

## RETAIL PRICE COMPETITION OF DOMESTIC AND INTERNATIONAL COMPANIES: A BI-LEVEL GAME THEORETICAL OPTIMIZATION APPROACH

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**Abstract.** Drawing on the Stackelberg game approach to solving the pricing problem in a supply chain, this paper develops a bi-level model whereby a domestic company and a foreign manufacturer compete to gain more profit from the market of a retailer. The domestic company acts as the leader and the retailer as the follower. The domestic company has two manufacturers each of whom produces and sells a different quality of the product. The retailer decides to purchase products based on the prices offered by the low-quality manufacturer, the high-quality manufacturer, and the foreign manufacturer, known as an exogenous factor. In fact, the first level seeks to maximize its profits and the second level seeks to reduce the cost of purchasing. In this paper, the price of the products of each manufacturer is considered a contributing factor to the retailer's tendency to buy from each manufacturer. This assumption is designed by the multinomial logit model. As the proposed model has binary variables in its follower segment, a novel hybrid exact method based on explicit enumeration method and Lambert-W function is applied to solve it. In other words, to calculate the optimal selling price of domestic products and their profit first by using the explicit enumeration method, the bi-level model is transformed into a single-level problem. The problem is, then, solved precisely by applying the Lambert-W function. The efficiency of the proposed model is proven by the results obtained from solving the model and the sensitivity analysis of the main parameters of the model. Moreover, to have a detailed managerial analysis of each manufacturer's profit on the competitive market environment, the market is studied in view of three different scenarios: (1) when there is a sense of patriotism regarding domestic manufacturers; (2) when customers have low incomes; and (3) when customers have high incomes. Finally, the study results conclude that if the domestic company has two manufacturers that produce a different type of quality can lead to an increase in the profit of the domestic company. Indeed, the proposed model can increase the competitive power of the domestic company against imported products by providing appropriate pricing on its products.

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## 1. INTRODUCTION

Today's fast-evolving marketplace often means that quality-differentiated brands of a certain product are being introduced onto the market by leaps and bounds, and retailers can, consequently, sell multiple brands of the same product with differing quality as supplied by different manufacturers. On the other hand, the emergence of global supply chains (SC), mostly as a natural corollary of ever-increasing connectivity in today's globalized world, also means that all manner of foreign manufacturers have carved a niche in the domestic market to compete with domestic manufacturers. As a result, considering the increasing number of manufacturers producing a specific product with differing quality and brands, retailers are torn between foreign and domestic manufacturers. Market globalization has inexorably given rise to increasingly fierce competition between domestic and foreign manufacturers. In such a competitive market, wherein the market share and profit of a given product are highly influenced by the price offered by other competing manufacturers, the importance of pricing decisions can hardly be overemphasized. Not for nothing have competitive pricing models received significant attention in both academic and practical contexts recently. One case in point can be the competition between domestic and foreign manufacturers in the cosmetics industry. Yoon *et al.* [1] stated that to gain more market share and profit in local and international markets *vis-a-vis* their foreign competitors, South Korean cosmetics companies improve their competitive edge by manufacturing products with different qualities and prices to better cater to their prospective customers' needs and tastes. Pursuing this policy has helped these Korean companies increase their share of the global and the local market and profitability.

In this paper, we consider a domestic company with two domestic products competing over price with a foreign manufacturer, with each trying to maximize their market share. In real-world situations, retailers consider various factors before making their purchase decision. Among others, the price of the products, the product quality level, and lead time contributing to such decisions. Some social factors such as the propensity for buying domestic goods or the average income level of the community are also among the contributing factors. For instance, when customers set great store by buying domestic products for patriotic reasons, domestic manufacturers are likely to take over from foreign manufacturers in terms of market share. As a further illustration, taking the average income of a community into consideration, if the average income level is high, the manufacturer producing products of higher quality stands a better chance of gaining the market share.

To show the effect of different factors contributing to a given manufacturer's market share, researchers widely use the concept of the utility function. In the literature on the topic, different types of utility functions such as multinomial logit (MNL) function, Huff function, linear utility function, etc. are used. In this paper, we consider two local manufacturers competing with a foreign manufacturer. Of the two local manufacturers, one produces items of high quality, and the other produces the same product but with low quality. The competition between these two manufacturers and a foreign manufacturer is quite understandable. In this paper, a multinomial logit (MNL) function determines the market share of each manufacturer. In a multi-echelon SC, the members of that SC may adopt different approaches to decision-making. These approaches are mainly classified as centralized and decentralized decision-making. In centralized decision-making, a unique decision-maker decides on the optimal solutions, while in decentralized decision-making, individual members of the SC separately make their own optimal decisions. This paper assumes that the retailer and domestic manufacturers operate in a decentralized system where the domestic manufacturers act as the leaders and the retailer the follower.

In a decentralized system, the SC's members have different ways to address their problem. One common approach to addressing constrained decentralized problems, where two SC participants are non-cooperative, is bi-level optimization where decisions are made hierarchically. In fact, this type of mathematical model consists of two levels name upper (leader) and lower (follower) level [2,3]. Each level of this problem is able to make the decision on some part of decision variables based on its objective function. Also, there may be some common constraints for both levels [4]. Furthermore, the lower level makes the optimal decision based on the decision of the upper level. This means that these two levels based on their objective functions and interaction can affect the decision of each other [5]. A bi-level model could be generally expressed as follows:

$$\begin{aligned}
& \min_{x \in X, Y} Z(x, y) \\
& \text{s.t. } C(x, y) \leq 0 \\
& \quad \min z(x, y) \\
& \quad \text{s.t. } c(x, y) \leq 0.
\end{aligned} \tag{1}$$

This research defines an SC problem as one involving a retailer, one foreign manufacturer offering imported products, and two domestic manufacturers offering a single product under different brands and varying product quality (low and high quality). The MNL model is utilized to express the market share of each manufacturer, which is assumed to depend on their proposed prices. The retailer supplies the market demand based on the manufacturers' offered prices. Each manufacturer's market share is a function of the price they quote for their commodities. Drawing on a bi-level programming approach, a manufacturer-Stackelberg game model is established and solved. Regarding the leader-follower competitive pricing model in an SC operating on a global scale, the following questions are addressed:

- What is the optimal pricing strategy of the domestic manufacturers to maximize their profit, against the pricing of an imported product offered by the foreign manufacturer?
- How do the foreign manufacturer's pricing decisions affect the pricing strategy of the domestic manufacturers and the retailer's decisions?
- How does the retailer's sensitivity to manufacturer's price affect the optimal decisions of each manufacturer, their market share, and profit?
- How does customers' sensitivity to the price of the product influence the optimal pricing decision of the domestic manufacturers and the retailer's decision?
- What is the optimal pricing decision for each manufacturer when faced with strongly patriotic customers?
- How does the manufacturing technology affect the market share of each manufacturer and their optimal pricing?
- How does the customers' average income level affect the optimal pricing decisions of the domestic manufacturers and, subsequently, their profits and market shares?
- What is the optimal strategy of the retailer to minimize costs, against the pricing decisions of the domestic and foreign manufacturers?

The most important contribution of the present research is designing a competitive pricing model for a global market scenario, wherein a domestic company, with two manufacturers, each with different brands, is competing with a foreign manufacturer to gain the maximum market share and profit. This contribution takes on added significance against this backdrop that despite the advancement of globalization by leaps and bounds in recent decades and foreign manufacturers featuring more prominently than ever in local markets, little scholarship has been done on pricing problems either on a global or local scale. To fill this gap, this study focuses on the issue of competitive pricing between the domestic and foreign manufacturers for selling their products to the retailer. Here, we assume the price offered by each brand, besides the retailer's inclination to buy from either the foreign or domestic manufacturers, to be the most important factors, influencing each manufacturer's market share. Previous studies have mainly used linear utility functions to capture the effect of these factors on manufacturers' market shares; this study differs in that it has adopted a logit model to express the utility of each manufacturer from the retailer's point of view. Furthermore, to the best of our knowledge, few studies have been done on the bi-level programming approach to addressing the pricing model in a constrained Stackelberg game. In this research, we have applied a bi-level optimization model to address this leader-follower pricing model. Also, to solve this bi-level problem, we have used a novel hybrid exact method based on the explicit enumeration method and Lambert-W function. More specifically, we have used the explicit enumeration method to transform the proposed model into a single-level problem. Then, we have applied the Lambert-W function to solve our problem exactly.

The rest of the present study is structured as follows. Section 2 reviews the relevant literature. Section 3 offers a definition and formulation of the problem. It is followed by a presentation of a solution procedure to

the developed problem in Section 4. Section 5, then, illustrates a numerical example and computational results. Section 6 explains managerial insights, with Section 7 drawing conclusions. Finally, all proofs of the propositions are shown in the Appendix A.

## 2. LITERATURE REVIEW

Recent decades have seen many analytical and research-based studies on different aspects of SC such as inventory management, network design, and pricing. Among these, pricing and revenue management have received more attention. This paper mainly focuses on the pricing decision in a global SC framework. Our research pertains to four streams of literature including pricing competition in an SC, global SC, bi-level programming, and considering multi-brands of a single product.

One critical aspect of marketing strategies, which considerably affects customers' preferences, is pricing decisions. Nowadays, the emergence of various manufacturers, both domestic and foreign, has fueled competition in terms of pricing decisions. Recently, many studies have investigated the competitive pricing issue in SC, among which the following works are more related to our study. By applying one Nash game theory and two Stackelberg models, Choi [6] analyzed a competitive SC where one retailer was selling two products of two competing manufacturers. Their results indicate that the less differentiated the products, the greater the prices and the profits of the SC members. Porteus *et al.* [7] studied a competitive pricing model characterized by quality differentiation and limited capacity. Their results indicate that the leaders use the low-pricing strategy to lower competition among followers and increase their own profits. Anderson and Bao [8] examined the effect of the level of price competition on the profits of the SC members and demonstrated that this factor can determine whether a decentralized system can outdo a centralized one. Li *et al.* [9] also developed a competitive pricing model composed of two suppliers and one retailer to discuss the optimal sourcing and pricing decisions made by suppliers and the retailer, respectively. Similarly, Lin *et al.* [10] introduced a competition SC problem with three levels where the manufacturer acts as the main decision-maker and has three integration strategies: forward, backward, and no integration. Ba *et al.* [11] presented an oligopoly-pricing model in online retailing competitive markets to explore the effect of adverse prices. Xiao *et al.* [12] also explored the outsourcing decision-making model for two manufacturers competing on retail prices and product quality. They examined the impact of various factors such as production cost on the optimal outsourcing decisions. On the other hand, by considering order size constraints, Ekici *et al.* [13] established a two-stage SC in a pricing model, composed of a retailer and two suppliers, to optimize pricing decisions of the two suppliers who compete for the market share. Xiao *et al.* [14] presented a model based on game theory to examine pricing and lead-time where an SC and an outside manufacturer compete with each other. They considered the presence of brand differentiation between two chains, leading to fierce competition for gaining better market shares. More recently, by considering two quality-differentiated brands, Li and Chen [15] have discussed a backward integration model in a retailer SC where the retailer has three options for integration with low-quality and high-quality manufacturers.

