

RESEARCH ON SUPPLY CHAIN PRICING DECISIONS CONSIDERING THE ADVERTISING EFFECT UNDER MARKET ENCROACHMENT

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Abstract. Based on a Stackelberg game, this paper establishes supply chain models in which an incumbent manufacturer invests in advertising and the retailer invests in advertising when there is a new entrant manufacturer. By solving the model, the subgame perfect equilibrium under different conditions is obtained, and then the influences of the advertising coefficient and the degree of differentiation of two brands on the pricing decisions of supply chain members are investigated. The results show that: in the incumbent manufacturer advertising model, the wholesale prices and retail prices of the incumbent manufacturer and the encroaching manufacturer change as the advertising coefficient and the degree of differentiation of the two brands change. In the retailer's advertising model, the wholesale prices, retail prices and profits of the incumbent manufacturer and the encroaching manufacturer are all higher than those of the benchmark model within the limited scope. Some valuable information could be provided for supply chain enterprises to develop collaborative strategies and promote supply chain management practices.

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1. INTRODUCTION

With the development of global economic integration, the development prospects of domestic and foreign retail companies are improving, which has also promoted the sound development of the supply chain. However, some unscrupulous merchants are selling counterfeit products in the market, and this phenomenon has become an important factor influencing supply chain decision-making. In the early days, counterfeit products were only some high-luxury products, such as luxury watches, jewelery, and fashionable clothes. Currently, fakes have been involved in many fields, including aerospace parts, auto parts, and even infant formula and medicines. The appearance of these fakes will affect the credibility and the reputation of the authentic brand among consumers.

In order to eliminate some of the negative effects of counterfeit products on genuine products in the market, many retail companies have launched anti-counterfeiting activities. For example, retail giant Amazon and China's largest e-commerce company Alibaba have taken many measures and invested considerable money in anti-counterfeiting. The existing international anti-counterfeiting alliances spend hundreds of millions of dollars on anti-counterfeiting each year.

Keywords. Supply chain management, Manufacturer competition, Market encroachment, Advertising, Pricing strategy.

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As counterfeit products become increasingly more rampant, their impact on the supply chain has aroused wider attention from the industry and scholars. It is important to discuss the impact of fakes on the supply chain for authentic companies to optimize anti-counterfeiting actions. Therefore, how to handle market encroachment has become a key issue facing enterprises. In response to market encroachment, incumbent firms will take a series of measures to alleviate the negative impact of market encroachment. As an important marketing strategy, advertising plays an important role in economic activities. The advertising effect can effectively enhance consumers' brand awareness and consumer demand and effectively suppress the intrusion of the counterfeit market. Therefore, this article studies the effect of advertising and the influence of counterfeit intrusion outside the supply chain on the decisions of different members. This paper researches the following questions:

- (1) The encroaching manufacturer entered the market, the structure of the supply chain would be changed, then how are supply chain decisions affected and how is supply chain equilibrium affected?
- (2) Different from the current papers that study the encroachment problem stemming from the cost difference between retail selling and direct selling, we neglect this difference and study how factors (such as advertising) affect the performance of manufacturer encroachment.
- (3) When introducing the advertising. How do different advertising strategies affect supply chain decisions and how is supply chain equilibrium affected?

To answer the above questions, this paper investigates a supply chain consisting of an incumbent manufacturer, an encroaching manufacturer and a retailer and constructs a supply chain model in which the incumbent manufacturer invests in advertising and the retailer invests in advertising during the market encroachment of the encroaching manufacturer. By solving the model, the subgame perfect equilibria in different situations are obtained; furthermore, the influence of the advertising coefficient and the degree of differentiation between the two brands on the pricing decisions of supply chain members is studied. The research in this paper provides some theoretical guidance for the selection of advertising models and the development of pricing strategies under market encroachment.

The rest of the paper is organized as follows. Section 2 provides a review of the relevant literature for this paper. Section 3 gives the problem description and basic assumptions. Section 4 develops the models is developed and gives solution methodology. Section 5 analyzes the equilibrium decisions. Numerical example is provided in Section 6. Managerial insights are discussed in Section 7. Then Section 8 concludes the paper.

2. LITERATURE REVIEW

This paper mainly studies two aspects: market encroachment and the advertising effect. Scholars have studied market encroachment from different perspectives. Li *et al.* [1] investigated the impact of supplier encroachment on supply chain members' decisions in direct and resale channels under asymmetric information and nonlinear pricing. Yoon's [2] research conclusions are different from the traditional view that manufacturers' encroachment into the retail market will harm retailers' profits [3] and occupy retailers' market share [4]. The research has found that encroachment will cause manufacturers to make investments to reduce costs, which in turn lower wholesale prices, ultimately benefiting retailers. Based on the wholesale price contract, Wang *et al.* [5] study the channel selection and decision-making of supply chain members in the face of market encroachment. When retailers face encroachment from incumbent manufacturers, they can improve their profits by choosing to sell products from potential intruding manufacturers, and incumbent manufacturers will choose to open direct sales channels when facing market encroachment. Ji [6] discussed the impact of encroaching manufacturers on supply chain members under different entry costs and product quality conditions. Guo *et al.* [7] found that when retailers face uncertain market demand and manufacturer encroachment, they can improve their own profits by appropriately collecting and disclosing demand information. When faced with a manufacturer's encroachment, a retailer will take measures to counter the encroachment. Zhang *et al.* [8] found that retail service investment by retailers in a retailer-led supply chain can effectively prevent manufacturers from invading, and retail service investment can improve the benefits of supply chain members and consumers. In

a remanufacturing supply chain, Zheng *et al.* [9] analysed the intrinsic relationship between remanufacturing, channel competition, and manufacturer channel encroachment decisions in the no-remanufacturing and remanufacturing scenarios. On this basis, Cao *et al.* [10] further considered the recycling remanufacturing to manufacturer channel encroachment strategy and encroachment timing. Choi *et al.* [11] focused on an intelligent dual channel (online-to-offline) strategy in industry to arrange the optimal services for customers and introduce two important factors, backorder and lead-time through marginal value. Zheng *et al.* [12] studied the impact of channel competition on market encroachment in a remanufacturing closed-loop supply chain under demand determination and uncertainty. Different from the encroachment of manufacturers in the supply chain into the retail market, the encroachment of enterprises outside a supply chain makes the channel structure more complicated. Choi *et al.* [13] studied an advanced retailing strategy for a single item for a retailer and a manufacturer with variable lead time. Padiyar *et al.* [14] studied a multi-echelon inventory model for deteriorating multi items with imperfect production under the environment of fuzzy and inflation. Cao *et al.* [15] studied the impact of product quality differences under different entry modes on the decision-making of supply chain members when upstream suppliers are outside the supply chain encroachment. Jin *et al.* [16] studied the impact of market encroachment and brand differentiation on the supply chain equilibrium under different power structures.

In the marketplace, advertising not only influences consumer choice preferences but is also one of the main strategies for companies to stimulate market demand and increase product consumption. Huang [17] and He *et al.* [18] classified advertisements into national advertisements for raising brand awareness and local advertisements for short-term stimulation of consumption according to their different roles. Scholars have considered the impacts of advertising inputs on supply chain decisions from different perspectives. Based on the information dissemination context, Zhang *et al.* [19] discuss the impact of advertising costs on corporate decisions and profits, and the results show that an increase in advertising costs can increase corporate effectiveness under certain circumstances. Oryani *et al.* [20] performed a new dynamic ARDL model and then test the financial resources curse hypothesis in Iran. Wang *et al.* [21] compared advertising strategies and product pricing under three electronic supply chain operating models and found that an increase in advertising stimulated demand, resulting in profit growth for supply chain members. During the implementation of local advertising, upstream manufacturers share a portion of the advertising costs of downstream retailers, which is known as cooperative advertising [22]. Shu *et al.* [23] develop four models of cooperation between manufacturers and e-tailing platforms to study cooperative advertising decisions under different power structures. The above research mainly focuses on the positive supply chain. Yi *et al.* [24] and Yao *et al.* [25] considered the effects of advertising and the advertising sharing rate on the pricing of supply chain members and supply chain coordination problems in closed-loop supply chains. Huang [26] considered the market encroachment problem when the remanufacturer faces the incumbent manufacturer's advertising investment; the study found that the incumbent manufacturer's investment in advertising will consolidate its position while the remanufacturer needs to further reduce costs to enter the market. Table 1 illustrates the summary of the most relevant studies.

