional $PI^{\lambda}D^{\mu}$ controller

While a supervisory controller manages the resonance of the risk factor, an active controller needs to manage the system volatility due to its dynamic behavior. Since the supply chain fluctuates increasingly across upstream, the control algorithm must be self-adaptive to cope with those fluctuations. The adaptive fractional $PI^{\lambda}D^{\mu}$ controller is proposed to ensure stability and performance, and this algorithm can securely support *the* decision-making process to guarantee more sustainable and resilient supply chains. In addition, fractional order calculus is introduced as a natural extension of differentiation and integration to the entire real number order, denoting fundamental operator as ${}_aD_t^r$ [52]. The continuous integral-differential operator is defined as follows:

$${}_aD_t^r = \begin{cases} \mathrm{d}^r/\mathrm{d}t^r & \mathrm{Re}(r) > 0 \\ 1 & \mathrm{Re}(r) = 0 \\ \int_a^t (\mathrm{d}\tau)^{-r} & \mathrm{Re}(r) < 0 \end{cases} \tag{21}$$

where $\mathrm{Re}(r) > 0$ corresponds to differentiators while $\mathrm{Re}(r) < 0$ yields integrators; a and t are the boundaries. This study assumes that the fractional order is a real number ($r \in \Re^+$) for simplicity. There are several definitions for fractional order calculus. Caputo's formula is commonly utilized in system analysis and control synthesis. Based on Caputo's formula, the fractional order differentiation and integral can be defined in a unified way such that

$$D^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{D^m f(t)}{(t-\tau)^{\alpha+1-m}} \,\mathrm{d}\tau \tag{22}$$

where $m - 1 \leq \alpha \leq m$; m is the smallest integer that is equal or greater than α. Using a non-integer order α, the mathematical model can capture more system dynamics. The Laplace transform of the Caputo fractional derivative is defined by

$$\int_0^{\infty} e^{-st} D^{\alpha} f(t) \,\mathrm{d}t = s^{\alpha} F(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1} D^k f(0) \tag{23}$$

where $\Gamma(\alpha) = \int_0^t e^{-t} t^{\alpha-1} \,\mathrm{d}t$ is the Gamma function, and $F(s)$ is a Laplace transform of $f(t)$. For simulation purposes, one of the common ways to convert them with an integer-order transfer function is Oustaloup's recursive approximation. This recursive algorithm is used to describe the system dynamics in the frequency domain,

$$s^{\alpha} \simeq K \prod_{k=-N}^{N} \frac{s + \omega_k'}{s + \omega_k}. \tag{24}$$

The algorithm is intended to offer a finite approximation of fractional-order systems in a desired range of frequencies. Then the sets of synthesis formulas for zeros, poles, and gains are defined by

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1+\alpha)}{2N+1}}, \omega_k' = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1-\alpha)}{2N+1}}, K = \omega_h^{\alpha} \tag{25}$$

where ω_h and ω_b are the high and low transitional frequencies, α is the order of the differential integration, and $(2N+1)$ is the order of the filter. If any function $f(t)$ is satisfied with the filter (25), the filter output can be considered an approximation of the fractional integral or differentiated signals. The PID controller (or three-term controller) is the most common form of feedback scheme and is used for solving a wide range of control problems. This study uses fractional order calculus on PID controller ($PI^{\lambda}D^{\mu}$) for supply chain management.

The conventional PID was improved by adding fractional-order parameters for increased freedom in control synthesis. Then the fractional-order control algorithm in the time domain is described by

$$u_{\mathrm{FOPID}} = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^{\mu} e(t). \tag{26}$$

This structure includes five independent parameters for the tuning controller, *i.e.*, proportional, integral, and derivative gains (K_p, K_i, K_d), respectively, and additional integration and differentiation orders (λ, μ). Compared to the traditional PID controller, the fractional order PID (FOPID) algorithm utilizes fractional calculus, providing more control synthesis flexibility to enhance performance. The gradient rule derives an adaptation mechanism for tuning control gains (K_p, K_i, K_d). This adaptive law is a gradient approach to minimize the cost function, which includes a squared model error function [58]. According to strict performance requirements, adaptive control is developed to estimate unknown parameters in real-time. For managing the supply chain network, the adaptive fractional order PID control (AFOPID) algorithm is specified below

$$u_{\mathrm{AFOPID}} = \left[\hat{K}_p e + \hat{K}_i D^{-\lambda} e + \hat{K}_d D^{\mu} e\right] \tag{27}$$

where the updated gains by adaptive laws are denoted by $\hat{K}_p$, $\hat{K}_i$, and $\hat{K}_d$. Specifically, the adaptive rules for tuning gains are given by

$$\dot{\hat{K}}_p = -\alpha_1 e^T e, \;\; \dot{\hat{K}}_i = -\alpha_2 e^T \int e \, \mathrm{d}t, \;\; \dot{\hat{K}}_d = \alpha_3 e^T \dot{e} \tag{28}$$

where $\alpha_i > 0$ is a learning rate, $(i = 1, 2, 3)$. The control gains are adjusted to minimize the cost function, and it is reasonable to change these gains in the negative gradient direction. The adaptive mechanism is designed to guarantee the stability of the closed-loop system and the convergence of tracking errors to zero.

Theorem 1. *Consider the uncertain supply chain system with parameter variations in equation* (9)*. Then the two-stage hybrid approach in equation* (15) *assures asymptotic stability for the perturbed error dynamics in equation* (14)*. The time convergence of tracking error is guaranteed against uncertainties or* $\lim_{t\to\infty} \|e(t)\| = 0$.