In view of globalization and the introduction of foreign products into the domestic market in today's market, an increasing number of foreign manufacturers have emerged in national markets. Meixell and Gargeya [16] proposed an exhaustive review of global SC. Research has shown that patriotic sentiments can influence the perception of customers towards imported products. Therefore, consumer sentiments of nationalism or patriotism can considerably affect the manufacturers' market share [17–19]. In addition, individuals who are intensively consumer-ethnocentric perceive the purchase of foreign products as a wrong decision, believing it to be detrimental to the domestic economy and stoking unemployment. Elliott and Cameron [20] examined the effect of the quality of products and the country of origin on consumers' decision-making. Although the emergence of foreign manufacturers can significantly affect pricing strategies and, consequently, the profits of domestic manufacturers, little scholarship has been done on the pricing issue in the global SC. The following papers are noteworthy. Seppälä *et al.* [21] studied the transfer-pricing issue in an SC operating on a global scale. By considering a monopoly market, He and Xiao [22] developed a pricing model in a non-cooperative global SC to optimize the pricing decisions of two foreign manufacturers who compete for the market share. Matta

and Miller [23] also established a multinational SC to optimize transfer prices and the structure of the SC. Nagurney and Li [24] extended a quality and pricing competition game model characterized by globalization. In their model, they assumed products to be differentiated by different brands with a possibility for outsourcing. Recently, using a game-theoretic approach in a three-level SC, Noori-daryan *et al.* [25] have presented a joint optimization model of pricing, lead-time, ordering, and supplier selection. Song *et al.* [26] discussed optimal pricing decisions in the supply chain by developing a distribution channel in a Stackelberg game structure with seller and independent buyer frameworks. Mukherjee *et al.* [27] examined pricing problems by considering some important parameters like crisis and recovery likelihood that comes from product recall of manufacturers.

The third stream of the related literature focuses on the bi-level programming approach discussed by many researchers, a limited number of whom have considered this approach to pricing models. Gao *et al.* [28] examined two nonlinear bi-level programming approaches to investigate the optimal pricing decisions in a vendor–buyer setting. By considering the price and advertising-dependent demand expansion, Sadigh *et al.* [29] optimized inventory, pricing, and advertising strategies in an SC, manufacturing different products. To address their problem, they proposed two Stackelberg game models solved through a bi-level programming approach. Mokhlesian and Zegordi [30] extended a bi-level inventory-pricing optimization model in a multi-product SC with multiple retailers and a single manufacturer. Zhang *et al.* [31] also established a bi-level optimization approach to investigate the best price and replenishment cycle in a leader–follower SC for high-tech products. Similarly, using two bi-level models, Ma *et al.* [32] developed a joint lot sizing and pricing model in two leader-follower SCs. Based on a bi-level programming approach, Parvasi *et al.* [33] addressed a school bus routing problem with a possibility for outsourcing. Wang *et al.* [34] established two Stackelberg models consisting of a vender-led SC and buyer-led SC in a multi-product, eco-friendly SC. They applied a bi-level programming model to obtain the optimal decisions on ordering, pricing, advertising, and environmental efforts. Recently, Amirtaheeri *et al.* [35] have extended a manufacturer-led SC to investigate optimal pricing and advertising solutions by applying a bi-level optimization approach. Recently, Zhang *et al.* [36] implemented a bi-level pricing structure in energy generation systems. They considered in the supply side the generation is stochastic. Also, they used a multi-agent learning algorithm to get the optimal price.

From the consumers' perspective, products of different brands and, consequently, different product quality are distinguishable [37,38]. Therefore, the differences among various brands highly affect manufacturers' market share. For example, selling products with higher quality or at lower prices can help the manufacturer secure more market share. As a result, different features of the various brands of a given product contribute to each brand's market share. By considering the quality and product features, Choi and Coughlan [39] discussed the retailer's decision-making problem regarding brand differentiation. By considering one manufacturer and two retailers, Rajagopalan and Xia [40] developed a pricing model in an SC to probe the impact of product variety and differentiation on the optimal decisions of the SC's participants. Pang and Tan [41] proposed four different game-theoretic models of pricing and quality of a single product with different brands produced by two competing manufacturers. Xiao *et al.* [42] also discussed the optimal strategy of the channel format and product variety in a retailer-led SC. Xiao *et al.* [14] explored the impact of brand differentiation on the lead-time and optimal price strategies of a competitive environment wherein an SC competes with an outside manufacturer. Zhou and Lin [43] extended an advertising model in a single-retailer multiple-manufacturer SC to investigate the issue of brand competition. Similarly, Giri *et al.* [44] developed a centralized and manufacturer-Stackelberg model to examine the optimal pricing and quality decisions of an SC containing multiple manufacturers and one retailer who make a single product under multiple brands with varying quality. More recently, by considering brand differentiation and profits, Giri *et al.* [45] have optimized the quality and pricing decisions of multiple manufacturers and a single retailer competing to differentiate their brand and gain more market share. Li and Chen [15] discussed the optimal price and quality strategies in a competitive retailer-led SC with two manufacturers producing two different high- or low-quality brands. Recently, Taleizadeh *et al.* [46] extended a model by studying optimal decisions and operational strategies in an SC network. Their model considered two manufacturers that produce goods of different quality and compete with each other to gain more market share and profit from a retailer that has fixed demand.

To capture the effect of different factors such as pricing on the market share of each member of the SC, researchers have drawn on various types of utility functions including linear function, logit function, and Huff function. This paper has adopted a logit model to introduce the price effect into each manufacturer's market share. Considering the historical perspective of utility functions, we found the following works more relevant to our study. Huang [47] developed three transport pricing models where the mode choice behavior was formulated using the logit model. Lüer-Villagra and Marianov [48] studied the optimal pricing and hub location decisions in a competitive atmosphere between two companies. In their research, they used a logit discrete choice model to formulate the customers' behavior. Čvokić *et al.* [49] extended a joint pricing-hub location model with competition raging between the leader and follower with market shares represented by logit functions. More recently, Zhang [50] has developed an optimization model of pricing and location in a competitive atmosphere between retailers. To express the impact of travel cost and mill price on each retailer's market share, he has used the multinomial logit (MNL) function. Mahmoodjanloo *et al.* [51] by using the MNL model developed a hub location pricing model by considering customer loyalty and assuming the demands are elastic. Parvasi and Taleizadeh [52] recently developed a bi-level programming model where the domestic manufacturer is the leader, and the foreign manufacturer is the follower. In their model, manufacturers intend to achieve the highest profit and market share in three different local markets by setting appropriate pricing strategies, where the customers of each market have different sales levels than the customers of the other markets. Accordingly, the authors determined the profit of each manufacturer for all markets by developing a desirability function depending on the price and quality of the goods.

Table 1 presents a concise review of the related literature. A close review of the literature reveals that no study has drawn on the bi-level programming approach to address pricing competition between two domestic manufacturers with different brands and a foreign manufacturer selling products to a common retailer. Furthermore, very few studies have looked into the effect of patriotic sentiments on the purchase decision. We have considered this issue by incorporating the effect of each manufacturer's price and the fact that the MNL model can determine the retailer's inclination to buy products from each of these manufacturers. In other words, the MNL model is used to find out how much the retailer buys from each manufacturer. Also, it bears noting that the retailer's tendency to buy from each manufacturer is determined by the price offered by the domestic manufacturer and foreign manufacturer as well as the customers' preference.

### 3. PROBLEM DEFINITION AND MATHEMATICAL MODEL

Nowadays, the rise in the number of manufacturers offering similar products with differing quality in a specific area has given rise to a competitive market. Foreign manufacturers release their products to local markets, with an inevitable decrease in profitability for domestic manufacturers operating on those local markets. Hence, to maintain their profit, domestic manufacturers and companies should offer an appropriate selling price to retailers. On the other hand, to preserve their market share in this fiercely competitive environment, retailers should cater to customers' needs. Therefore, in a given area, based on customers' preferences for each manufacturer's product, retailers determine the quantity of purchase from these manufacturers.

Against this backdrop, this study examines the competition between manufacturers and a retailer in an SC network (Fig. 1). As Figure 1 shows, there are three manufacturers named H, L, and F such that the domestic company has two manufacturers (Manufacturers H and L). The products of Manufacturers H and L are of high quality and low quality, respectively. Manufacturer F also sells imported products to the retailer.

By considering the bi-level programming application, the decision-making environment is designed based on a Stackelberg game (Fig. 2). In this regard, the domestic company and the retailer are introduced as the leader and follower, respectively. In this structure, the domestic company owns the two manufacturers so that the company makes a central decision for both domestic manufacturers. Regarding the product price of each domestic manufacturer (L and H) and the product price of the foreign manufacturer (F), the follower seeks to minimize its cost of purchasing products. Indeed, by estimating the customer's demand for each manufacturer's product (L, H, and F) along with each manufacturer's product price, the retailer decides on how much to buy

TABLE 1. A brief review of the literature.

Author(s)	Assumptions											
	Competitive pricing	Different brands	Bi-level programming	Global SC	Utility function		Game model				Multiple manufacturers/ suppliers	
					Multinomial logit (MNL)	Other functions	Centralized	Manufacture/ vendor Stackelberg	Retailer/buyer Stackelberg	Other		Decentralized
[6]	✓								✓		✓	
[11]	✓	✓										✓
[8]	✓							✓	✓			✓
[9]	✓							✓	✓			✓
[10]	✓											✓
[12]	✓	✓										✓
[14]	✓	✓						✓	✓			✓
[53]	✓								✓			✓
[13]	✓											✓
[48]	✓					✓					✓	✓
[50]	✓					✓					✓	✓
[22]	✓			✓							✓	✓
[24]	✓	✓		✓		✓					✓	✓
[54]	✓			✓							✓	✓
[25]	✓			✓			✓					✓
[28]	✓									✓	✓	✓
[29]	✓			✓						✓	✓	✓
[30]	✓			✓						✓	✓	✓
[33]	✓			✓								✓
[34]	✓			✓						✓	✓	✓
[35]	✓			✓						✓	✓	✓
[43]		✓								✓	✓	✓
[44]	✓	✓						✓		✓	✓	✓
[41]	✓	✓								✓	✓	✓
[45]	✓	✓						✓	✓		✓	✓
[15]	✓	✓								✓	✓	✓
[52]	✓	✓	✓	✓	✓						✓	✓
This paper	✓	✓	✓	✓	✓				✓			✓

from each manufacturer. To carry out this task, a utility function (MNL model) that can specify the market share of each manufacturer is used. According to the production cost of each domestic manufacturer, the leader seeks to maximize the sum of manufacturers' profit by determining an appropriate price for its own manufacturers' products. In effect, the income of the domestic company depends on the quantity demanded by the retailer from this same company.