In summary, most of the existing researchers focus on the impact of direct sales channels opened by manufacturers in the supply chain on the retail market, while there are few studies on the impact of advertising investment and market encroachment outside the supply chain on the decisions of different members.

3. PROBLEM DESCRIPTION AND BASIC ASSUMPTIONS

Consider a two-tier supply chain consisting of an incumbent manufacturer I and a retailer R . The incumbent manufacturer produces a product I with a premium brand value that is sold to the market through the retailer, while there is an encroaching manufacturer E in the market producing a product with a common brand value that is also sold to the same market through the retailer. In a manufacturer-led supply chain game system, the manufacturer is the Stackelberg game leader, and the retailer is the follower.

TABLE 1. Comparison among recent relevant studies.

Literature	Supply chain structure	Channel encroachment	Manufacturer competition	Advertising
Li <i>et al.</i> [1]	MR	✓	—	—
Yoon [2]	MR	✓	—	—
Ha <i>et al.</i> [3]	MR	✓	—	—
Zhang <i>et al.</i> [8]	MR	✓	—	—
Zheng <i>et al.</i> [12]	CMR	✓	✓	—
Choi <i>et al.</i> [11]	MR	—	—	—
Huang and Li [17]	MR	—	✓	✓
Wang and Yu [21]	MP	✓	—	✓
Zhang <i>et al.</i> [27]	MR	✓	—	✓
This paper	MMR	✓	✓	✓

Notes. MR – a manufacturer and a retailer; CMR – a closed-loop supply chain with a manufacturer and a retailer; MP – a manufacturer and a platform; MMR – two manufacturers and a retailer.

TABLE 2. List of notations.

Notation	Description
v	Consumers' willingness to pay for product I (follows certain distribution)
θv	Consumers' willingness to pay for product E (follows certain distribution)
θ	The degree of differentiation between the two brands (or the consumer identification with product E)
α	Advertising marketing efficiency coefficient
β	Advertising cost coefficient
s	Advertising level
w	Wholesale Price, w_i , $i = I, E$, represent wholesale price of products I and E , respectively (\$/unit)
p	retailer Price, p_i , $i = I, E$, represent retailer price of products I and E , respectively (\$/unit)
U_i	Net utility, U_i , $i = I, E$, represent products I and E , respectively
	Consumer demand, q_i , $i = I, E$, represent products I and E , respectively (unit)
π_j^h	Profits of supply chain member j in model h (respectively indicate an incumbent manufacturer, an encroaching manufacturer, and retailer. $h = M, R$ respectively indicate that incumbent manufacturers invest in advertising and retailers invest in advertising.) (\$)

To define the research question and to facilitate the construction of the subsequent model, we make the following assumptions:

- (1) In the process of the game, the manufacturer and the retailer are information symmetric, and they are risk neutral.
- (2) Different consumers have different levels of willingness to pay for the same product, so assume that v is uniformly distributed in interval $[0, 1]$.
- (3) Manufacturers and retailers make advertising investments in order to maximize profits. The advertising costs are $\frac{1}{2}\beta s_j^2$, and parameters $\alpha, \beta \in [0, 1]$.
- (4) Assume that the costs of the incumbent manufacturer, the encroaching manufacturer, and the retailer are 0.

To enhance the readability, a list of notations is presented in Table 2.

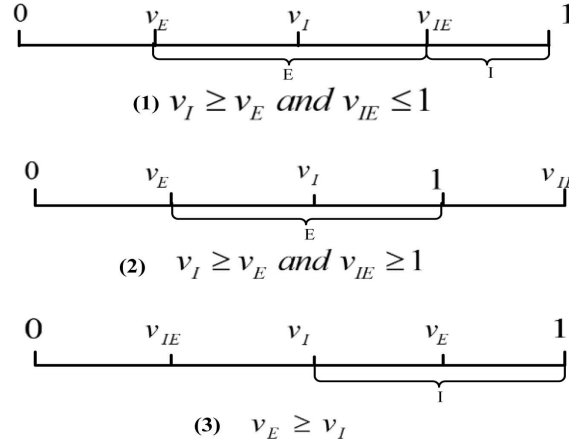


FIGURE 1. Consumer product choice space under different conditions. (a) $v_I \geq v_E$ and $v_{IE} \leq 1$, (b) $v_I \geq v_E$ and $v_{IE} \geq 1$ and (c) $v_E \geq v_I$.

4. MODELS

4.1. Benchmark model

In the case of market encroachment, the net utility of consumers buying different products is:

$$\begin{aligned} U_I &= v - p_I, \\ U_E &= \theta v - p_E. \end{aligned}$$

Consumers compare the sizes of U_I , U_E and 0 to buy products or give up. That is, consumers choose $\max(U_I, U_E, 0)$ which specifically include the following: (1) when $U_I, U_E < 0$, consumers give up buying; (2) when $U_I > U_E \geq 0$, consumers prefer to purchase product I ; (3) when $U_E > U_I \geq 0$, consumers prefer to purchase product E ; and (4) when $U_I = U_E \geq 0$, there is no difference in whether consumers buy product I or product E .

For the sake of generality, suppose that the demand market is the unit market, consumers' product value v obeys a normal distribution of $[0, 1]$, and the number of consumers is also evenly distributed in this interval. Obviously, there are three critical states of consumer choice: (1) $v - p_I = 0$. At this point, the consumer valuation is defined as: $v_I = p_I$. (2) $\theta v - p_E = 0$. At this point, the consumer valuation is defined as: $v_E = \frac{p_E}{\theta}$. (3) $v - p_I = \theta v - p_E$. At this point, the consumer valuation is defined as: $v_{IE} = \frac{p_I - p_E}{1 - \theta}$.

When $v_I \geq v_E$ that is, $v_{IE} \geq v_I \geq v_E$ and when $v_E \geq v_I$ that is, $v_E \geq v_I \geq v_{IE}$; consumers' choices of products are shown in the following figure:

As seen from the figure, we can deduce that:

- (1) When $v_I \geq v_E$ and $v_{IE} \leq 1$, that is, $\frac{p_E}{\theta} \leq \theta \leq 1 - p_I + p_E$, both product I and product E have demands, and the demands at this time are as follows: $q_I = 1 - \frac{p_I - p_E}{1 - \theta}$ and $q_E = \frac{p_I - p_E}{1 - \theta} - \frac{p_E}{\theta}$.
- (2) When $v_I \geq v_E$ and $v_{IE} \geq 1$, that $1 - p_I + p_E \leq \theta \leq 1$, consumers purchase product E . At this time, the product demand is $q_E = 1 - \frac{p_E}{\theta}$.
- (3) When $v_E \geq v_I$, that is, $0 \leq \theta \leq \frac{p_E}{p_I}$, consumers purchase product I . At this time, the product demand is $q_I = 1 - p_I$.

In order to make the research conclusions more comprehensive, we only consider the situation when two product demands exist at the same time, that is, when $\frac{p_E}{p_I} \leq \theta \leq 1 - p_I + p_E$, $q_I = 1 - \frac{p_I - p_E}{1 - \theta}$ and $q_E = \frac{p_I - p_E}{1 - \theta} - \frac{p_E}{\theta}$.

At this time, the profit functions of the incumbent manufacturer, the encroaching manufacturer, and the retailer are:

$$\begin{aligned}\pi_I &= \omega_I \left(1 - \frac{p_I - p_E}{1 - \theta} \right), \\ \pi_E &= \omega_E \left(\frac{p_I - p_E}{1 - \theta} - \frac{p_E}{\theta} \right), \\ \pi_R &= (p_I - \omega_I) \left(1 - \frac{p_I - p_E}{1 - \theta} \right) + (p_E - \omega_E) \left(\frac{p_I - p_E}{1 - \theta} - \frac{p_E}{\theta} \right).\end{aligned}$$

The sequence of the supply chain game is as follows. First, each manufacturer determines the wholesale price ω_I and ω_E of their respective products simultaneously. Then, the retailer determines the retail price (p_I, p_E) of both products simultaneously according to (ω_I, ω_E) .