Proof. To prove the closed-loop stability, the Lyapunov function candidate for the error dynamics is considered as

$$V_e(e) = \frac{1}{2} e^T H e \tag{29}$$

where H is selected as an identity matrix for simplicity, or $H = I_{4\times 4}$. The time derivative of equation (29) along the solution of equation (14) can be calculated as

$$\dot{V}_e = e^T [A(e + v_d) + \Delta v(e + v_d) + f(e + v_d) + \delta + u_s + u_{\mathrm{AFOPID}}]. \tag{30}$$

Substituting (30) into (19) leads to the following relation:

$$\begin{aligned} \dot{V}_e &= e^T [A(e + v_d) + \Delta v(e + v_d) + f(e + v_d) + \delta] \\ &\quad + e^T \left[-\mathrm{sgn}\left(e^T\right)(A v^u(e + v_d) + v^u(e + v_d) + \Delta v^u(e + v_d) + \beta + |u_{\mathrm{AFOPID}}|) + u_{\mathrm{AFOPID}}\right]. \end{aligned} \tag{31}$$

Now, two conditions (C1 and C2) will be considered for the stability of the closed-loop system.

<u>C1.</u> For $e^T > 0$, equation (31) can be bounded as,

$$\begin{aligned} \dot{V}_e &= e^T [A(e + v_d) + \Delta v(e + v_d) + f(e + v_d) + \delta] \\ &\quad - e^T [(A v^u(e + v_d) + v^u(e + v_d) + \Delta v^u(e + v_d) + \beta + |u_{\mathrm{AFOPID}}|] + u_{\mathrm{AFOPID}} \\ &= \left[e^T A(e + v_d) - e^T A v^u(e + v_d)\right] + \left[e^T \Delta v(e + v_d) - e^T \Delta v^u(e + v_d)\right] \end{aligned}$$

$$
\begin{aligned}
&+ \left[e^T f(e+v_d) - e^T v^u(e+v_d)\right] + \left[e^T\delta - e^T\beta\right] + \left[-e^T|u_{\text{AFOPID}}| + e^T u_{\text{AFOPID}}\right]\\
&\le e^T[|A(e+v_d)| - Av^u(e+v_d)] + e^T[|\Delta v(e+v_d)| - \Delta v^u(e+v_d)]\\
&+ e^T[|f(e+v_d)| - v^u(e+v_d)] + e^T[|\delta| - \beta] + e^T[-|u_{\text{AFOPID}}| + |u_{\text{AFOPID}}|] \le 0.
\end{aligned} \tag{32}
$$

<u>C2.</u> For $e^T < 0$, equation (31) can be bounded as

$$
\begin{aligned}
\dot{V}_e &= e^T[A(e+v_d) + \Delta v(e+v_d) + f(e+v_d) + \delta]\\
&\quad - \left[\left|e^T\right|(Av^u(e+v_d) + \left|e^T\right|v^u(e+v_d) + \left|e^T\right|\Delta v^u(e+v_d) + \left|e^T\right|\beta + \left|e^T\right||u_{\text{AFOPID}}|\right] + \left|e^T\right|u_{\text{AFOPID}}\\
&= \left[e^T A(e+v_d) - \left|e^T\right|(Av^u(e+v_d)\right] + \left[e^T\Delta v(e+v_d) - \left|e^T\right|\Delta v^u(e+v_d)\right]\\
&\quad + \left[e^T f(e+v_d) - \left|e^T\right|v^u(e+v_d)\right] + \left[e^T\delta - \left|e^T\right|\beta\right] + \left[\left|e^T\right|u_{\text{AFOPID}} - \left|e^T\right||u_{\text{AFOPID}}|\right]\\
&\le \left|e^T\right|[|A(e+v_d)| - (Av^u(e+v_d)] + \left|e^T\right|[|\Delta v(e+v_d)| - \Delta v^u(e+v_d)]\\
&\quad + \left|e^T\right|[|f(e+v_d)| - v^u(e+v_d)] + \left|e^T\right|[|\delta| - \beta] + \left|e^T\right|[|u_{\text{AFOPID}}| - |u_{\text{AFOPID}}|] \le 0.
\end{aligned} \tag{33}
$$

From equations (32) and (33), the control synthesis given in equation (15) with adaptive laws (28) can guarantee asymptotic stability or $\lim_{t\to\infty}\|e(t)\| = 0$. More importantly, the presented algorithm is very robust under uncertainty. □

4.3. Synchronization strategy

The synchronization algorithm facilitates today's supply chains to react quickly to erratic demand and product design changes. Volatile markets change unpredictably and rapidly. Small triggers may result in significant changes in prices or demands. This scheme is particularly suitable for just-in-time supply chain management. All supply chain constituents should come together as a group under rapidly changing business environments. Then the shipment sent could be delivered in the right amounts and at the right time to meet customer demand. If a business enterprise has a synchronized scheme in place, the management policy will quickly bring many incredible benefits to your business. Factory managers can prepare their production plans with realistic and data-driven shipment lead times in their minds. Distributors need to know what shipping services are required and how quickly goods can be delivered. Retailers must accurately understand the entire product lifecycle to pinpoint purchasing needs in real-time. It's about being proactive, identifying the next step in a process, and minimizing potential risks long before they happen. In this paper, risk management and optimization can be realized using an active control scheme. Establishing synchronization algorithms depends on the master-slave systems' arrangement. The reference supply chain model, by fixing key parameters, is described as a master system, and the perturbed system is defined as a slave system incorporating the external fluctuations. To observe the switching synchronization for a chaotic supply chain system, the master-slaver system is constructed as