Indeed, this section describes the various features of the problem, followed by proposing its mathematical model. The main objective of a bi-level SC model is to create an SC network to maximize the profit of the domestic manufacturers by considering the possibility of manufacturer selection by the retailer.

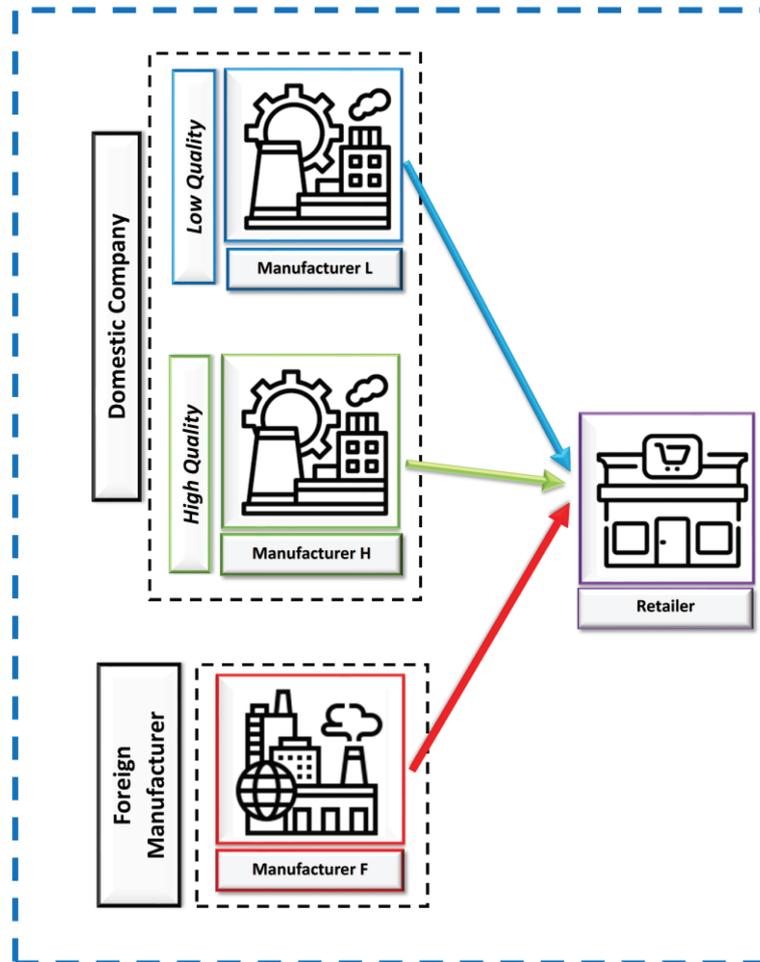


FIGURE 1. The proposed SC network.

It should be noted that the leader, as the level problem (ULP) decision-maker, has complete knowledge of the foreign manufacturer's product price and the retailer's demand for each manufacturer's product. Also, in this paper, the product price of the foreign manufacturer is considered as an exogenous factor. This is because we have assumed that the domestic company knows the price of foreign goods offered in the market to determine an appropriate price for their products.

More specifically, this paper primarily aims to determine a proper pricing strategy for domestic company manufacturers, so that they can compete with the foreign manufacturer and achieve the maximum share and profit of the retailer market. To this end, a mathematical model has been designed, using the concept of the Stackelberg game model based on bi-level programming. Also, to determine the share and the profit of each manufacturer, the MNL model has been used.

To maximize the profit of the domestic company, in view of the stated assumptions, a non-linear mixed integer-mathematical model is proposed as follows:

#### Sets

$N$  Set of domestic manufacturers ( $i, l \in \{1, 2\}$ )

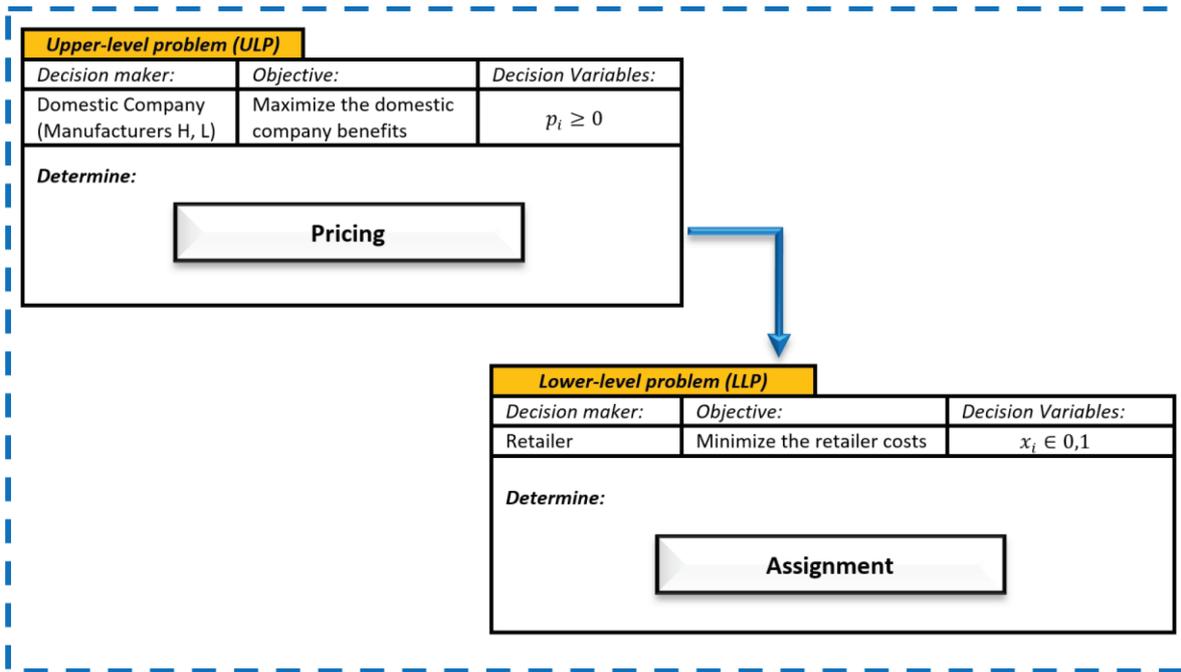


FIGURE 2. The perceptual structure of bi-level SC network.

**Parameters**

- $c_i$  Unit cost of producing one product by the  $i$ th domestic manufacturer
- $c_F$  Unit cost of producing one product by the foreign manufacturer
- $P_{max_i}$  Maximum price of the  $i$ th manufacturer's product due to government control policies
- $d$  Retailer demand
- $p_F$  The product price of the foreign manufacturer (imported product)
- $\theta_i$  Sensitivity parameter of the retailer to the product price of the  $i$ th domestic manufacturer
- $\theta_F$  Sensitivity parameter of the retailer to the price of the imported product
- $u_F$  Utility of the retailer from the foreign manufacturer

**Decision variables**

- $p_i$  Product price of the  $i$ th manufacturer
- $x_i$  1 if the retailer purchases from the  $i$ th manufacturer, 0 otherwise
- $u_i$  The utility of the retailer from the  $i$ th domestic manufacturer
- $M_i$  Market share of the  $i$ th domestic manufacturer
- $M_F$  Market share of the foreign manufacturer

In this paper, the retailer can decide to buy their products from whichever manufacturer that it desires. The retailer's sensitivity to the price of each manufacturer ( $\theta_i$  and  $\theta_F$ ) is considered as the factor affecting the retailer's decision. Therefore, the desirability of the domestic and foreign manufacturer for the retailer in the market is defined as follows:

$$u_i = -\theta_i \times p_i \quad \forall i \in N \tag{2}$$

$$u_F = -\theta_F \times p_F \tag{3}$$

where  $\theta_i$  and  $\theta_F$  are positive parameters.

Considering the factors contributing to the retailer's attraction to a manufacturer, the MNL function, which is applied to calculate the probability of the retailer's attraction to a manufacturer, is stated as:

$$M_i = \frac{\exp(u_i)}{\sum_{l=1}^N \exp(u_l) + \exp(u_F)} \quad \forall i \in N \quad (4)$$

$$M_F = \frac{\exp(u_F)}{\sum_{i=1}^N \exp(u_i) + \exp(u_F)} = 1 - \sum_{i=1}^N M_i. \quad (5)$$

Expressions (4) and (5) are utility functions. Indeed, these expressions calculate the retailer's desire toward the products of domestic and foreign manufacturers, respectively. In other words,  $M_i$  and  $M_F$  show the manufacturers' market share. It is important to mention that,  $M_i$  and  $M_F$  are measured by the retailer's sensitivity to the price of each manufacturer ( $\theta_i$  and  $\theta_F$ ).

It is also important to note that the costs of manufacturing each domestic product depend on some important expenses such as manufacturing technology, labor costs, setup costs, and R&D unit-related costs ( $c_i$ ). Apart from these expenses, to export its product to the domestic manufacturers' market, the foreign manufacturer must consider the costs of globalization fees such as international shipping costs, tariffs, and customs fees ( $c_F$ ). Hence, the product price of the foreign manufacturer is calculated as follows:

$$p_F = \alpha_F \times c_F \quad (6)$$

where  $\alpha_F$  is greater than 1.

In fact,  $\alpha_F$  is a coefficient that shows the marginal profit rate of the foreign manufacturer according to the costs of production and globalization fees of that product. For example, if  $X$  is equal to 1.1, it means the selling price of the product is 10% higher than the cost of making the product. Therefore, the marginal profit rate of the foreign manufacturer is 1.1.

Lastly, to find the optimal price for each domestic manufacturer and the retailer's optimal decision, the problem is formulated as:

$$Z = \text{Max}_p \left( \sum_{i=1}^N (p_i - c_i) \times M_i \times d \times x_i \right) \quad (7)$$

s.t.

$$p_i \leq P \max_i \quad \forall i \in N \quad (8)$$

$$p_i \geq 0 \quad \forall i \in N \quad (9)$$

$$W = \text{Min}_x \left( \sum_{i=1}^N p_i \times M_i \times d \times x_i + \left( d - \sum_{i=1}^N M_i \times d \times x_i \right) \times p_F \right) \quad (10)$$

s.t.

$$x_i \in \{0, 1\} \quad \forall i \in N. \quad (11)$$

Objective (7) and constraints (8) and (9) indicate the first level (ULP); objective (10) and constraints (11) depict the second level (LLP) of the model. The objective of the ULP ( $Z$ ) maximizes the gain of the domestic company. The objective function of the ULP is shaped by two sections. The first section involves the profits of Manufacturer L ( $i = 1$ ). Similarly, the second section includes the profits of Manufacturer H ( $i = 2$ ). It is important to note that, since the products' quality of Manufacturer H is better than that of Manufacturer L, the production cost of Manufacturer H would be greater than that of Manufacturer L ( $c_2 \geq c_1$ ). Constraint (8) guarantees that the price of the manufacturers' products cannot exceed a specific price. Indeed, this price ( $P \max_i$ ) is determined according to the governmental market control policy. Constraint (9) indicates that

product price is a positive and continuous variable. The second level objective function ( $W$ ) aims to minimize the retailer's charges. In fact, being a fraction of the lower-level problem (LLP) regarding each manufacturer's (H, L, and F) price and total customer demand ( $d$ ), the retailer decides how much of the demand to source from each manufacturer. Finally, constraint (11) shows that  $x_i$  is considered to be the binary variable of the second level.