Using the inverse induction method to solve the Hessian matrix H of π_R , we can obtain:

$$\begin{aligned}H &= \begin{pmatrix} -\frac{2}{1-\theta} & \frac{2}{1-\theta} \\ \frac{2}{1-\theta} & -\frac{2}{(1-\theta)\theta} \end{pmatrix}, \\ |H| &= \frac{4}{(1-\theta)\theta} > 0.\end{aligned}$$

H is negative definite, that is, there is a unique optimal solution. Therefore, based on the first-order condition, the reaction function of the retailer can be obtained as:

$$\begin{aligned}p_I^* &= \frac{1 + \omega_I}{2}, \\ p_E^* &= \frac{\theta + \omega_E}{2}.\end{aligned}$$

Substituting p_I^* and p_E^* into π_I and π_E , respectively, it is easy to know that π_I and π_E are concave functions with respect to ω_I and ω_E . Therefore, from the first-order condition, it follows that:

$$\begin{aligned}\omega_I^* &= \frac{2(\theta - 1)}{\theta - 4}, \\ \omega_E^* &= \frac{\theta(\theta - 1)}{\theta - 4}.\end{aligned}$$

Substituting ω_I^* and ω_E^* into the retailer's reaction function, we can obtain:

$$\begin{aligned}p_I^* &= \frac{3\theta - 6}{2\theta - 8}, \\ p_E^* &= \frac{\theta(2\theta - 5)}{2\theta - 8}.\end{aligned}$$

4.2. Incumbent manufacturers invest advertising models

When an incumbent manufacturer invests in advertising (indicated by the superscript “ M ”), the increase in visibility resulting from the advertising effect is beneficial to the manufacturer. Therefore, the net utility of consumers buying different products is:

$$\begin{aligned}U_I &= v - p_I + \alpha s_I, \\ U_E &= \theta v - p_E.\end{aligned}$$

Additionally, consider only the situation in which two product requirements exist at the same time, that is, $\frac{p_E}{p_I - \alpha s_I} \leq \theta \leq 1 - p_I + p_E + \alpha s_I$. At this time, the demand functions are $q_I = 1 - \frac{p_I - p_E - \alpha s_I}{1 - \theta}$ and $q_E = \frac{p_I - p_E - \alpha s_I}{1 - \theta} - \frac{p_E}{\theta}$. The profits of incumbent manufacturers, encroaching manufacturers, and retailers are:

$$\begin{aligned}\pi_I^M &= \omega_I \left(1 - \frac{p_I - p_E - \alpha s_I}{1 - \theta} \right) - \frac{1}{2} \beta s_I^2, \\ \pi_E^M &= \omega_E \left(\frac{p_I - p_E - \alpha s_I}{1 - \theta} - \frac{p_E}{\theta} \right), \\ \pi_R^M &= (p_I - \omega_I) \left(1 - \frac{p_I - p_E - \alpha s_I}{1 - \theta} \right) + (p_E - \omega_E) \left(\frac{p_I - p_E - \alpha s_I}{1 - \theta} - \frac{p_E}{\theta} \right).\end{aligned}$$

The sequence of the supply chain game is as follows. First, the incumbent manufacturer determines the wholesale price ω_I^M of its own product and the level of advertising input s_I^M . Next, the encroaching manufacturer determines the wholesale price ω_E^M of its own product. Then, the retailer determines the retail price p_I^M and p_E^M of both products according to ω_I^M , s_I^M and ω_E^M .

Using the inverse induction method to solve the Hessian matrix H of π_R^M , we can obtain:

$$\begin{aligned}H &= \begin{pmatrix} -\frac{2}{1-\theta} & -\frac{\frac{1}{1-\theta}}{1-\theta} - \frac{2}{\theta} \end{pmatrix}, \\ |H| &= \frac{4}{\theta(1-\theta)} > 0.\end{aligned}$$

H is negative definite, that is, there is a unique optimal solution. Therefore, based on the first-order condition, the reaction function of the retailer can be obtained as:

$$\begin{aligned}p_I^M &= \frac{\alpha s_I + \omega_I + 1}{2}, \\ p_E^M &= \frac{\theta + \omega_E}{2}.\end{aligned}$$

By substituting p_I^M and p_E^M into π_I^M and π_E^M respectively, from the first order condition, we can obtain:

$$\begin{aligned}s_I^M &= \frac{6\alpha(5\theta^2 - 8\theta + 3)}{27\alpha^2\theta - 14\beta\theta^2 - 18\alpha^2 + 22\beta\theta - 8\beta}, \\ \omega_I^M &= -\frac{4\beta(\theta - 1)(5\theta^2 - 8\theta + 3)}{27\alpha^2\theta - 14\beta\theta^2 - 18\alpha^2 + 22\beta\theta - 8\beta}, \\ \omega_E^M &= \frac{\theta(9\alpha^2\theta + 2\beta\theta^2 - 9\alpha^2 - 4\beta\theta + 2\beta)}{27\alpha^2\theta - 14\beta\theta^2 - 18\alpha^2 + 22\beta\theta - 8\beta}.\end{aligned}$$

Substituting s_I^M , ω_I^M , and ω_E^M into p_I^M and p_E^M , we can obtain:

$$\begin{aligned}p_I^M &= \frac{\left((-27\theta + 18)\alpha^3 + (-27\theta + 18)\alpha^2 + ((14\beta - 30)\theta^2 + (-22\beta + 48)\theta + 8\beta - 18)\alpha + 2(\theta - 1)(2\theta - 1)(5\theta - 2)\beta \right)}{(36 - 54\theta)\alpha^2 + 4(\theta - 1)(7\theta - 4)\beta}, \\ p_E^M &= \frac{3\theta(12\alpha^2\theta - 4\beta\theta^2 - 9\alpha^2 + 6\beta\theta - 2\beta)}{54\alpha^2\theta - 28\beta\theta^2 - 36\alpha^2 + 44\beta\theta - 16\beta}.\end{aligned}$$

4.3. Retailers invest advertising model

When retailers invest in advertising (represented by superscript “ R ”), the increase in visibility resulting from the advertising effect is beneficial to the product sales of both the incumbent manufacturer and the encroaching

manufacturer. At this time, the net utility of consumers purchasing the two products is:

$$\begin{aligned} U_I &= v - p_I + \alpha_1 s_R, \\ U_E &= \theta v - p_E + \alpha_2 s_R. \end{aligned}$$

For the convenience of the discussion, α_1 and α_2 are regarded as equal parameters α . Therefore, the net utility of consumers buying the two products are:

$$\begin{aligned} U_I &= v - p_I + \alpha s_R, \\ U_E &= \theta v - p_E + \alpha s_R. \end{aligned}$$

Consistent with the above, we only consider the situation where two product requirements exist at the same time, that is, $\frac{p_E - \alpha s_R}{p_I - \alpha s_R} \leq \theta \leq 1 - p_I + p_E$. The demand functions are: $q_I = 1 - \frac{p_I - p_E}{1 - \theta}$ and $q_E = \frac{p_I - p_E}{1 - \theta} - \frac{p_E - \alpha s_R}{\theta}$. The profits of incumbent manufacturers, encroaching manufacturers, and retailers are:

$$\begin{aligned} \pi_I^R &= \omega_I \left(1 - \frac{p_I - p_E}{1 - \theta} \right), \\ \pi_E^R &= \omega_E \left(\frac{p_I - p_E}{1 - \theta} - \frac{p_E - \alpha s_R}{\theta} \right), \\ \pi_R^R &= (p_I - \omega_I) \left(1 - \frac{p_I - p_E}{1 - \theta} \right) + (p_E - \omega_E) \left(\frac{p_I - p_E}{1 - \theta} - \frac{p_E - \alpha s_R}{\theta} \right) - \frac{1}{2} \beta s_R^2. \end{aligned}$$

The sequence of the supply chain game is as follows. First, each manufacturer determines the wholesale price ω_I^R and ω_E^R of their products at the same time. Then, the retailer determines the retail price p_I^R and p_E^R and the advertising level s_R^R of the two products at the same time according to ω_I^R and ω_E^R .