$$
\begin{cases}
\dot{v}_m = Av_m + f(v_m): & \text{Master system}\\
\dot{v} = Av + f(v) + \Delta v + \delta + u: & \text{Slave system}
\end{cases} \tag{34}
$$

where the state vector for the master system with subscript m is denoted by $v_m = [x_m, y_m, z_m, w_m]^T \in \Re^4$; the vector of nonlinear terms in the master system is given by $f(v_m) = [0, -x_m z_m, x_m y_m, 0]^T$. It is convenient to define the differences between the reference and perturbed systems to ascertain the active control functions. Let $e = [e_1, e_2, e_3, e_4]^T \in \Re^4$ be the vector for synchronization errors between the master and slave system. Then it can be described as follows:

$$
e = v - v_m = [(x - x_m), (y - y_m), (z - z_m), (w - w_m)]^T. \tag{35}
$$

In turn, the error dynamics vector is given by

$$
\dot{e} = A(e + v_m) + \Delta v(e + v_m) + f(e + v_m) + \delta + u + \dot{v}_m. \tag{36}
$$

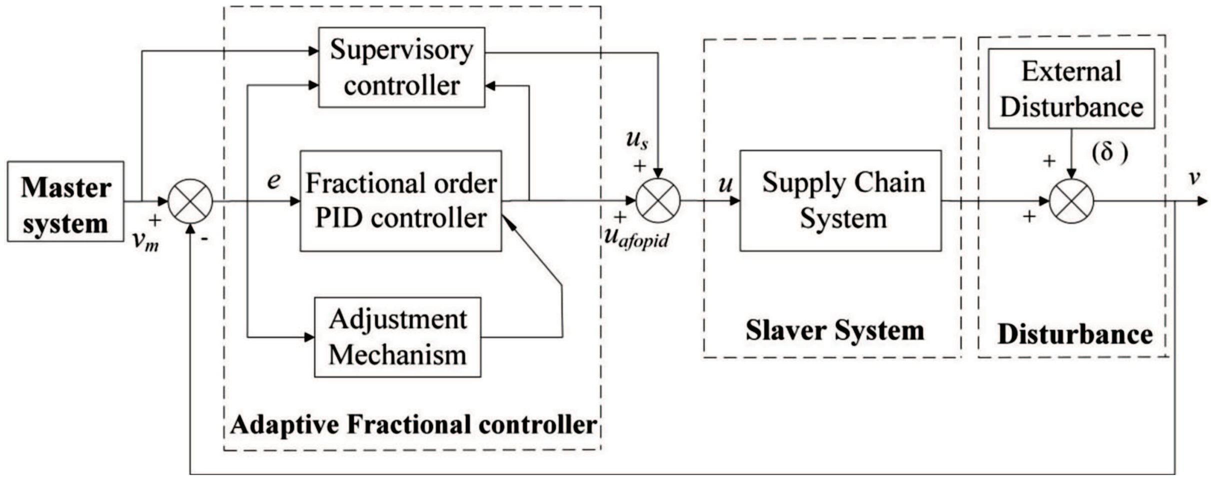


FIGURE 7. Schematic diagram for chaos synchronization scheme.

For the synchronization algorithm, the active controller is designed as equation (15), where u_{AFOPID} is given in equation (27). The supervisory controller u_s is given by

$$u_s = -\operatorname{sgn}\big(e^T\big)\big(Av^u(e+v_m) + v^u(e+v_m) + \Delta v^u(e+v_m) + \beta + |u_{\text{AFOPID}}| + |\dot{v}_m|\big). \tag{37}$$

The supply chain system synchronization setup for ever-changing market conditions is shown in Figure 7.

Theorem 2. *Consider a set of master/slave supply chain systems given in equation* (34)*. Then synchronization errors of closed-loop error dynamics* (36) *for any initial conditions will be asymptotically stable by employing a two-state hybrid algorithm described in equation* (15)*.*

Proof. The proof is similar to that of Theorem 1. The detailed proof is omitted for the sake of brevity. □

4.4. Performance assessment

To evaluate quantitative measures of the closed-loop system performance, a suitable objective function that represents supply chain requirements is based on some performance specifications. For example, the desired output specifications show the overshoot when the supply exceeds the customer's demand. For a rising time, how fast your supply chain provides the required product quantity of customers while the supply chain network is a chaotic system. Two kinds of performance criteria, including the ISE and the ITSE, are used to evaluate the performance comparisons of control algorithms. The system parameters are adjusted so that the performance index reaches an extreme, typical minimum value. A performance index to be useful in criteria must be a number that is always positive or zero. Specifically, the performance indices and the cost functions, which are functions of error signals, are defined by

$$\text{ISE} = \int_0^\infty \|e(t)\|^2 \, \mathrm{d}t \tag{38}$$

$$\text{ITSE} = \int_0^\infty t\|e(t)\|^2 \, \mathrm{d}t. \tag{39}$$

It should be pointed out that the first index weighs the error equally over the entire interval and penalizes significant errors more than the smaller one. In contrast, the last index gives higher weight to the error at later times. Hence, the control system is not penalized for having a significant initial error. Both performance criteria

TABLE 4. Numerical values of the model parameters.