#### 4. SOLUTION METHOD

When the decision of actors interacts and can affect each other, usually game theory approach is often implemented to design this competition. Indeed, game theory structure establishes a theoretical basis to track how actors affect each other and what through interaction, they can and are trying to acquire something [33]. Since, this research wants to study the interaction between components of SC and reach the optimal decision, use the game theory framework. In addition, based on the problem structure of this study (hierarchy decision between the domestic manufacturer and the retailer), this paper applied the Stackelberg game. Since the bi-level programming approach can design the Stackelberg game structure mathematically, this research implemented this programming method to develop the problem [52]. Therefore, this paper addresses a pricing model in a game theory framework based on a bi-level optimization problem (BOP) for a competitive SC network. BOP problems have been used in various applications and theories such as Stackelberg game, Hierarchical planning, SC, Pricing, Electricity, Transportation, Environmental Economics, and Machine Learning [55]. Many researchers have attempted to solve this problem. In general, approaches developed to solve this problem can be classified into two major groups [55].

**A. Classical approaches:** they consist of four types of approaches, including single-level reduction, descent methods, penalty function methods, and trust-region methods.

- The single-level reduction is used when the LLP is convex. By utilizing the Karush–Kuhn–Tucker (KKT) conditions in the LLP, the goal is to transform the BOP to a single-level problem and, afterward, handle the single-level formulation.
- A descent method in a bi-level optimization problem causes a decrease in the upper-level objective function for as long as it takes to maintain the feasibility of the new point. Since the feasibility of a point is assured only when its lower level is optimal, finding a proper the descent method can be extremely difficult. To solve the BOP, scientists have proposed several methods to approximately calculate the gradient of the objective of the ULP. To decide the orientation of descent, they have also considered formulating auxiliary programs [56].
- The penalty function methods generally involve the KKT conditions used for the LLP and then, a penalizing approach is applied to resolve the single-level formulation. In fact, in this method, a penalizing value is added to the objective function of the ULP to fulfill the LLP's optimization [57].
- In the trust-region method, the algorithm based on an iterative technique estimates a certain area of the objective function using a model function. In fact, the algorithm optimizes the current iteration and the radius of the trust region simultaneously, with the process continuing until convergence occurs [58].

**B. Evolutionary approaches:** apart from the above methods, BOPs have been addressed using some evolutionary algorithms. One of the more successful and popular methods is the nested evolutionary algorithm. The nested method uses two common approaches to solve the BOPs. One of them uses an evolutionary algorithm at the ULP and a classical algorithm at the LLP. The other one utilizes evolutionary algorithms at the ULP and LLP, simultaneously. Some examples of this method's successful application for solving the BOPs are found in the works of [33, 59–63]. Besides, the evolutionary approaches addressing BOPs include other ways such as single-level reduction, meta-modeling-based methods, and the auxiliary bi-level meta-model [55].

In this paper, applying an exact solution to address the proposed model involves the determination of the best strategy for the ULP, carried out by considering all LLP responses against the upper-level decisions. In other

TABLE 2. LLP's response scenarios.

Strategy 1 ( $W_1$ )	Strategy 2 ( $W_2$ )	Strategy 3 ( $W_3$ )	Strategy 4 ( $W_4$ )
$x_1 = 0$ and $x_2 = 0$	$x_1 = 1$ and $x_2 = 0$	$x_1 = 0$ and $x_2 = 1$	$x_1 = 1$ and $x_2 = 1$

words, we are faced with two decisions here. The first one is the pricing decision for two domestic manufacturers in the ULP. Then, in the LLP, we should determine which manufacturer the retailer buys their product from and how much of it. To solve this problem, the first single-level reduction has been applied. More specifically, since the LLP has a binary variable, it can be regarded as an integer programming challenge at this level. Consequently, the exact solution of the model can be denoted by straightforward enumeration in the LLP. In other words, to solve the problem, we examine all response scenarios for the LLP *vis-a-vis* the upper-level decisions. A mathematical proof is used to confirm that one of these scenarios will always provide the best answer (profit) to the upper level of the model. Finally, to address the single-level problem, the Lambert-W function has been used. It should be noted that the Lambert-W function is an exact method that is practical and commonly used in solving problems containing the MNL function.

#### 4.1. Reformulation of the bi-level competitive pricing problem

As the lower level has a binary variable, there are four response scenarios in LLP. Table 2 presents these response scenarios.

Proposition 1 shows that for each decision in the ULP, there is a specified response scenario at the LLP.

**Proposition 1.** *The retailer's optimal response strategies are as follows:*

- (A) *When  $p_1 \geq p_F$  and  $p_2 \geq p_F$ .  
The retailer's optimal response strategy is Strategy 1.*
- (B) *When  $p_1 \leq p_F \leq p_2$ .  
The retailer's optimal response strategy is Strategy 2.*
- (C) *When  $p_2 \leq p_F \leq p_1$ .  
The retailer's optimal response strategy is Strategy 3.*
- (D) *When  $p_1 \leq p_F$  and  $p_2 \leq p_F$ .  
The retailer's optimal response strategy is Strategy 4.*

*Proof.* See Appendix A. □

The optimal strategy is demonstrated in Figure 3.

Proposition 1 and Figure 3 show that when the product prices of the domestic manufacturers are higher than those of the foreign manufacturer (A), the retailer does not purchase from the domestic manufacturer. Noticeably, if the product prices of the domestic manufacturers are lower than those of the imported products (D), owing to the utility function of each manufacturer, the retailer decides on the quantity of purchase from each manufacturer. Finally, modes B and C show that if the prices of Manufacturers H or L are higher than those of the foreign manufacturer, the domestic manufacturer has no chance of selling its products to the retailer.

Regarding Proposition 1, to solve the original problem, we can decompose the original model into four single-level models as follows:

- (1) If the LLP selects the  $W_1$  strategy, the original model objective function would be as follows:

$$\begin{aligned}
 Z_1 &= (p_1 - c_1) \times M_1 \times d \times x_1 + (p_2 - c_2) \times M_2 \times d \times x_2 \\
 &= (p_1 - c_1) \times M_1 \times d \times 0 + (p_2 - c_2) \times M_2 \times d \times 0 = 0.
 \end{aligned} \tag{12}$$

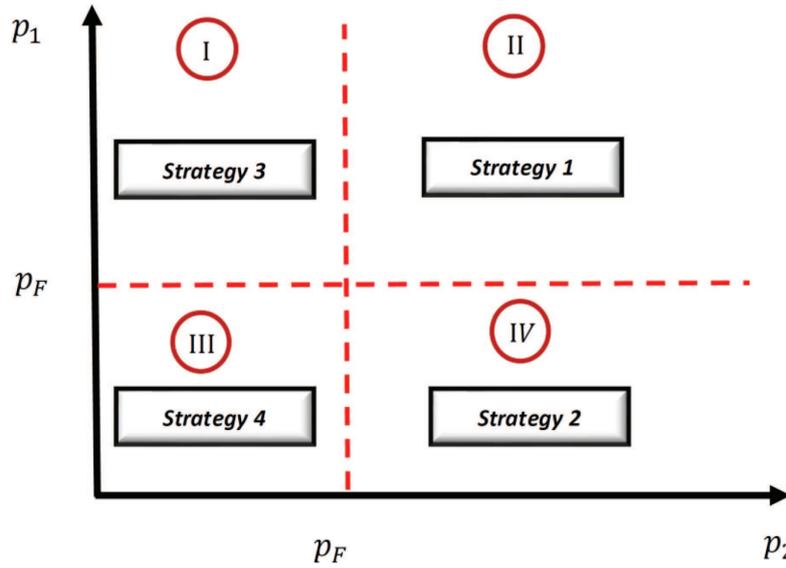


FIGURE 3. The retailer’s optimal response strategy *vis-a-vis* the pricing decision of each domestic manufacturer.

(2) If the LLP selects the  $W_2$  strategy, the original model would be as follows:

$$Z_2 = \text{Max}_p(p_1 - c_1) \times M_1 \times d. \tag{13}$$

Equations (8) and (9).

$$M_1 = \frac{\exp(u_1)}{\exp(u_1) + \exp(u_F)} \tag{14}$$

$$p_1 \leq p_F. \tag{15}$$

Note that constraint  $p_2 \geq p_F$  is not added to the above model since the retailer does not purchase from Manufacture H. Thus, practically, Manufacture H does not produce any product for the retailer.

(3) If the LLP selects the  $W_3$  strategy, the original model would be as follows:

$$Z_3 = \text{Max}_p(p_2 - c_2) \times M_2 \times d. \tag{16}$$

Equations (8) and (9).

$$M_2 = \frac{\exp(u_2)}{\exp(u_2) + \exp(u_F)} \tag{17}$$

$$p_2 \leq p_F. \tag{18}$$

Also, because the retailer does not purchase from Manufacture L, constraint  $p_1 \geq p_F$  must not be added to the above model. Practically, Manufacture L does not produce any product for the retailer.

(4) If the LLP selects the  $W_4$  strategy, the original model would be as follows:

$$Z_4 = \text{Max}_p \left( \sum_{i=1}^N (p_i - c_i) \times M_i \times d \right). \tag{19}$$

Equations (4), (8), (9), (15) and (18).

Since the ULP has complete information about the LLP response strategies, we can decompose the space problem into four smaller spaces based on the LLP response strategies, such that each response strategy for the LLP is added as a constraint to the upper-level objective function. As a result, to determine the best decision strategy for the BOP, we are faced with four single-level math problems ( $Z_1 - Z_4$ ).

Then, among these single-level problems, the model that would provide the most profit for the ULP should be selected. Indeed, that single-level model should identify the best leader decision and the best follower response strategy to the ULP decision. Proposition 2 shows that the most profit for the domestic company is always found in Space III (Fig. 3).

**Proposition 2.** *If the domestic company can search the entire decision space according to the model parameters and constraints, then the best profit of the domestic company will always be found in Space III. In other words:*

$$z_4 \geq z_j \quad \forall j = 1, 2, 3. \quad (20)$$

Proposition 2 shows that diversification in product quality can attract a wider range of customers. Obviously, absorbing more customers will increase the system's profit, too. Indeed, according to Proposition 2, to attract more customers, the domestic company should always seek to sell both of its products to be able to compete with the foreign manufacturer.