Using the inverse induction method to solve the Hessian matrix H of π_R^R , we can obtain:

$$\begin{aligned} H &= \begin{pmatrix} -\frac{2}{1-\theta} & -\frac{2}{1-\theta} & 0 \\ \frac{2}{1-\theta} & -\frac{2}{1-\theta} - \frac{2}{\theta} & \frac{\alpha}{\theta} \\ 0 & \frac{\alpha}{\theta} & -\beta \end{pmatrix}, \\ |H| &= \frac{2(2\beta\theta - \alpha^2)}{\theta^2(\theta - 1)}. \end{aligned}$$

From the above matrix, it can be obtained that when $2\beta\theta - \alpha^2 > 0$, the matrix is negatively definite. Therefore, the retailer's reaction function can be obtained according to the first-order condition:

$$\begin{aligned} p_I^R &= \frac{(\theta - \omega_E - \omega_I - 1)\alpha^2 + 2\beta\theta(\omega_I + 1)}{4\beta\theta - 2\alpha^2}, \\ p_E^R &= \frac{\alpha^2\omega_E - \beta\theta^2 - \beta\theta\omega_E}{\alpha^2 - 2\beta\theta}, \\ s_R^R &= \frac{\alpha(\theta - \omega_E)}{2\beta\theta - \alpha^2}. \end{aligned}$$

Substituting p_I^R and p_E^R into π_I^R and π_E^R , respectively, it is easy to know that π_I^R and π_E^R are concave functions with respect to ω_I^R and ω_E^R . Therefore, from the first-order condition, it follows that:

$$\begin{aligned} \omega_I^R &= \frac{(\theta - 1)(4\beta - \alpha^2)}{(2\theta - 8)\beta + 3\alpha^2}, \\ \omega_E^R &= \frac{(\theta - 1)(\alpha^2 + 2\beta\theta)}{3\alpha^2 + 2\beta\theta - 8\beta}. \end{aligned}$$

Substituting ω_I^R and ω_E^R into the retailer's reaction function can obtain:

$$\begin{aligned} p_I^R &= \frac{(12\theta^2 - 24\theta)\beta^2 - 2\alpha^2(\theta^2 + 2\theta - 6)\beta + 3\alpha^4(\theta - 1)}{8(\theta - 4)\theta\beta^2 + 8\alpha^2(\theta + 2)\beta - 6\alpha^4}, \\ p_E^R &= \frac{4\beta^2\theta^3 + (2\alpha^2\beta - 10\beta^2)\theta^2 + \alpha^2(\beta - \alpha^2)\theta + \alpha^4}{(3\alpha^2 + 2\beta\theta - 8\beta)(2\beta\theta - \alpha^2)}, \\ s_R^R &= \frac{(2\theta + 1)\alpha^3 - 6\beta\alpha\theta}{-3\alpha^4 + 4\alpha^2(\theta + 2)\beta + 4(\theta - 4)\theta\beta^2}. \end{aligned}$$

5. MODEL COMPARISON AND ANALYSIS

Proposition 1. *In the incumbent manufacturer's investment model, the wholesale price of the incumbent manufacturer's product I is affected by the degree of differentiation between the two brands and the advertising coefficient. In the case where the difference between the two brands is small (that is, $0.591 < \theta < \frac{3}{5}$) and the advertising coefficient is small ($\frac{\alpha^2}{\beta} < \frac{-10\theta^3 + 70\theta^2 - 92\theta + 32}{27\theta - 18}$), the wholesale price of the incumbent manufacturer's product I is less than the wholesale price in the benchmark model. In the case of a larger advertising coefficient ($\frac{\alpha^2}{\beta} > \frac{-10\theta^3 + 70\theta^2 - 92\theta + 32}{27\theta - 18}$), the wholesale price of the incumbent manufacturer's product I is greater than the wholesale price in the benchmark model. When the difference between the two brands is large (that is, $\frac{3}{5} < \theta < \frac{2}{3}$) and the advertising coefficient is small, the wholesale price of product I of the incumbent manufacturer is greater than the wholesale price in the benchmark model. In the case of a large advertising coefficient, the wholesale price of product I of the incumbent manufacturer is less than the wholesale price in the benchmark model.*

Proof. $\omega_I^M - \omega_I = -\frac{2(\theta-1)(10\beta\theta^3 + 27\alpha^2\theta - 70\beta\theta^2 - 18\alpha^2 + 92\beta\theta - 32\beta)}{(27\alpha^2\theta - 14\beta\theta^2 - 18\alpha^2 + 22\beta\theta - 8\beta)(\theta-4)}$ when $0.591 < \theta < \frac{3}{5}$, $\frac{\alpha^2}{\beta} < \frac{-10\theta^3 + 70\theta^2 - 92\theta + 32}{27\theta - 18}$, $\omega_I^M - \omega_I < 0$; when $\frac{\alpha^2}{\beta} > \frac{-10\theta^3 + 70\theta^2 - 92\theta + 32}{27\theta - 18}$, $\omega_I^M - \omega_I > 0$; and when $\frac{3}{5} < \theta < \frac{2}{3}$, $\frac{\alpha^2}{\beta} < \frac{-10\theta^3 + 70\theta^2 - 92\theta + 32}{27\theta - 18}$, $\omega_I^M - \omega_I < 0$. $\square$

Proposition 1 shows that in the incumbent manufacturer input advertising model, when the advertising coefficient is small, the advertising costs are large, and the difference between the two brands is small. That is, consumers are less willing to pay for product E at this time, and the incumbent manufacturer lowers the wholesale price in order to attract consumers. As the advertising coefficient increases, the advertising costs decrease, the advertising efficiency increases, and the popularity of product I further improves. In order to obtain as large of profits as possible, incumbent manufacturers increase the wholesale price. When consumers are more willing to pay for product E, the advertising coefficient of the incumbent manufacturer is smaller. That is, the advertising costs are higher, and the incumbent manufacturer increases the wholesale price in order to increase profits. When the advertising coefficient is large, higher advertising and marketing efficiency and consumers' higher willingness to pay for product E enable manufacturers to attract consumers to obtain sufficient profits at low wholesale prices.

Proposition 2. *In the incumbent manufacturer's input advertising model, when the advertising coefficient is small ($\frac{\alpha^2}{\beta} < \frac{14\theta^2 - 22\theta + 8}{27\theta - 18}$), the wholesale price of product E is greater than the wholesale price in the benchmark model. When the advertising coefficient is in the middle range ($\frac{14\theta^2 - 22\theta + 8}{27\theta - 18} < \frac{\alpha^2}{\beta} < \frac{-\theta^2 + 2\theta - 1}{-\frac{9}{8}(\theta+1)}$), the wholesale price of product E is less than the wholesale price in the benchmark model. When the advertising coefficient is large ($\frac{\alpha^2}{\beta} > \frac{-\theta^2 + 2\theta - 1}{-\frac{9}{8}(\theta+1)}$), the wholesale price of product E is greater than the wholesale price in the benchmark model. When the advertising coefficient is low ($\frac{\alpha^2}{\beta} < \frac{14\theta^2 - 22\theta + 8}{27\theta - 18}$), the retail price of product E is greater than the retail price in the benchmark model. When the advertising coefficient is in the middle range*

$\left(\frac{14\theta^2-22\theta+8}{27\theta-18} < \frac{\alpha^2}{\beta} < \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}\right)$, the retail price of product E is less than the retail price in the benchmark model. When the advertising coefficient is large $\left(\frac{\alpha^2}{\beta} > \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}\right)$, the retail price of product E is greater than the retail price in the benchmark model.

Proof. $\omega_E^M - \omega_E = -\frac{16\theta(\theta-1)(\beta\theta^2+(-\frac{9}{8}\alpha^2-2\beta)\theta-\frac{9}{8}\alpha^2+\beta)}{(-27\alpha^2\theta+14\beta\theta^2+18\alpha^2-22\beta\theta+8\beta)(\theta-4)}$ when $0.5714 < \theta < \frac{2}{3}$, $\frac{14\theta^2-22\theta+8}{27\theta-18} < \frac{\alpha^2}{\beta} < \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}$, $\omega_E^M - \omega_E < 0$; when $\frac{\alpha^2}{\beta} < \frac{14\theta^2-22\theta+8}{27\theta-18}$, $\omega_E^M - \omega_E > 0$; and when $\frac{\alpha^2}{\beta} > \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}$, $\omega_E^M - \omega_E > 0$.