Parameters	Values	Parameters	Values
m	0.1	Δc	$0.4\sin(2t)$
a	1.1	Δd	$0.5\sin(5t)$
b	0.05	Δm	$0.2\cos(7t)$
c	26	δ_1	$0.1\sin(1t)$
d	1.5	δ_2	$0.4\cos(3t)$
Δa	$0.7\sin(0.5t)$	δ_3	$0.7\sin(5t)$
Δb	$0.5\cos(3t)$	δ_4	$0.1\sin(2t)$

are essential in assessing the supply chain capacity for the real market. More effective control will be guaranteed when the smaller ISE and ITSE are obtained in the closed-loop system. If the system lacks resilience and cannot eliminate the risks, the criteria on ISE and ITSE will tend to infinitely [29]. However, one should note that risks cannot be eliminated entirely, only managed by effective strategies.

5. Numerical simulation

In this numerical analysis, market conditions constantly change with volatility, becoming more unstable or chaotic. Volatile markets are characterized by wide and rapid price or demand fluctuations. As stated, today's supply chains are in turmoil, described by chaotic dynamics that contain many forms of nonlinearities and physical constraints. Numerical simulations have been conducted to investigate nonlinear dynamical behaviors for the multi-stage supply chains' chaotic suppression and synchronization strategies. The numerical system parameters are selected for the perturbed supply chain system, as shown in Table 4.

5.1. Chaos suppression

Risk mitigation is demonstrated by suppressing chaotic behaviors against uncertainties. Numerical simulation is carried out with the following initial conditions: $(x_0, y_0, z_0, w_0) = (0.040, 0.020, 0.035, 0.025)$. All simulations' controllers have been activated at $t = 25$ (time periods). The observed results for suppressing four stages of a chaotic supply chain system are shown in Figure 8. Three active controllers, including PID, FOPID, and AFOPID, have been implemented to investigate performance comparisons through ISE and ITSE. To examine the effect of each controller, the same gain values (K_p, K_i, and K_d) are applied for all control systems. Moreover, the same fractional orders of λ and μ are chosen in FOPID and AFOPID. The transient response can be quantified with the following properties: rise time, overshoot, settling time, and delay time. The transient response tests show that with PID and FOPID, it takes some time to track the desired target. The AFOPID algorithm obtains the desired responses due to the adaptive mechanism. The performance evaluations have been illustrated in greater detail in Figure 9. System tracking errors are asymptotically converged to zero after activating the AFOPID controller. In cases of PID and FOPID, the system responses are slowly moved on the way to zero. Despite some disruptions, the adaptive fractional-order controller can appropriately eliminate oscillatory trajectories of demand, inventory, and produced quantities, ensuring robust stability.

The comparison results based on the performance indices are presented in Figure 10. Active decision support systems can be implemented by a control algorithm that minimizes the index and provides practical management [42]. Also, the control activities of decision-makers are illustrated for the manufacturer stage in Figure 11. For brevity, the analysis has focused on the manufacturer stage because this is the last stage of the supply chain, which is most strongly influenced by the bullwhip effect. The performance of the manufacturer stage determines the system's ability to adapt to customer needs as well as endogenous and exogenous factors. According to the test results shown in Figure 10, all system performances with similar settings have achieved different performance levels. In detail, the AFOPID controller provides lower costs compared with PID and FOPID in performance