*Proof.* See Appendix A. □

## 4.2. Optimizing the single-level problem

Given that  $u_F$  is an exogenous and constant parameter, if  $\eta$  is defined as:

$$\eta = \exp(u_F) = \exp(-\theta_F \times p_F). \quad (21)$$

By using equations (19) and (21), the objective function of the single-level problem is as follows:

$$Z_4 = \text{Max}_p \left( \sum_{i=1}^N (p_i - c_i) \times \frac{\exp(u_i)}{\sum_{l=1}^N \exp(u_l) + \eta} \times d \right). \quad (22)$$

Therefore, by performing differentiation on expression (22) with respect to  $p_i$  and putting the expression equal to zero, the following expressions for the problem are obtained:

$$\frac{\partial Z_4}{\partial p_i} = 0 \quad (23)$$

$$\left[ \sum_{l=1}^N \exp(-\theta_l \times p_l) + \eta \right] \times [1 - \theta_i(p_i - c_i)] + \theta_i \left[ \sum_{l=1}^N (p_l - c_l) \times \exp(-\theta_l \times p_l) \right] = 0 \quad \forall i \in N \quad (24)$$

$$p_i^* = c_i + \frac{\left[ \sum_{l=1}^N \exp(-\theta_l \times p_l) + \eta \right] + \theta_i \left[ \sum_{l=1}^N (p_l - c_l) \times \exp(-\theta_l \times p_l) \right]}{\theta_i \times \left[ \sum_{l=1}^N \exp(-\theta_l \times p_l) + \eta \right]} \quad \forall i \in N. \quad (25)$$

According to the exponential behavior of the domestic manufacturers' market share, it is clear that expression (25) can only show a function of the price and cannot show the price quantitatively. However, it is possible to obtain an optimum solution by using the Lambert-W function [64]. The domestic manufacturers' optimum price at each manufacturer is stated in Proposition 3.

**Proposition 3.** *The optimal price of the domestic manufacturers is given by the following closed expression:*

$$p_i^* = c_i + \frac{\left\{ 1 + W \left[ \frac{1}{\eta} \times \left( \sum_{l=1}^N \exp(-1 - (\theta_l \times c_l)) \right) \right] \right\}}{\theta_i} \quad \forall i \in N \tag{26}$$

where the Lambert-W function can be defined as the inverse function of  $f(w) = we^w$  [48]. Therefore, based on Proposition 3, the optimal price and the objective function can be calculated if the proposed single level does not have any constraint. Given expressions 8, 15, and 18,  $p_i^*$  is not accurate if  $p_i^*$  exceeds the limitation of these expressions. Condition 1 can fix this problem.

**Condition 1.** The optimal price of the domestic manufacturers is calculated under this condition:

$$\begin{cases} \text{if } p_i^* \leq \text{Min}(p_F, P_{\text{max}_i}) \rightarrow p_i^* \text{ is acceptable} \\ \text{else } p_i^* > \text{Min}(p_F, P_{\text{max}_i}) \rightarrow p_i^* = \text{Min}(p_F, P_{\text{max}_i}). \end{cases}$$

Since in the MNL function, the objective function grows up to a certain level with an increase in the price only to decline later [64]. Hence, based on Condition1, if  $\text{Min}(p_F, P_{\text{max}_i})$  is smaller than  $p_i^*$ , the maximum value of the objective function is obtained where  $p_i^*$  is equal to  $\text{Min}(p_F, P_{\text{max}_i})$ .

Against this backdrop, in this section, first with the help of Propositions 1 and 2, the proposed bi-level problem is transformed into the single-level model. Then, by using the Lambert-W function, the optimal value of the price is obtained (Prop. 3). Finally, the optimum profit of the domestic manufacturers is calculated in accordance with their respective prices.

### 5. COMPUTATIONAL RESULTS

This section mainly aims to analyze the proposed model and the solution method. To offer a better understanding of the model behavior, a sensitivity analysis is conducted for different parameters of the proposed model so that we can offer some relevant managerial insights in this study at Section 6. Hence, this section analyzes the sensitivity of the cardinal parameters  $c_i$ ,  $p_F$ ,  $\theta_F$ , and  $\theta_i$ , to scrutinize the response of the proposed model and its far-reaching implications for the pricing policy, and total expected profit of the domestic company, and the foreign manufacturer.

Concerning the validation of the presented model, we run some tests. Given that in the literature there is no model sample, we have generated some random problems. For the experiments, we have used the following setting:  $d = 10\,000$ ,  $c_1 = 3$ ,  $c_2 = 5$ ,  $c_F = 6$ ,  $\theta_F = 0.5$ ,  $p_F = 20$ ,  $P_{\text{max}_1} = 25$ , and  $P_{\text{max}_2} = 30$ . It should be mentioned that the unit of all parameters which are related to the expense and price is the same (*i.e.*, Euro or Dollar, etc.). In addition, since the proposed model is a non-linear mixed integer, we have submitted the written model to MATLAB 2019a. The codes ran on a computer with the following configuration, a 2/90 GHz Intel Core i5 CPU and 8 GB RAM and Window10 OS.

#### 5.1. Sensitivity analysis on the $c_i$ parameter

In this part, the effects of different values of parameter  $c_i$  on the domestic company’s profit, the manufacturers’ profit (L and H), and the foreign manufacturer’s profit are examined. Hence, to determine the foreign manufacturer’s profit, we have stated equation (27) as follows:

$$\text{Foreign manufacturer profit} = (p_F - c_F)(u_F \times d). \tag{27}$$

We generated five sample problems with different values of  $\theta_i$  to examine the effect of parameter  $c_i$  on the aforementioned variables accurately and from the different angles; moreover, we have studied these issues in 6 different sizes of  $c_i$ . It should be noted that  $c_i$  is calculated based on equation (28).

$$c_{i(\text{new})} = c_i + \Theta. \tag{28}$$

TABLE 3. Analysis of the effect of  $c_i$  on the domestic company and the foreign manufacturer.

Data set	$\Theta$	Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
		$\theta_1 = 0.5$		$\theta_1 = 0.3$		$\theta_1 = 0.5$		$\theta_1 = 0.3$		$\theta_1 = 0.7$	
		$\theta_2 = 0.5$		$\theta_2 = 0.5$		$\theta_2 = 0.3$		$\theta_2 = 0.3$		$\theta_2 = 0.7$	
		Domestic company profit	Foreign manufacturer profit								
1	-2	137 681	17 757	177 492	1851	166 535	2340	178 367	1270	94 699	18 351
2	-1	128 989	18794	171 324	2087	156 924	2362	168 457	1270	86 065	19 930
3	0	120 369	19 947	165 269	2364	147 281	2382	158 548	1270	77 553	21 777
4	1	111 839	21 238	156 820	2474	137 604	2401	148 639	1270	691 84	23 961
5	2	103 407	22 689	146 996	2474	127 893	2419	138 730	1270	60 986	26 570
6	3	95 085	24 330	137 173	2474	118 150	2435	128 820	1270	52 993	29 727

Table 3 illustrates the value of  $\Theta$  in all 6 samples.

According to Table 3, as expected, with an increase in the manufacturing costs of each product unit, the profit of the domestic company decreases, resulting in an increased profit for the foreign manufacturer. To take a closer look at the results of Problems 1–5, you can refer to the appendix section (Tabs. A.1–A.5).

Figure 4 illustrates the trend of the domestic manufacturer’s profit (L) in Problems 1 and 2. The results indicate that if the market is more interested in the product of a specific manufacturer (lower  $\theta_i$ ), an increase in the costs of manufacturing that product, and, consequently, an increase in the price of that commodity has a far more devastating impact on the loss of the domestic manufacturer’s profit than when the market is less interested in that product. This is because the demand of the market is more sensitive to the price of that commodity.

Given that the behavior of profit reduction is linear, we used linear regression to show the accuracy of this claim (dotted line). As can be seen in Figure 4, the slope of the linear regression of Problem 2 is greater than that of Problem 1 ( $|-7826.1| > |-6229.7|$ ). Let us note that in Problems 1 and 2, the only difference is in parameter  $\theta_1$ . In Figures 5 and 6, we have examined the slope of the profit reduction for manufacturer H and the domestic company in Problems 1, 2 and 1, 4, and 5, respectively. The results reveal that the asserted claim is correct.

### 5.2. Sensitivity analysis on the $\theta_F$ parameter

This section deals with the profit of the domestic company, the foreign manufacturer’s profit, and the foreign manufacturer’s market share in view of the impact of patriotism on the purchase of domestic commodities. This study defines the patriotic behavior of the market as either the market’s tendency to buy domestic goods (less  $\theta_i$ ) or imported goods (more  $\theta_F$ ). Therefore, we have defined two problems with different values of parameter  $\theta_i$  (Problems 6 and 7) for accurately understanding the market behavior with respect to the above-mentioned conditions. We have also created 5 data sets with different values of  $\theta_F$ . The computational results of this data set are available in Tables 4 and 5. As expected, the market share of the domestic company increases when the market avoids purchasing the foreign commodity. On the other hand, this helps the domestic manufacturers raise the sale price of their products to retailers and, as a result, turn more profit. The profitability trend for the domestic and foreign companies in Problems 6 and 7 are depicted in Figures 7 and 8, respectively.

Figure 9 suggests that in addition to purchase restrictions of foreign commodities, if the market tends to show greater willingness to buy domestic commodities, the domestic company certainly makes more profit. In fact, as shown in Figure 9, in a scenario where the market is inclined towards local commodities (Problem 7), the

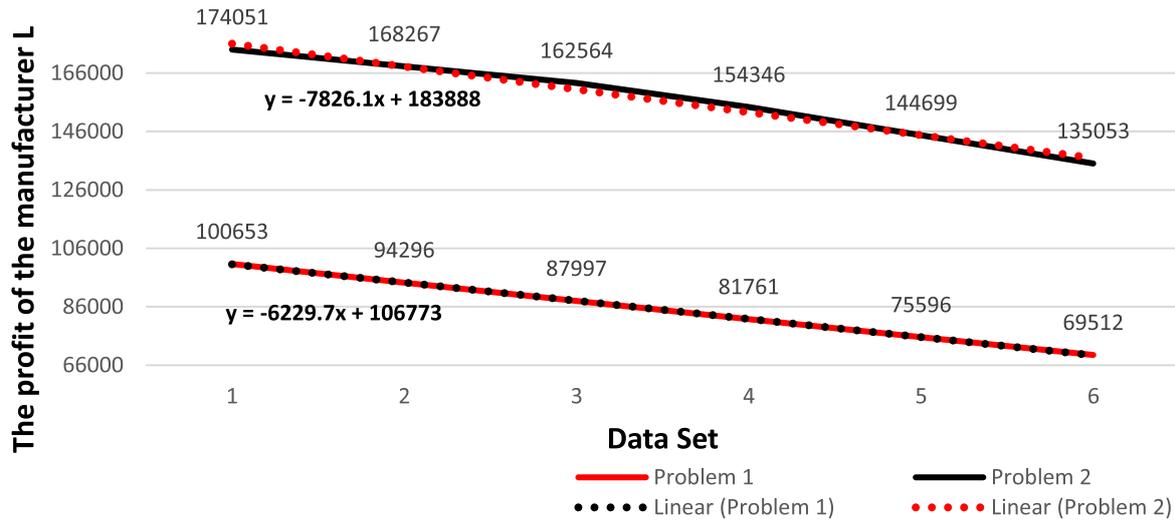


FIGURE 4. Consequence analysis of  $c_i$  on Manufacturer L's profit.