$p_E^M - p_E = -\frac{8\theta(\theta-1)(\beta\theta^2-\frac{9}{8}\alpha^2\theta-2\beta\theta-\frac{9}{8}\alpha^2+\beta)}{(-27\alpha^2\theta+14\beta\theta^2+18\alpha^2-22\beta\theta+8\beta)(\theta-4)}$ when $0.5714 < \theta < \frac{2}{3}$, $\frac{\alpha^2}{\beta} < \frac{14\theta^2-22\theta+8}{27\theta-18}$, $p_E^M - p_E > 0$; when $\frac{14\theta^2-22\theta+8}{27\theta-18} < \frac{\alpha^2}{\beta} < \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}$, $p_E^M - p_E > 0$; and when $\frac{\alpha^2}{\beta} > \frac{-\theta^2+2\theta-1}{-\frac{9}{8}(\theta+1)}$, $p_E^M - p_E > 0$. $\square$

Proposition 2 shows that in the incumbent manufacturer's advertising investment model, the wholesale price and retail price of the manufacturer's product E are affected by the incumbent manufacturer's advertising coefficient. In the incumbent manufacturer input advertising model, when the advertising coefficient is small, that is, when the incumbent manufacturer's advertising and marketing is less efficient and the advertising costs are larger, the incumbent manufacturer has limited advertising effects, the encroaching manufacturer is able to make some profit by increasing the wholesale price, and the retail price increases as the wholesale price increases.

As the advertising coefficient increases, the incumbent manufacturer's advertising marketing efficiency coefficient increases, and the advertising costs decrease. The stronger advertising effect makes consumers have a higher willingness to pay for the incumbent manufacturer's product I , the encroaching manufacturer can only reduce their own wholesale price to obtain profits, and the retail price is also reduced. When the incumbent manufacturer's advertising coefficient is large, it will bring certain benefits to the replacement brand for the encroaching manufacturer. The encroaching manufacturer can obtain profits through higher wholesale prices, and the retailer will also increase its retail price.

Proposition 3. When the incumbent manufacturer invests in advertising, when the advertising coefficient is small $\left(\frac{\alpha^2}{\beta} < \frac{14\theta^2-22\theta+8}{27\theta-18}\right)$, the wholesale price of the incumbent manufacturer's product I is less than the wholesale price of the incumbent manufacturer's product E . When the advertising coefficient is in the middle range $\left(\frac{14\theta^2-22\theta+8}{27\theta-18} < \frac{\alpha^2}{\beta} < \frac{-22\theta^2+34\theta-12}{9\theta}\right)$, the wholesale price of the incumbent manufacturer's product I is greater than the wholesale price of the encroaching manufacturer's product E . When the advertising coefficient is large $\left(\frac{\alpha^2}{\beta} > \frac{-22\theta^2+34\theta-12}{9\theta}\right)$, the wholesale price of the incumbent manufacturer's product I is less than the wholesale price of the encroaching manufacturer's product E . When the difference between the two brands is small or large $(0 < \theta < \frac{3}{5} \text{ or } \frac{2}{3} < \theta < 1, \text{ respectively})$, the wholesale prices of the in-place manufacturers continue to increase as the advertising coefficient increases. When the degree of differentiation between the two brands is in the middle range $(\frac{3}{5} < \theta < 1\frac{2}{3})$ the wholesale price of the incumbent manufacturer keeps decreasing as the advertising coefficient increases. When the degree of differentiation between the two brands is low $(0 < \theta < \frac{3}{5})$, the wholesale price for the manufacturer increases as the manufacturer invests in advertising. When the differentiation between the two brands is high $(\frac{3}{5} < \theta < 1)$, the wholesale price for the manufacturer decreases as the advertising investment of the manufacturer increases.

Proof. $\omega_I^M - \omega_E^M = \frac{(\theta-1)(9\alpha^2\theta+22\beta\theta^2-34\beta\theta+12\beta)}{14\beta\theta^2-27\alpha^2\theta-22\beta\theta+18\alpha^2+8\beta}$ when $0.5714 < \theta < \frac{2}{3}$, $\frac{\alpha^2}{\beta} < \frac{14\theta^2-22\theta+8}{27\theta-18}$, $\omega_I^M - \omega_E^M < 0$; when $\frac{14\theta^2-22\theta+8}{27\theta-18} < \frac{\alpha^2}{\beta} < \frac{-22\theta^2+34\theta-12}{9\theta}$, $\omega_I^M - \omega_E^M > 0$; when $\frac{\alpha^2}{\beta} > \frac{-22\theta^2+34\theta-12}{9\theta}$, $\omega_I^M - \omega_E^M < 0$, $\frac{\partial\omega_I^M}{\partial\frac{\alpha^2}{\beta}} = \frac{36(3\theta-2)(\theta-1)^2(5\theta-3)}{(14\theta^2-27\theta\frac{\alpha^2}{\beta}-22\theta+18\frac{\alpha^2}{\beta}+8)^2}$; when $0 < \theta < \frac{3}{5}$ or $\frac{2}{3} < \theta < 1$, $\frac{\partial\omega_I^M}{\partial\frac{\alpha^2}{\beta}} > 0$; and when $\frac{3}{5} < \theta < \frac{2}{3}$, $\frac{\partial\omega_I^M}{\partial\frac{\alpha^2}{\beta}} < 0$. $\frac{\partial\omega_E^M}{\partial\frac{\alpha^2}{\beta}} = -\frac{36\theta(5\theta-3)(\theta-1)^2}{(14\theta^2-27\theta\frac{\alpha^2}{\beta}-22\theta+18\frac{\alpha^2}{\beta}+8)^2}$ when $0 < \theta < \frac{3}{5}$, $\frac{\partial\omega_E^M}{\partial\frac{\alpha^2}{\beta}} > 0$ and when $\frac{3}{5} < \theta < 1$, $\frac{\partial\omega_E^M}{\partial\frac{\alpha^2}{\beta}} < 0$. $\square$

Proposition 3 shows that when the advertising coefficient is small, that is, when the advertising marketing efficiency is low and the advertising costs are high, the incumbent manufacturer will prevent the manufacturer

from encroaching the market at a low price. As the advertising coefficient increases, advertising and marketing efficiency increases, advertising costs decrease, and incumbent manufacturers are able to earn higher profits by setting higher wholesale prices due to their higher brand value. When the advertising coefficient is large, lower wholesale prices from incumbent manufacturers can attract retailers to expand demand. When the advertising coefficient is small, the incumbent manufacturer invests in advertising so that the wholesale price of the incumbent manufacturer and the wholesale price of the encroaching manufacturer both increase with the advertising coefficient, indicating that the incumbent manufacturer's advertising investment can benefit the substitute product to some extent.