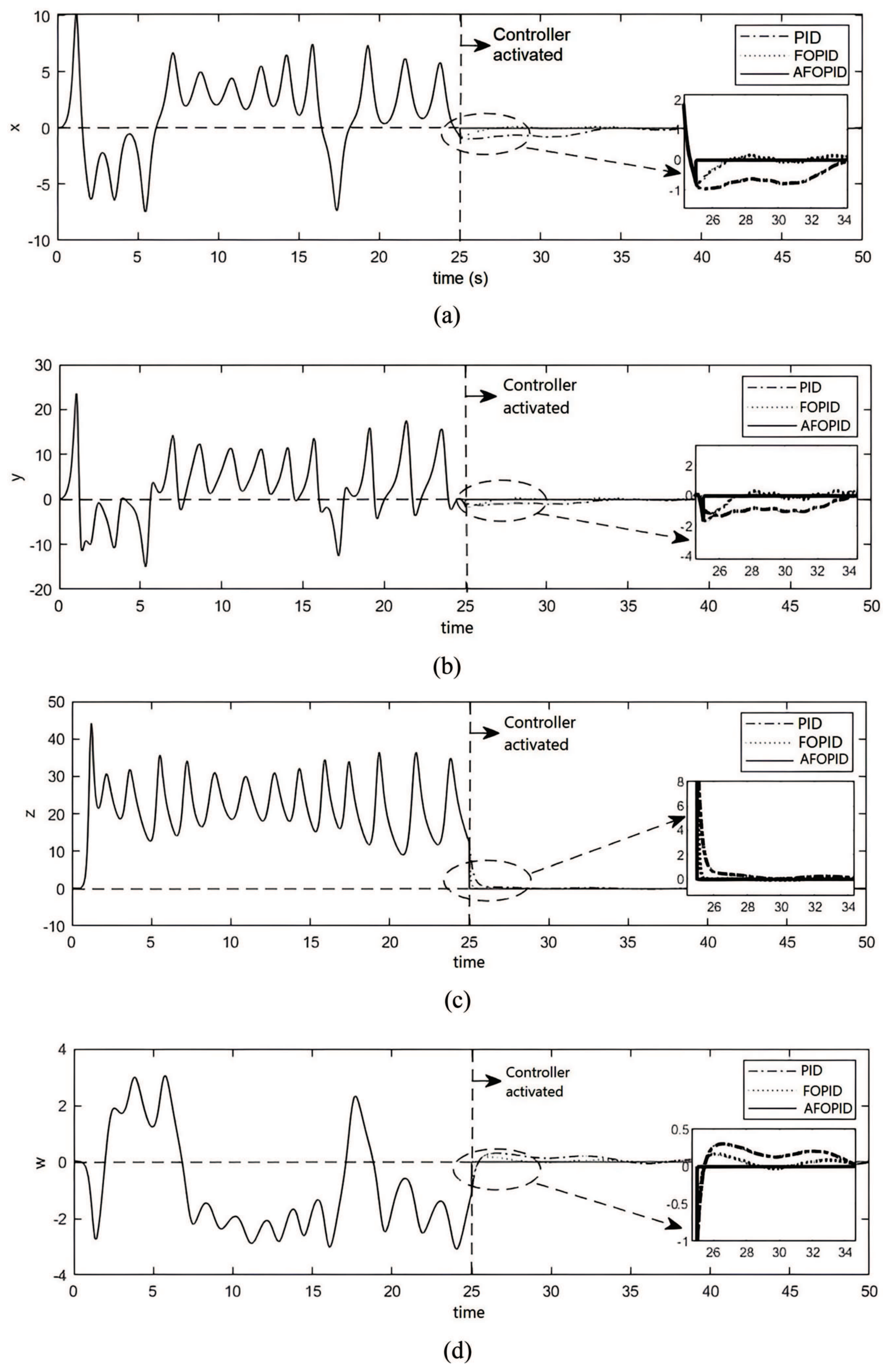


FIGURE 8. Time histories of state variables for chaos suppression where controllers are activated at $t = 25$ (time periods): (a) $x(t)$, (b) $y(t)$, (c) $z(t)$, and (d) $w(t)$.

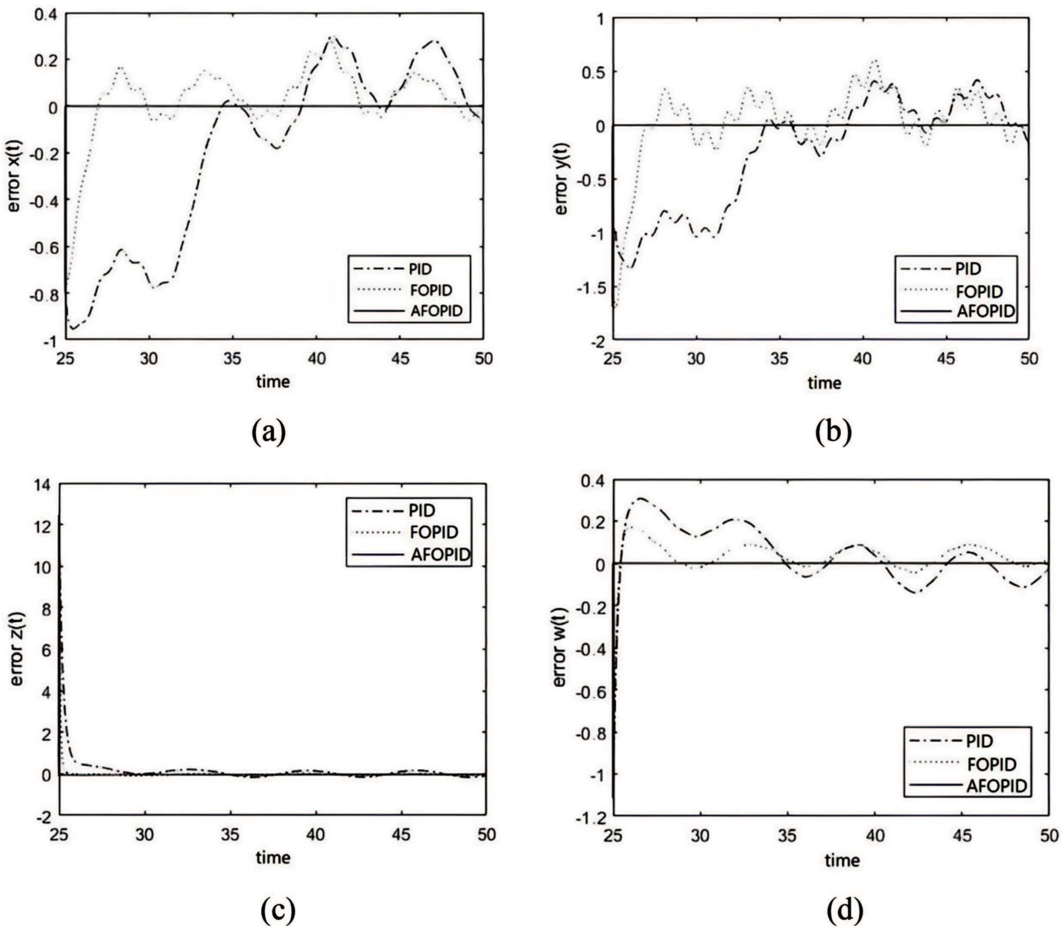
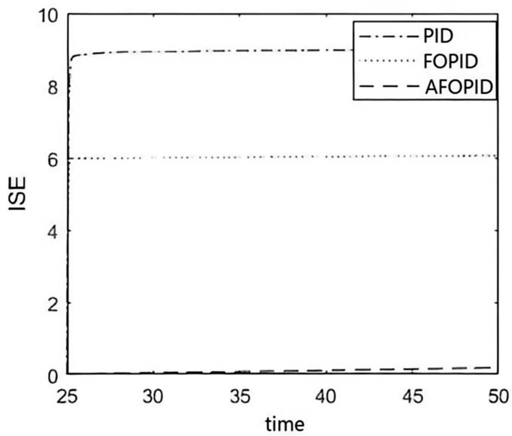


FIGURE 9. Time histories of error signals for chaos suppression where controllers are activated at $t = 25$ (time periods).

indicators. The dynamical analysis demonstrates that the transient responses under the AFOPID controller have the fastest rising time, quickest setting time, and the smallest maximum percentage of overshoot. Thus, the performance index evaluates the AFOPID scheme as the most effective controller.