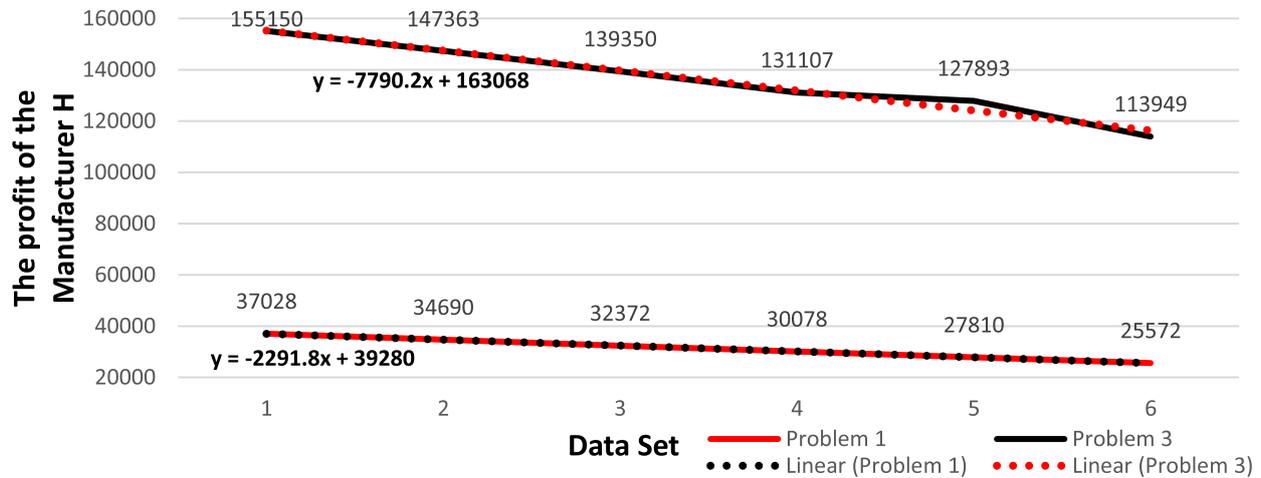


FIGURE 5. Consequence analysis of  $c_i$  on Manufacturer H's profit.

domestic company makes greater profit than otherwise (Problem 6), and the profit of the foreign manufacturer is much lower. As noted above, when a market refrains from buying foreign goods, the market share of the foreign manufacturer starts shrinking. However, this decline in the market share is even more severe when the market shows a greater tendency toward purchasing domestic products. Indeed, if the market shows a greater preference for domestic commodities, the foreign manufacturer's product becomes less competitive than the domestic one's. Figure 10 illustrates the accuracy of this claim ( $|-0.0574| > |-0.044|$ ).

### 5.3. Sensitivity analysis of the $\theta_i$ parameter

In this section, to investigate the impact of the  $\theta_i$  parameter on the proposed model, we examined the market in two different scenarios: when customers have a low average income, and when they enjoy a high average income. Considering the two different conditions, we have also analyzed and evaluated the market share

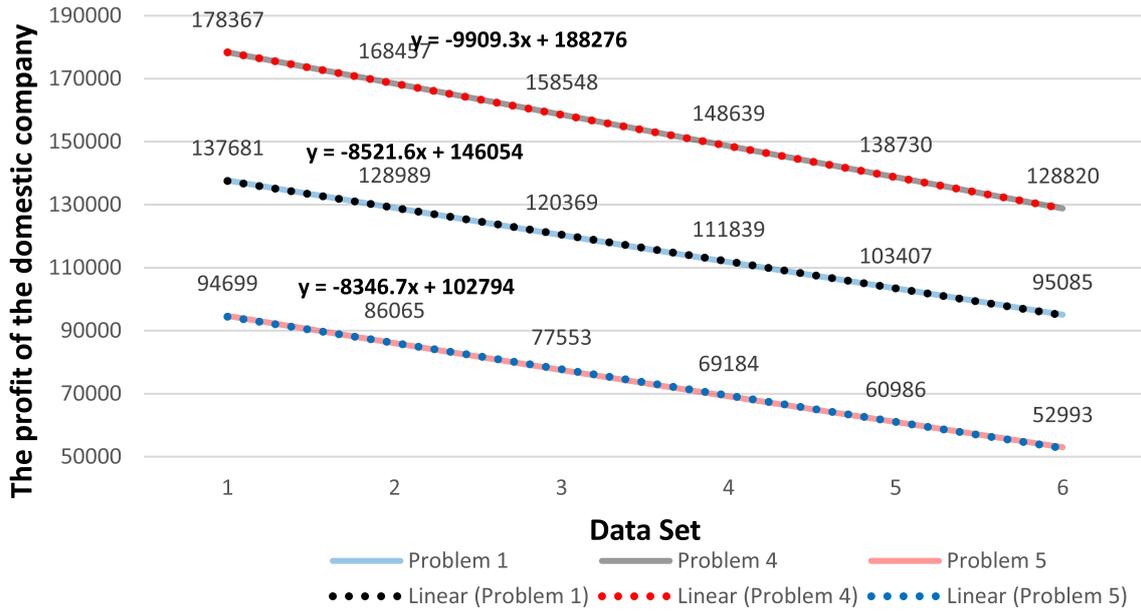


FIGURE 6. Consequence analysis of  $c_i$  on the domestic company’s profit.

TABLE 4. Analysis of the effect of  $\theta_F$  on the domestic company and the foreign manufacturer (Problem 6).

Data set	$\theta_F$	$\theta_i$	Domestic company			Foreign manufacturer			
			Manufacturer L profit	Manufacturer H profit	Total profit	$p_1$	$p_2$	$u_F$	Profit
1	0.3	0.4	52 688	23 674	76 362	13.14	15.14	0.24664	34 530
2	0.35	0.4	66 041	29 674	95 715	15.07	17.07	0.2071	28 994
3	0.4	0.4	79 986	35 940	115 926	17.09	19.09	0.1774	24 836
4	0.45	0.4	81 509	54 413	135 922	19.18	20	0.13345	18 683
5	0.5	0.4	79 613	70 247	149 860	20	20	0.063379	8873

TABLE 5. Analysis of the effect of  $\theta_F$  on the domestic company and the foreign manufacturer (Problem 7).

Data set	$\theta_F$	$\theta_i$	Domestic company			Foreign manufacturer			
			Manufacturer L profit	Manufacturer H profit	Total profit	$p_1$	$p_2$	$u_F$	Profit
1	0.3	0.3	71 743	39 373	111 115	17.44	19.44	0.23076	32 307
2	0.35	0.3	71 794	63 348	135 142	20	20	0.15536	21 751
3	0.4	0.3	79 613	70 247	149 860	20	20	0.063379	8873
4	0.45	0.3	82 935	73 178	156 113	20	20	0.024289	3400
5	0.5	0.3	84 229	74 319	158 548	20	20	0.009075	1270

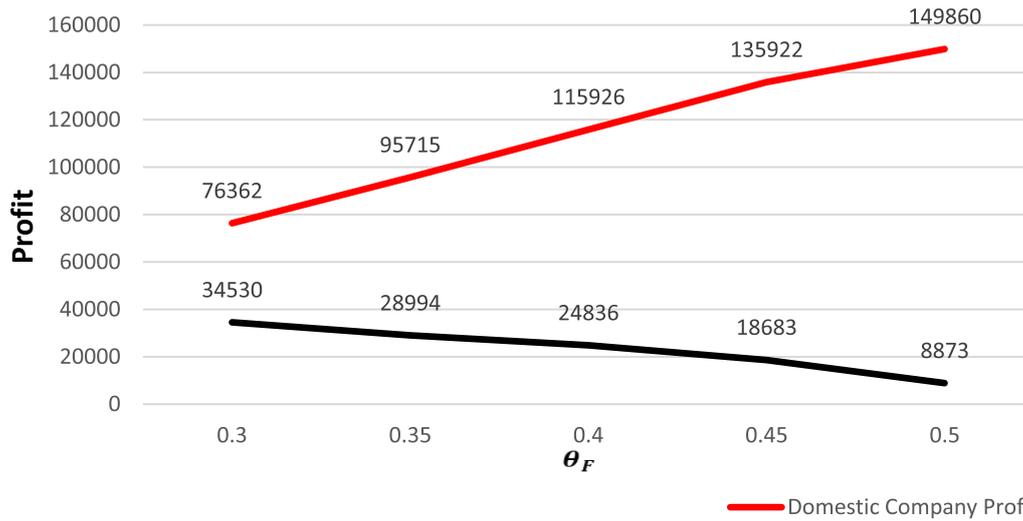


FIGURE 7. Effect analysis of  $\theta_F$  on the profit of the domestic company and the foreign manufacturer (Problem 6).

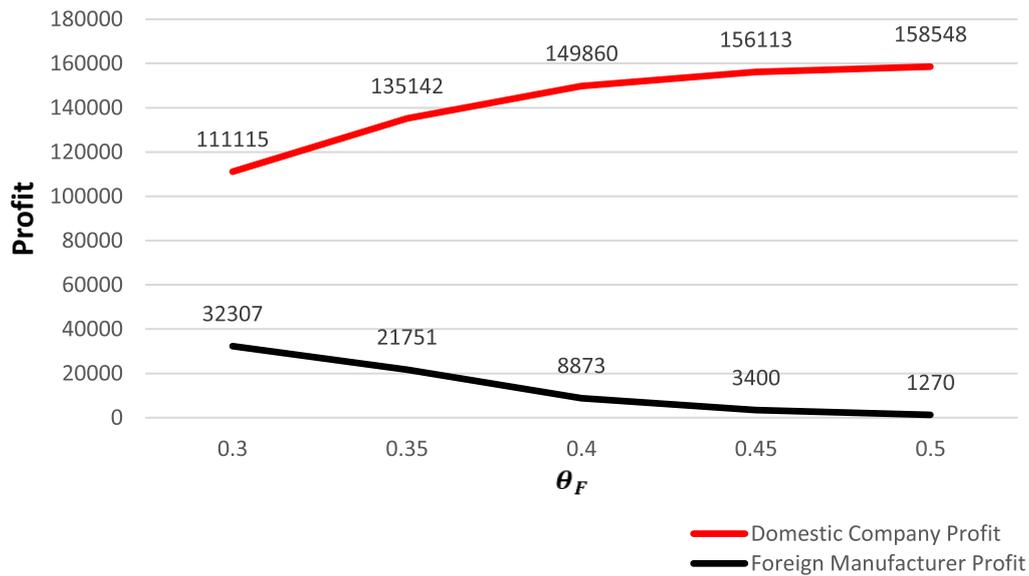


FIGURE 8. Effect analysis of  $\theta_F$  on the profit of the domestic company and the foreign manufacturer (Problem 7).

profitability of each manufacturer. To this end, we have investigated Problem 1 in two different modes. Results of Tables 6 and 7 refer to customers with a low average income and a high average income, respectively. As seen here, we have examined Problem 1 for five different data sets. In Tables 6 and 7, parameter  $\theta_1$  and  $\theta_2$  have also changed, respectively.