Proposition 4. *In the retailer advertising input model, within the constraints, the wholesale and retail prices of the incumbent manufacturer and the encroaching manufacturer are higher than those of the benchmark model. The wholesale prices of the incumbent manufacturer and the encroaching manufacturer both increase as the retailer's advertising coefficient increases, and the wholesale and retail prices of the incumbent manufacturer are higher than the wholesale and retail prices of the encroaching manufacturer.*

Proof. From $2\beta\theta - \alpha^2 > 0$, we can obtain $\frac{\alpha^2}{\beta} < 2\theta$.

$$\begin{aligned}\omega_I^R - \omega_I &= -\frac{\alpha^2(\theta^2 + \theta - 2)}{(3\alpha^2 + 2\beta\theta - 8\beta)(\theta - 4)} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } \omega_I^R - \omega_I > 0. \\ p_I^R - p_I &= -\frac{\alpha^2(3\alpha^2\theta^2 - 2\beta\theta^3 - 6\alpha^2\theta - 8\beta\theta^2 - 6\alpha^2 + 28\beta\theta)}{2(3\alpha^2 + 2\beta\theta - 8\beta)(\alpha^2 - 2\beta\theta)(\theta - 4)} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } p_I^R - p_I > 0. \\ \omega_E^R - \omega_E &= -\frac{2\alpha^2(\theta^2 + \theta - 2)}{(3\alpha^2 + 2\beta\theta - 8\beta)(\theta - 4)} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } \omega_E^R - \omega_E > 0. \\ p_E^R - p_E &= -\frac{\alpha^2(4\alpha^2\theta^2 - 4\beta\theta^3 - 5\alpha^2\theta - 10\beta\theta^2 - 8\alpha^2 + 32\beta\theta)}{2(3\alpha^2 + 2\beta\theta - 8\beta)(\alpha^2 - 2\beta\theta)(\theta - 4)} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } p_E^R - p_E > 0. \\ \omega_I^R - \omega_E^R &= -\frac{2((\theta - 2)\beta + \alpha^2)(\theta - 1)}{(2\theta - 8) + 3\alpha^2} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } \omega_I^R - \omega_E^R > 0. \\ p_I^R - p_E^R &= -\frac{4((\theta - 3)\beta + \frac{5}{4}\alpha^2)(\theta - 1)}{(4\theta - 16)\beta + 6\alpha^2} \text{ when } 0 < \theta < 1, 0 < \frac{\alpha^2}{\beta} < 2\theta, \text{ and } p_I^R - p_E^R > 0. \\ \frac{\partial \omega_I^R}{\partial \frac{\alpha^2}{\beta}} &= -\frac{2(\theta - 1)(\theta + 2)}{(2\theta - 8 + 3\frac{\alpha^2}{\beta})^2} > 0 \text{ and } \frac{\partial \omega_E^R}{\partial \frac{\alpha^2}{\beta}} = -\frac{4(\theta - 1)(\theta + 2)}{(2\theta - 8 + 3\frac{\alpha^2}{\beta})^2} > 0. \quad \square\end{aligned}$$

Proposition 4 shows that within the constraints of the retailer input advertising model, the retailer advertising input can benefit both the incumbent manufacturer and the encroaching manufacturer by setting a higher wholesale price than the benchmark model, and the wholesale price of the incumbent manufacturer is greater than the wholesale price of the encroaching manufacturer due to the high brand value of the incumbent manufacturer. The retailer will accordingly set a higher retail price than the benchmark model due to the high wholesale price.

Proposition 5. *Under the retailer's advertising investment, the profits of the incumbent manufacturer and the profits of the encroaching manufacturer are greater than the profit level under the benchmark model; when the advertising coefficient is small ($\frac{\alpha^2}{\beta} < \frac{2\theta}{1+\theta}$), the profits of the incumbent manufacturer are higher than the profits of the encroaching manufacturer. When the advertising coefficient is large ($\frac{2\theta}{1+\theta} < \frac{\alpha^2}{\beta} < 2\theta$), the incumbent manufacturer's profits are smaller than the encroaching manufacturer's profits.*

Proof. $\pi_I^R - \pi_I = \frac{\alpha^2(\theta + 2)(\theta - 1)(-\frac{1}{8}\alpha^2\theta + \beta\theta + \frac{5}{4}\alpha^2 - 4\beta)}{(\theta - 4)^2(\beta\theta + \frac{5}{2}\alpha^2 - 4\beta)^2}$ when $0 < \theta < 1$, $\frac{\alpha^2}{\beta} < 2\theta$, and $\pi_I^R - \pi_I > 0$.

$$\pi_E^R - \pi_E = \frac{2\alpha^2(\theta - 1)(\beta^2\theta^4 + (\alpha^2\beta - 5\beta^2)\theta^3 + (\frac{1}{4}\alpha^4 - 7\alpha^2\beta + 16\beta^2)\theta^2 + (-\frac{17}{4}\alpha^4 + 32\alpha^2\beta - 48\beta^2)\theta + 4\alpha^4 - 8\alpha^2\beta)}{(3\alpha^2 + 2\beta\theta - 8\beta)^2(2\beta\theta - \alpha^2)(\theta - 4)^2}.$$

when $0 < \theta < 1$, $\frac{\alpha^2}{\beta} < 2\theta$, and $\pi_E^R - \pi_E > 0$.

$$\pi_I^R - \pi_E^R = \frac{(\alpha^2\theta + \alpha^2 - 2\beta\theta)(\theta - 1)\beta}{(3\alpha^2 + 2\beta\theta - 8\beta)(\alpha^2 - 2\beta\theta)} \text{ when } \frac{\alpha^2}{\beta} < \frac{2\theta}{1+\theta}, \pi_I^R - \pi_E^R > 0 \text{ and when } \frac{2\theta}{1+\theta} < \frac{\alpha^2}{\beta} < 2\theta, \pi_I^R - \pi_E^R < 0. \quad \square$$

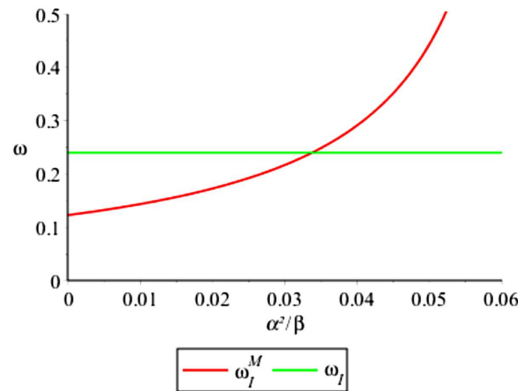


FIGURE 2. The wholesale price of the incumbent manufacturer when $0.591 < \theta < \frac{3}{5}$.

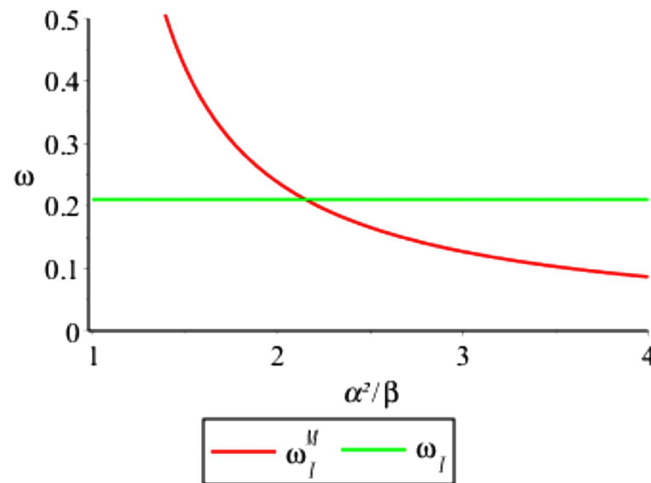


FIGURE 3. The wholesale price of the incumbent manufacturer when $\frac{3}{5} < \theta < \frac{2}{3}$.

Proposition 5 shows that the retailer's investment in advertising can increase the profits of the incumbent manufacturer and the encroaching manufacturer. When the advertising coefficient is small, the profits of the incumbent manufacturer are greater than the profits of the encroaching manufacturer; as the advertising coefficient increases, the advertising marketing efficiency increases, and the advertising effect is more favourable to the encroaching manufacturer. At this time, the profits of the encroaching manufacturer are greater than the profits of the incumbent manufacturer.

6. EXAMPLE ANALYSIS

In this section, the above model will be verified by the numerical analysis method. Similar to Choi *et al.* [11], Choi *et al.* [13], Padiyar *et al.* [14], parametric values are obtained from Zhang *et al.* [27]. First, let $\theta = 0.595$ and $\theta = 0.65$. The variation of the wholesale price of the incumbent manufacturer in the baseline model and the wholesale price of the incumbent manufacturer in the incumbent manufacturer advertising input model can be obtained for different scenarios, as shown in Figures 2 and 3.

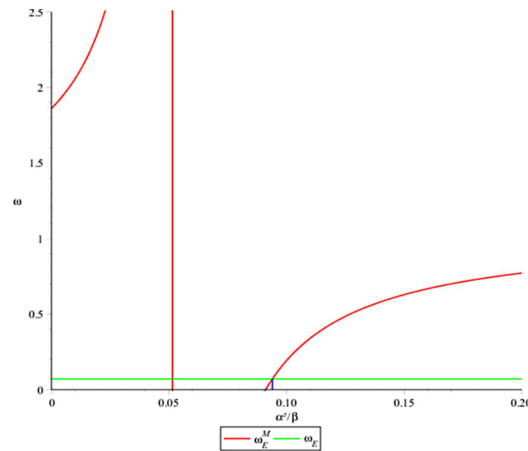


FIGURE 4. Encroaching manufacturer's wholesale price.