As illustrated in Figures 11a and 11b, the control activities have different behaviors after activating strategies. The control effort keeps changing in the time evolution of system states, creating a chattering phenomenon, possibly disrupting the supply chain system in a real case. On the contrary, according to Figure 11c, AFOPID control action is relatively smooth after activation and remains stable for given periods. Thus, control activity signals or efforts are somewhat related to relevant costs for decision-making. Table 5 shows the cost comparisons of the presented algorithms. The optimal solution under adaptive strategy can offer lower costs for each time period than other control strategies. In the whole time series, the FOPID controller results in a 15.33% higher total cost over the AFOPID, while the total cost with the PID algorithm is 29.26% higher than the AFOPID algorithm. Employing the AFOPID algorithm will reduce business costs to increase productivity.

In supply chain management, maintaining control systems at lower and fewer mutations contributes to reducing operating costs and can keep the material flowing smoothly so that the enterprise can offer a product or service satisfying customers' desires and needs. The validity and reliability of a fractional-order control synthesis



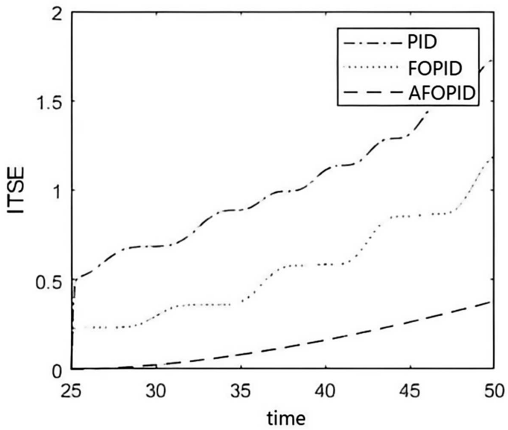


FIGURE 10. Performance index comparisons of active controllers for $z(t)$.

are confirmed with an adaptation mechanism for the chaotic suppression of supply chains. Despite disruption factors such as parameter perturbation, and external disturbance, the controlled supply chain system has effectively mitigated the oscillatory trajectories of the shipment at the factory, distributor, retailer, and customer.

5.2. Chaos synchronization

Logistic providers need to know intimately what shipping services are required and how quickly those can be delivered. All supply chain constituents must work together efficiently to meet their business goals. The same parameters with initial conditions are used for demonstrating synchronization strategy as in chaos suppression. For the master system, new initial conditions are given: $(x_{m0}, y_{m0}, z_{m0}, w_{m0}) = (5, -4, 8, -5)$. As depicted in Figure 12, the AFOPID controller is implemented for synchronizing chaotic supply chains. All state variables of the slave are completely converged to the master system as soon as the controller is activated $t = 25$ (time periods). Based on performance comparisons, an adaptive fractional order controller can offer more efficient synchronization for chaotic supply chains. If this synchronized scheme is in place, your company can benefit from implementing the strategy.

Despite some disruption factors, the synchronization scheme has made an adaptation possible for enterprises on account of obtaining the same shipment at the factory, distributor, retailer, and customer in an appropriate time period. Then the proposed approach might reduce the supply chain risks caused by parametric uncertainty and external disturbance under a volatile market environment. Numerical simulation and graphical analysis clearly show that the proposed approach is a simple and reliable strategy to implement suppression and synchronization for chaotic systems.

5.3. Managerial implications

Applying a system dynamics approach to the supply chain management problem using nonlinear techniques is first described in Section 3, such as stability analysis of equilibrium points, bifurcation diagrams, phase portraits, and time-series responses. The local stability near equilibrium points and chaotic behavior are discovered through eigenvalue analysis. There are significant changes in the dynamical behaviors when tiny adjustments of the quality coefficient in the range $[0, 3]$ are introduced. A similar dynamical pattern is also observed when the distortion coefficient varies over a wide range of $[0, 100]$. By increasing the distortion coefficient, the distributor stage exhibits more chaotic behaviors. The quality coefficient (r) is then considered a control parameter to