As expected and shown in Table 6, with a decrease in the average income level of the community, the market gradually tends to buy lower-quality goods. In other words, the retailer’s tendency to buy from Manufacturer L

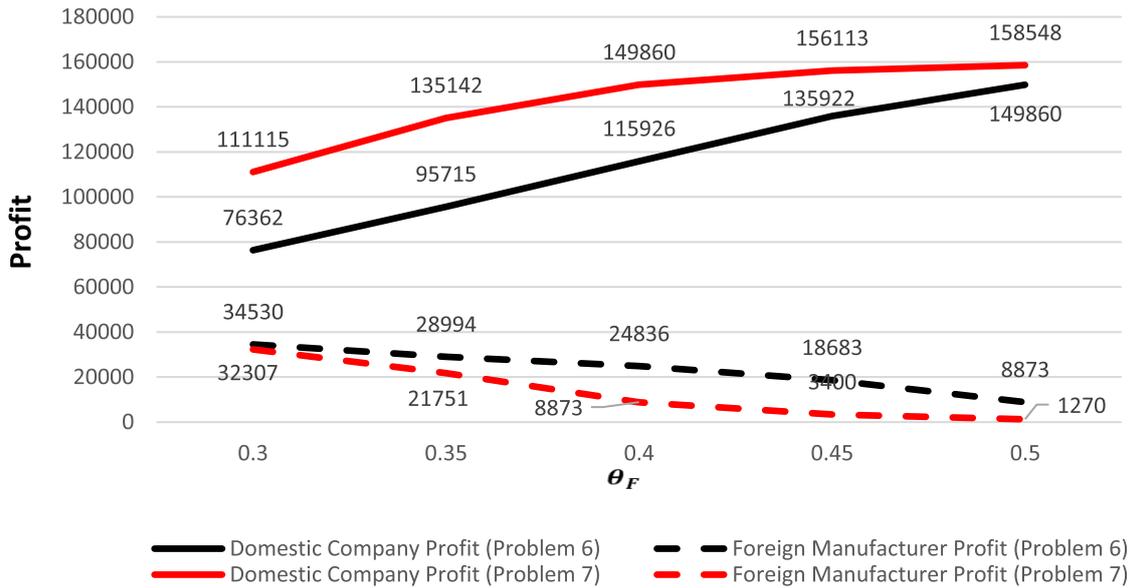


FIGURE 9. Effect analysis of  $\theta_F$  on the profit of the domestic company and the foreign manufacturer (Problem 6 and 7).

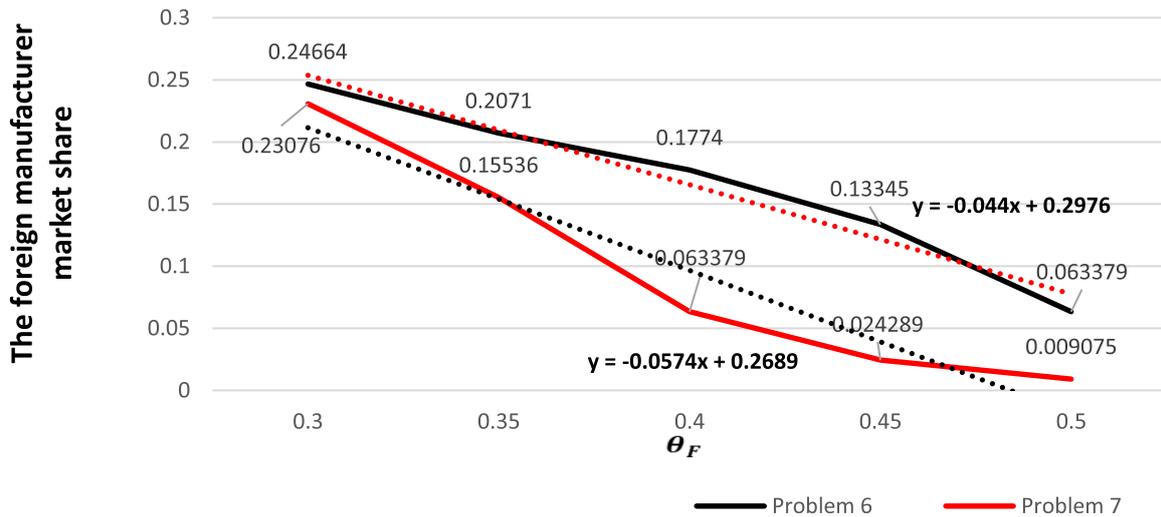


FIGURE 10. Effect analysis of  $\theta_F$  on the foreign manufacturer’s market share (Problems 6 and 7).

increases according to the market demand. As a result, the market share of Manufacturer L increases, in turn increasing this manufacturer’s profitability. It is also clear that as the Manufacturer L’s profitability increases, the profitability of Manufacturer H and the foreign manufacturer decrease (Fig. 11a) – an adverse correlation, in fact. The same thing applies to Manufacturer H when the average income level of the community increases. Figure 11b substantiate the validity of the results.

Furthermore, in a situation where the consumers’ economic condition dictates market behavior, the analysis findings show that when the retailer sources the product from one particular domestic manufacturer, that

TABLE 6. Analysis of the effect of  $\theta_1$  on the domestic company and the foreign manufacturer.

Data set	$\theta_1$	$\theta_2$	Domestic company						Foreign manufacturer		
			Manufacturer L profit	Manufacturer H profit	Total profit	$u_1$	$u_2$	$p_1$	$p_2$	$u_{co}$	Profit
1	0.6	0.5	65 022	38 747	103 769	0.571	0.283	14.40	18.68	0.146	20 472
2	0.55	0.5	75 489	35 491	110 980	0.599	0.256	15.59	18.85	0.144	20 212
3	0.5	0.5	87 997	32 372	120 369	0.627	0.231	17.03	19.03	0.142	19 947
4	0.45	0.5	103 199	29 409	132 608	0.653	0.207	18.80	19.23	0.140	19 678
5	0.40	0.5	131 325	16 528	147 853	0.799	0.115	19.42	19.42	0.086	12 050

TABLE 7. Analysis of the effect of  $\theta_2$  on the domestic company and the foreign manufacturer.

Data set	$\theta_1$	$\theta_2$	Domestic company						Foreign manufacturer		
			Manufacturer L profit	Manufacturer H profit	Total profit	$u_1$	$u_2$	$p_1$	$p_2$	$u_{co}$	Profit
1	0.5	0.6	98 930	16 132	115 062	0.731	0.140	16.53	16.53	0.129	21 117
2	0.5	0.55	92 745	24 156	116 901	0.666	0.191	16.93	17.66	0.143	20 098
3	0.5	0.5	87 997	32 372	120 369	0.627	0.231	17.03	19.03	0.142	19 947
4	0.5	0.45	74 535	51 950	126 485	0.526	0.346	17.16	20	0.128	17 837
5	0.5	0.40	44 881	90 691	135 572	0.314	0.605	17.31	20	0.081	11 455

manufacturer increases its product price in comparison to other competitors. For example, assume that the average income level of a community is improving (*i.e.*,  $\theta_2$  decreases gradually). In this case, the slope in the price of Manufacturer H's products is higher than that in the price of Manufacturer L's products. The accuracy of this statement is depicted for both market conditions under consideration in Figure 12.

#### 5.4. Model analysis in the presence of one domestic manufacturer

This part aims to show how a domestic company with two manufacturers can make more profit compared to a domestic company with only one manufacturer that produces goods with a quality on a par with that of imported ones. Hence, in this part, the domestic company's profit is investigated in two different scenarios: (1) with two domestic manufacturers (two different types of product quality) and (2) one domestic manufacturer (one level of product quality).

To this end, we have examined Problems 1, 2, and 3 in the two above-mentioned modes. We selected these samples because we can properly investigate each aspect of this segment with them. As seen in Table 8, Problem 1 depicts a state where the manufacturers have the same conditions in terms of sensitivity to the product price. Problem 2 also shows a state where the domestic manufacturer with a low-quality product has more desirable conditions than the other manufacturers in the aforementioned parameters. In contrast, Problem 3 represents a state where the domestic manufacturer with a high-quality product has more favorable conditions than the other manufacturers in the above-mentioned parameters.

It should be noted that in this section  $\theta D$  (the sensitivity parameter of the retailer to the product price of the domestic manufacturer) and  $cD$  (the unit cost of producing one product by the domestic manufacturer) for the domestic manufacturer in a mode with one manufacturer is calculated through the averaging method. For

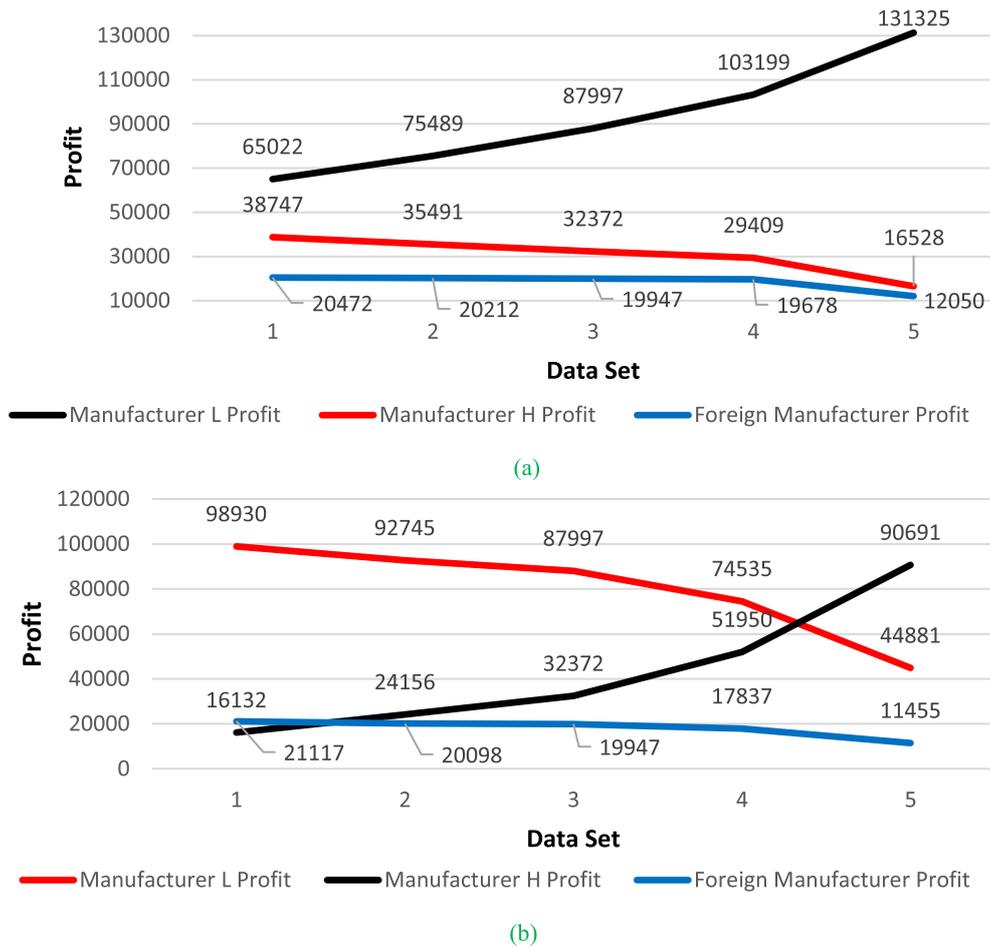


FIGURE 11. Analysis of the effect of  $\theta_i$  on the profit of Manufacturer L, Manufacturer H, and the foreign manufacturer. (a) Customers' low income. (b) Customers' high income.