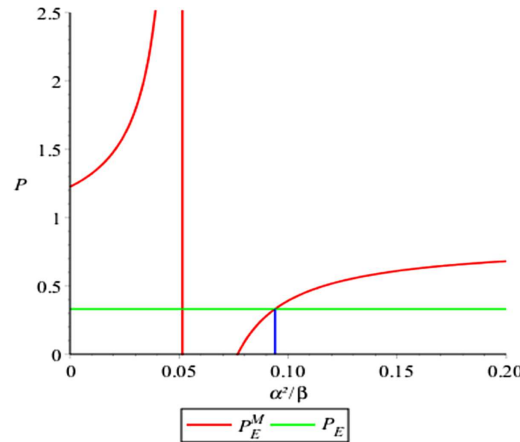
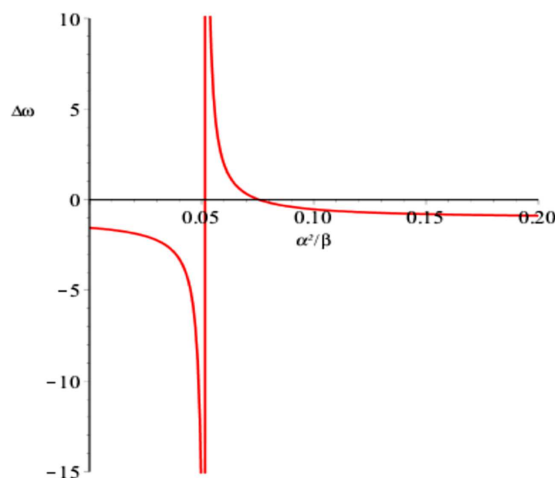


FIGURE 5. Encroaching manufacturer's retail price.

Figure 2 shows that when $0.59 < \theta < 0.6$ and $0 < \frac{\alpha^2}{\beta} < 0.33$, the incumbent manufacturer's wholesale price under the incumbent manufacturer's advertising input is less than the incumbent manufacturer's wholesale price in the benchmark model. When $\frac{\alpha^2}{\beta} > 0.33$, the incumbent manufacturer's wholesale price under the incumbent manufacturer's advertising input is greater than the incumbent manufacturer's wholesale price in the benchmark model. Figure 3 shows that when $0.6 < \theta < \frac{2}{3}$ and $0 < \frac{\alpha^2}{\beta} < 2.16$, the incumbent manufacturer's wholesale price under the incumbent manufacturer's advertising input is greater than the incumbent manufacturer's wholesale price in the benchmark model. When $\frac{\alpha^2}{\beta} > 2.16$, the incumbent manufacturer's wholesale price under the incumbent manufacturer's advertising input is less than the incumbent manufacturer's wholesale price in the benchmark model. Proposition 1 is proved.

Then, taking $\theta = 0.59$, it is possible to obtain the benchmark model and the encroaching manufacturer's wholesale price and retail price with the advertising input of the incumbent manufacturer, as shown in Figures 4 and 5.

Figure 4 shows that when $\frac{\alpha^2}{\beta} < 0.051$, the wholesale price of product E is greater than the wholesale price in the benchmark model. When $0.051 < \frac{\alpha^2}{\beta} < 0.094$, the wholesale price of product E is less than the wholesale

FIGURE 6. Incumbent manufacturers invest in the advertising model $\Delta\omega$.

price in the benchmark model. When $\frac{\alpha^2}{\beta} > 0.094$, the wholesale price of product E is greater than the wholesale price in the benchmark model. Figure 5 shows that when $\frac{\alpha^2}{\beta} < 0.051$, the retail price of product E is greater than the retail price in the benchmark model. When $0.051 < \frac{\alpha^2}{\beta} < 0.094$, the retail price of product E is less than the retail price in the benchmark model. When $\frac{\alpha^2}{\beta} > 0.094$, the retail price of product E is greater than the retail price in the benchmark model. These results are consistent with the formulation of Proposition 2.

Then, taking $\theta = 0.59$, the results of comparing the wholesale prices $\Delta\omega$ of the incumbent and encroaching manufacturers in the incumbent manufacturer input advertising model can be obtained, as shown in Figure 6.

Figure 6 shows that when $\frac{\alpha^2}{\beta} < 0.051$, $\Delta\omega < 0$, that is, the wholesale price of the incumbent manufacturer's product I is less than the wholesale price of the encroaching manufacturer's product E . When $0.051 < \frac{\alpha^2}{\beta} < 0.076$, $\Delta\omega > 0$, and the wholesale price of the incumbent manufacturer's product I is greater than the wholesale price of the encroaching manufacturer's product E . When $\frac{\alpha^2}{\beta} > 0.076$, $\Delta\omega < 0$, and the wholesale price of the incumbent manufacturer's product I is less than the wholesale price of the encroaching manufacturer's product E .

Taking $\theta = 0.59$, the wholesale and retail prices of the incumbent manufacturer and the encroaching manufacturer in the benchmark model and the retailer input advertising model can be obtained, as shown in Figures 7–12.

Figures 7–12 show that when $0 < \frac{\alpha^2}{\beta} < 1.18$, $\omega_I^R > \omega_I$, $p_I^R > p_I$, $\omega_E^R > \omega_E$, $p_E^R > p_E$, $\omega_I^R > \omega_E^R$, and $p_I^R > p_E^R$. Proposition 4 is proved.

Finally, consistent with the above, taking $\theta = 0.59$, the profit difference between the incumbent manufacturer, the encroaching manufacturer and the benchmark model comparison under the retailer's advertising input is obtained, as shown in Figure 13.

Figure 13 shows that when $0 < \frac{\alpha^2}{\beta} < 1.18$, the profit difference between the incumbent manufacturer, the encroaching manufacturer and the benchmark model comparison are all greater than 0, which shows that under the advertising investment of the retailer, the profits of the incumbent manufacturer and the profits of the encroaching manufacturer have increased compared with the profits under the benchmark model. When $\frac{\alpha^2}{\beta} < 0.74$, the profits of the incumbent manufacturer are greater than the profits of the incumbent manufacturer; when $0.74 < \frac{\alpha^2}{\beta} < 1.18$, the profits of the incumbent manufacturer are less than the profits of the incumbent manufacturer. Furthermore, the figure shows that the profit growth rate of the encroaching manufacturer is

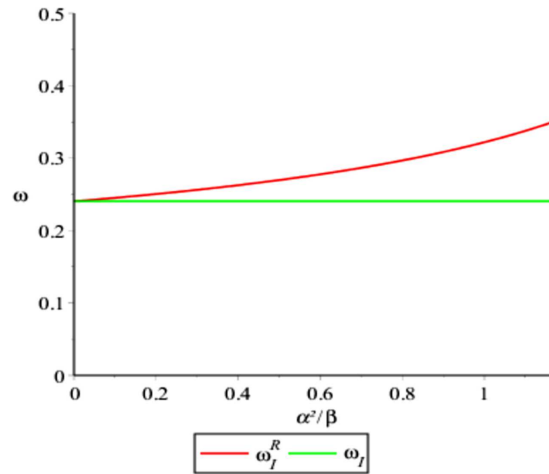


FIGURE 7. Incumbent manufacturer's wholesale prices in the benchmark and retailer input advertising models.

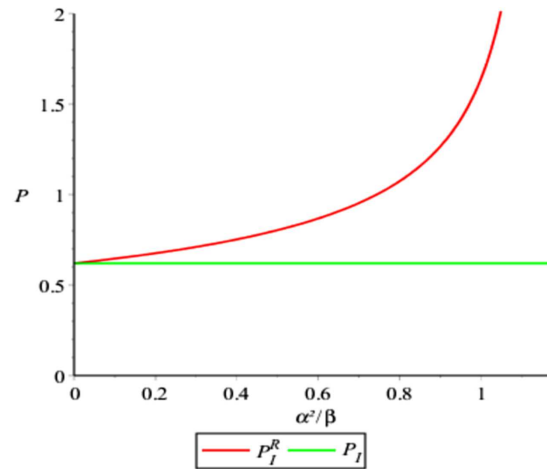


FIGURE 8. Incumbent manufacturer's retail prices in the benchmark and retailer input advertising models.

greater than the profit growth rate of the incumbent manufacturer. Therefore, the retailer's investment in advertising can greatly increase the profit levels of the encroaching manufacturer.