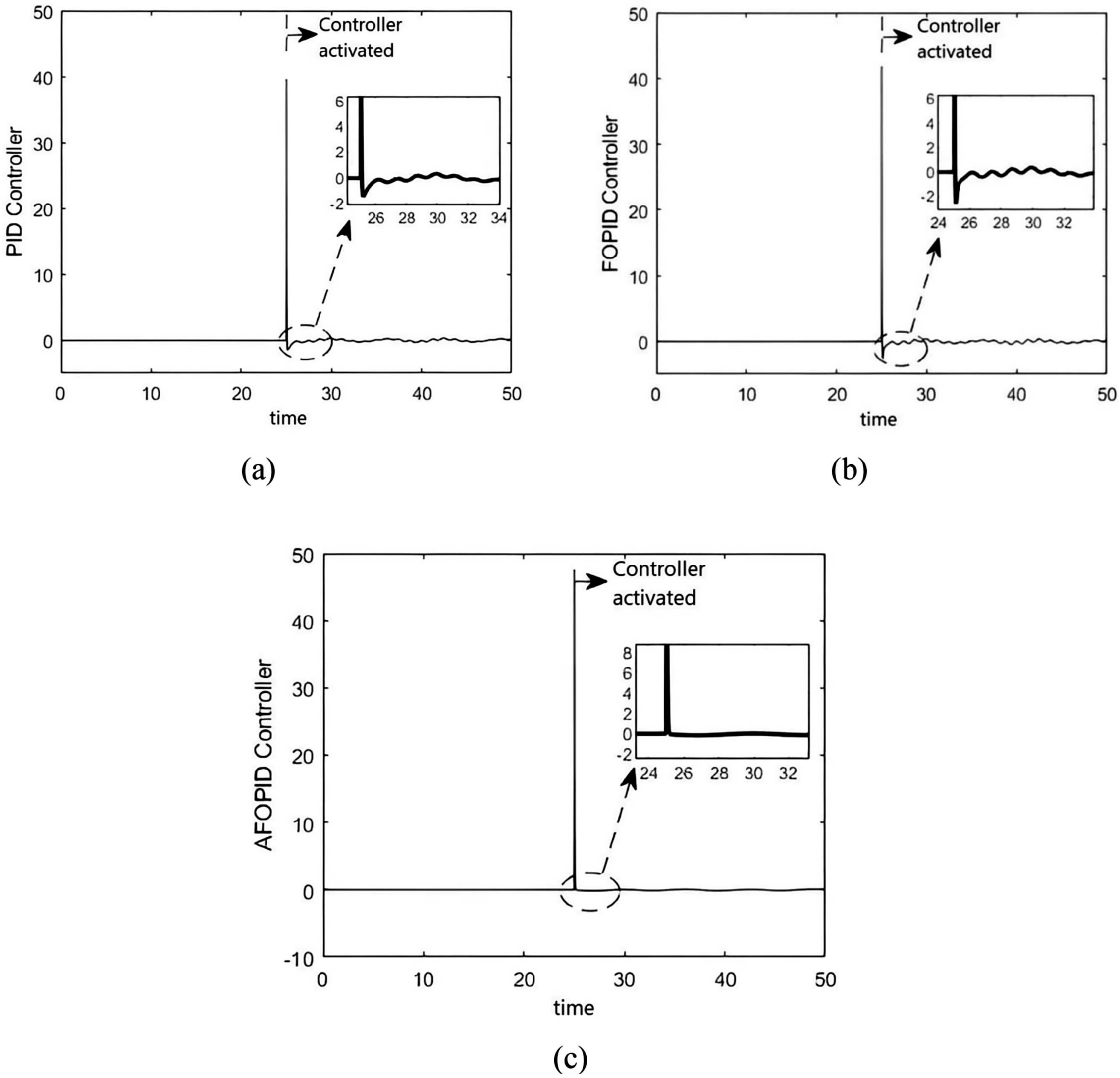


FIGURE 11. Control activities for the last stage of the supply chain network – retailer $z(t)$.

evaluate the bifurcation analysis of the manufacturing level. The findings demonstrate that the chaotic phenomena are initially severe and tend to disappear into periodic motion before remaining stable in the range of $r = [0, 3]$. Increasing safety factors influence the stability of the stage in the supply chain system. The bullwhip effect is investigated based on the magnitude of changes in market demand. The demand distortion travels upstream in the supply chain with amplified variability. This analysis will help policymakers understand distorted information's impact and apply active strategies for improving supply chain efficiency.

Supply chain managers may not completely understand the susceptibility of their supply chains and underlying business disruption. For embracing the need for adaptability and innovation in a rapidly changing business environment, the suppression and synchronization strategies have been realized in Section 4 using novel fractional-order control with adaptive law. The test results demonstrate that the proposed technique performs better than the current benchmarking methods. With the business world changing rapidly, the proposed hybrid