7. MANAGERIAL INSIGHTS

Some important management insights can be drawn from the above findings.

Firstly, for the incumbent manufacturer, to reduce the threat from the encroaching manufacturer, the incumbent manufacturer should not advertise blindly. It only invest in advertising to the extent that its advertising is effective in order to increase profits, otherwise, it may lead to greater profits for the encroaching manufacturer and the manufacturer needs to join forces with the retailer to counteract the encroaching manufacturer's influence on the market to protect its profits.

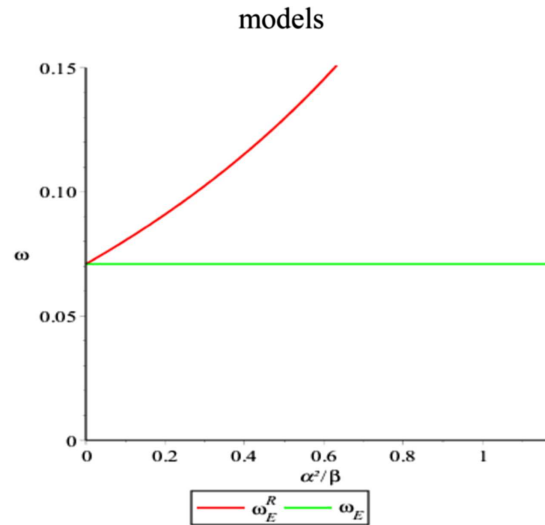


FIGURE 9. Encroaching manufacturer's wholesale prices in the benchmark and retailer input advertising models.

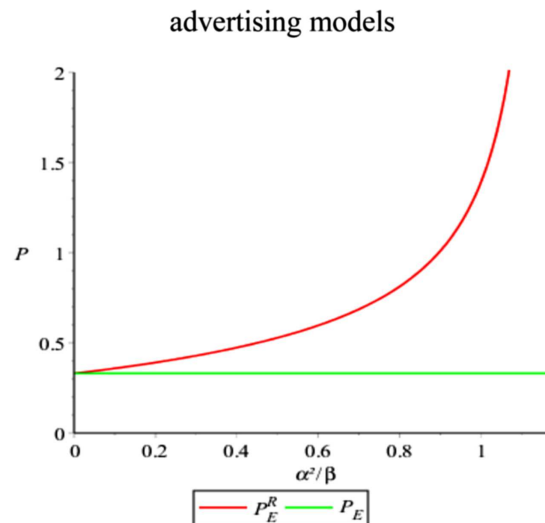


FIGURE 10. Encroaching manufacturer's retail price in the baseline model and retailer input advertising model.

Then, for the retailer, the addition of an encroaching manufacturer does not necessarily lead to increased profits, but both the retailer's advertising and the incumbent manufacturer's advertising can increase the retailer's profits. Under the retailer advertising model, both the incumbent manufacturer and the encroaching manufacturer increase their profits, but the encroaching manufacturer benefits more when the advertising is less effective, and only when the advertising is more effective does the incumbent manufacturer benefit more from the effectiveness of the retailer's advertising. Therefore, the manufacturer needs to cooperate with the retailer by giving the retailer a subsidy to increase the efficiency of the retailer's advertising.

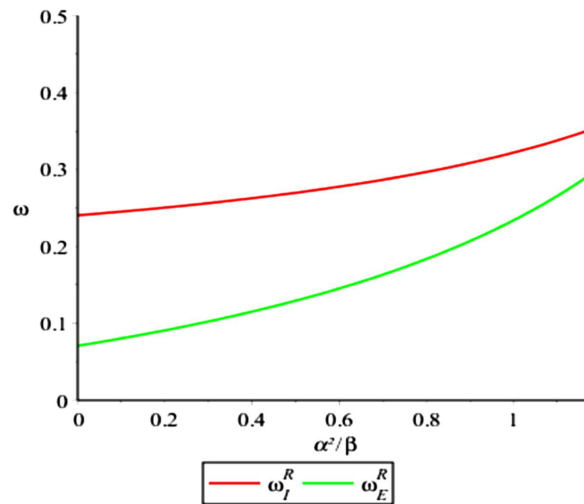


FIGURE 11. Wholesale prices for incumbent and encroaching manufacturers in the retailer advertising model.

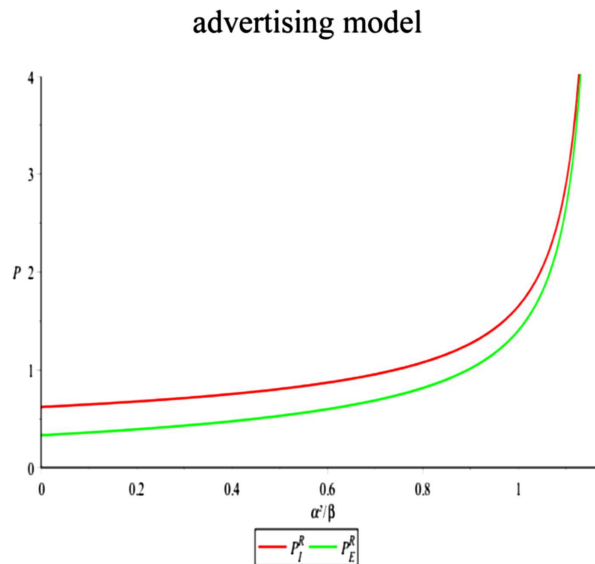


FIGURE 12. Retail prices for incumbent and encroaching manufacturers in the retailer advertising model.

8. CONCLUSION

This paper studies a supply chain consisting of an incumbent manufacturer, an encroaching manufacturer and a retailer. By solving the model, the subgame perfect equilibria in different situations are obtained, and then the influences of the advertising coefficient and the degree of differentiation of the two brands on the pricing decisions of supply chain members are investigated. Then, the conclusions are presented as follows: compared to the benchmark model, the changes in wholesale and retail prices of the incumbent manufacturer's product I and

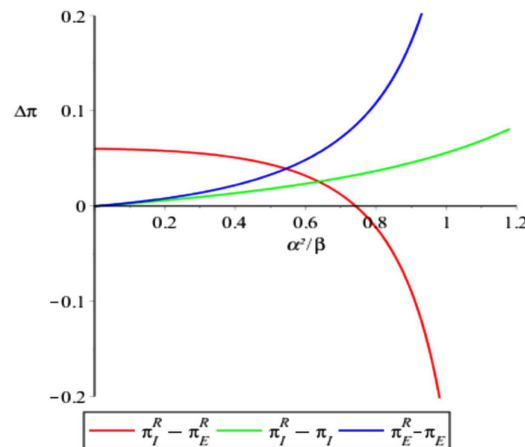


FIGURE 13. Profit comparison between the incumbent manufacturer and the encroaching manufacturer under retailer advertising investment.

the encroaching manufacturer's product E are more complex in the incumbent manufacturer's advertising model. The wholesale prices and retail prices of the incumbent manufacturer and the encroaching manufacturer change as the advertising coefficient and the degree of differentiation of the two brands change. In the retailer input advertising model, within limits, the wholesale prices, retail prices and profits of the incumbent manufacturer's product I and the encroaching manufacturer's product E are higher than those of the benchmark model.

The main limitation of this study was that in order to focus on the research problem and simplify the analysis process, the paper only considered a simple two-level supply chain, which does not fully take into account the structural complexity of the supply chain in realistic. In addition, this paper only considers that decision-makers are rational, which in reality may have multiple behavioral preferences. Behavioral characteristics can be considered in future research. Furthermore, the influence of government policy can be considered; thus, more detailed research must be conducted by extending the sophistication of the supply chain structure to render the model more consistent with the actual decision-making and reveal the social phenomena and laws.

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