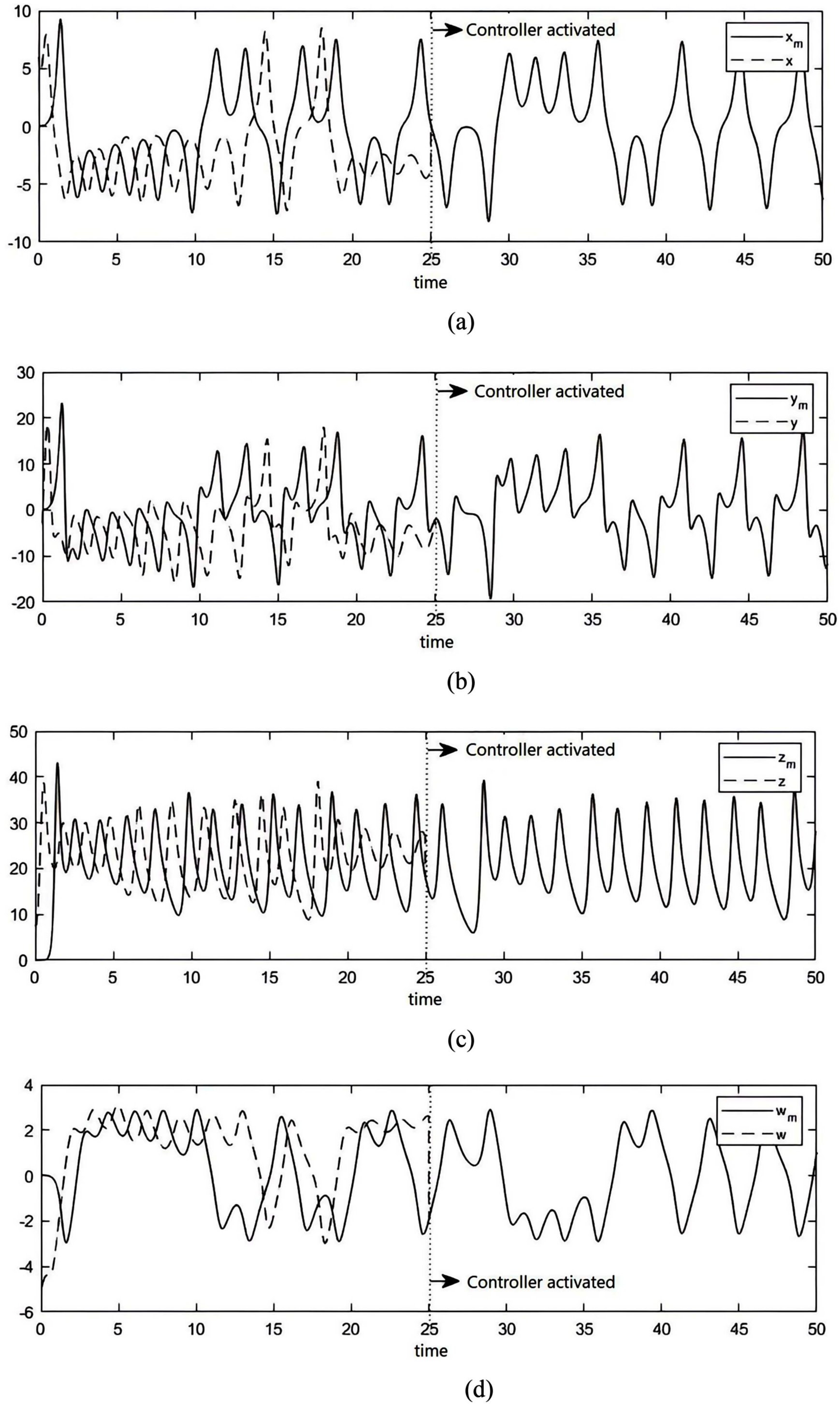


FIGURE 12. Time histories for chaos synchronization where controllers are activated at $t = 25$ (time periods): (a) $x(t)$, (b) $y(t)$, (c) $z(t)$, and (d) $w(t)$.

TABLE 5. Total cost analysis for control strategies.

Time periods	% increase in total control efforts compared with AFOPID	
	PID	FOPID
25	−14.58%	−10.42%
30	75.71%	58.29%
35	34.78%	23.48%
40	30.33%	8.52%
45	26.06%	5.88%
50	23.25%	6.25%

algorithm (AFOPID) delivered an outstanding performance for chaotic suppression. FOPID provides the second-best option, and the PID algorithm has been identified as the worst approach. For relevant costs used for decision making, in comparison to PID and FOPID control systems, utilizing the AFOPID approach results in the optimal solution being offered at a lower control activity, roughly 29.26% and 15.33%, respectively. The hybrid control scheme is the most effective scheme evaluated by performance indicators, such as ISE and ITSE.

Active management strategy uses control methods to form effective business solutions and achieve goals. This approach can help policymakers make strategic decisions based on controlled data rather than personal opinions or feelings. More importantly, the presented algorithm is very robust in a volatile market.

6. Conclusions

A business crisis results in the stability of a company or organization at risk and can be caused by internal or external factors. Under a tumultuous market, business enterprises try to reconcile their uncertain supply chains with vulnerability. Traditional logistics management software might not provide efficient tools to cope with supply chain disruptions imposed by a volatile market. A new four-dimensional supply chain model has been introduced to explore complex dynamic interactions between participants. The nonlinear characteristics are extensively investigated using the stability analysis of equilibrium points, bifurcation diagram, phase portrait, and time-series responses. Dynamical behaviors such as unpredictable or chaotic limit cycles are explored with varying control parameters. In particular, the supply chain system will be more oscillatory as transport risk and distortion increase. The bullwhip effect is examined based on the magnification of market demand variability. The variance amplification in the dynamical behaviors is to show that the demand distortion is significant along with information flow in the supply chain networks from retailers to manufacturers.

Next, novel adaptive fractional-order control with supervisory monitoring is designed to realize optimal operations for digital supply chain networks. The complete control system is to guarantee chaos suppression as well as a synchronization scheme with accuracy. The closed-loop system stability has been verified using the Lyapunov theory. Extensive numerical simulations have been carried out to validate the hybrid control synthesis through performance indices. The optimal solution using the AFOPID strategy can guarantee a lower control effort than PID and FOPID control strategies, approximately 29.26% and 15.33%, respectively. In addition, the hybrid control scheme is considered the most effective scheme evaluated by performance indicators, such as ISE and ITSE. As a result, a business that adapts to the new digital transition through efficient management software will be positioned to readily embrace the supply chain risks with ensuring resilience and sustainability. The multi-echelon optimization software can help the enterprise reduce the volatility of supply chain networks and improve its market performance, becoming more agile during uncertain times.

The current study's limitations include its design and methodology, which should be addressed in future research. The dynamic analysis should consider more system variables, such as inventory, shipment sent, manufacturing rate, and unfilled order, to gain more insights into the dynamic interactions. On the methodological side, more advanced control algorithms might be exploited for efficient management, such as intelligent control-based neural network prediction and metaheuristic optimization techniques. In system modeling, actual data and deep learning algorithms can be used to construct the supply chain dynamics system parameters to ensure better accuracy. In addition, different actors for each level of the supply chain network can be considered to bring the system closer to reality.

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Data availability statement. All data generated or analyzed during this study are included in this article (and its supplementary information files).

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