example, the parameters mentioned in Problem 2 are calculated as follows:

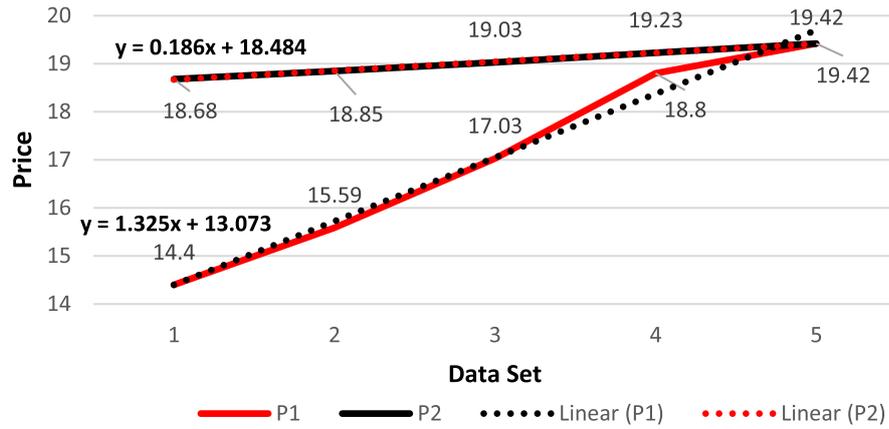
$$\theta D = \frac{0.3 + 0.5}{2} = 0.4 \quad (29)$$

$$cD = \frac{3 + 5}{2} = 4. \quad (30)$$

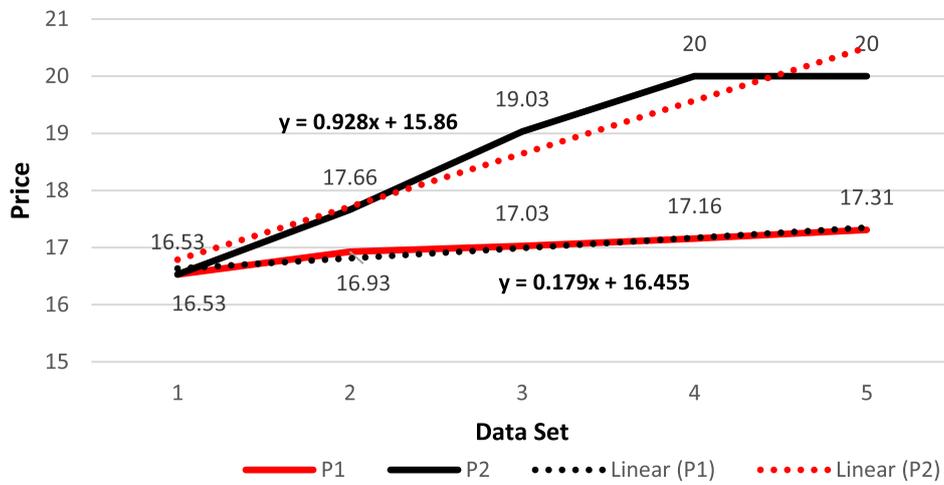
As illustrated in Figure 13, the domestic company's profit in all instances improves when the company decides to produce products with different qualities. Also, the foreign manufacturer's profit significantly decreases when the domestic company has two manufacturers (Problem 2 and 3).

## 6. MANAGERIAL INSIGHTS

This study can be used in various applications in the real world especially in the auto parts industry and electronic home appliances industry. The reason behind this is that in these industries the foreign and domestic manufacturers compete to acquire more profit from the retail market, as a result, the optimal pricing strategy



(a)



(b)

FIGURE 12. Analysis of the effect of  $\theta_i$  on the price of Manufacturer L and Manufacturer H. (a) Customers' low income. (b) Customers' high income.

TABLE 8. Comparison between the domestic company with two manufacturers and one manufacturer.

Instances	Two domestic manufacturers		One domestic manufacturer	
	Domestic company profit	Foreign manufacturer profit	Domestic company profit	Foreign manufacturer profit
Problem 1	120 369	19 947	106 544	22 127
Problem 2	165 269	2364	140 928	16 688
Problem 3	147 281	2382	140 928	16 688

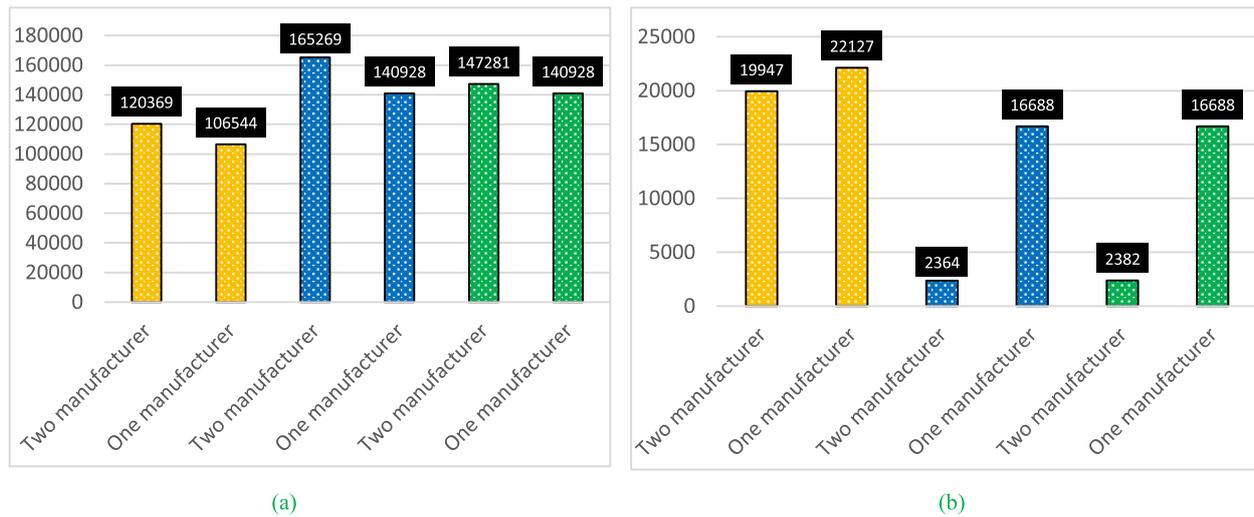


FIGURE 13. Comparison of the domestic company with two manufacturers and one manufacturer. (a) Domestic company profit. (b) Foreign manufacturer profit.

for the domestic manufacturer to keep their market share would be inevitable [65]. Hence, this study can help domestic manufacturer by determining the best competitive pricing strategy by identifying customer preferences. In addition, our result reveals the fact that one way by which a domestic company can enhance its competitive advantage is to increase the level of response to customer preferences by producing products of different qualities. In other words, by producing products with different qualities, the domestic company can meet the needs of high-income customers as well as low-income customers (Fig. 13). Following are some important points in which managers and beneficiaries can benefit from the results of this study.

### 6.1. Managerial decisions on the cost parameter

Due to analysis of Section 5.1, with an increase in the manufacturing costs of each domestic product, the domestic company becomes less competitive in pricing products against the foreign manufacturer. As a result, to maintain the profit, the domestic company increases the price of its products; this price rise decreases the desirability of the products, and as its utility function declines, so, too, does its profit. Therefore, the results of Section 5.1 indicate that domestic companies need to upgrade their manufacturing technology to maintain the profitability of their products and their market share and they can manufacture products at a lower cost. Thus, they will be able to increase the pricing ability of their products in today's competitive environment. Practically, the obtained results reveal that in case of purchasing manufacturing technology, companies should shift their perspective from a cost-oriented one to an investment-oriented one. Of course, it is crucial to bear in mind that the decision to increase the level of technology for the production depends on several factors such as the size of the company, market needs, and investment costs; therefore, given these factors, a company should choose the optimum level of technology.

### 6.2. Managerial decisions on price sensitivity parameter

It is important to note that in analyzing the results of Section 5.3, we hit upon an interesting managerial implication. In a market where the average income is low, customers prefer to buy products of Manufacturer L, thus the market share of Manufacturer L increases (Fig. 14). This phenomenon leads to a monopoly in the market and results in Manufacturer L increasing the price of its goods to increase profitability. At some point in time, the price of Manufacturer L's products is even likely to be close to that of Manufacturer H's products.

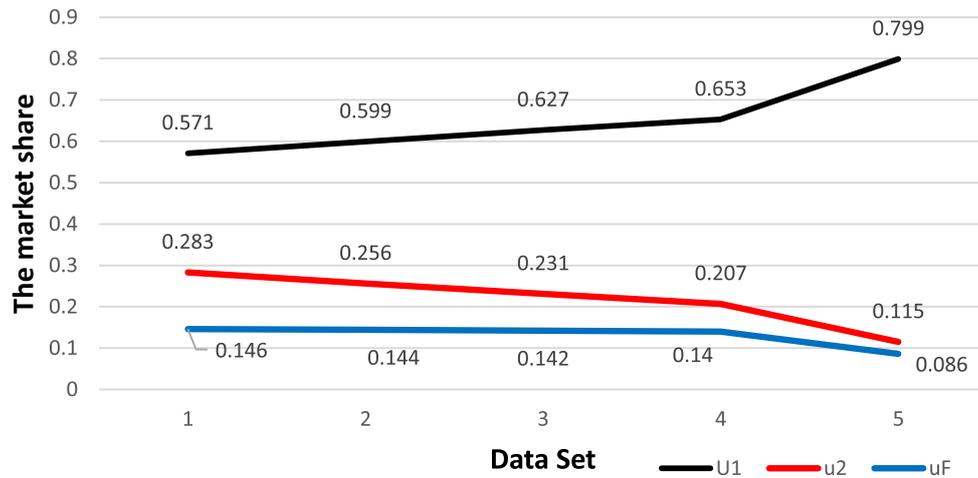


FIGURE 14. Consequence analysis of  $\theta_1$  on the market share of Manufacturer L, Manufacturer H, and the foreign manufacturer.

Hence, one can conclude that an excessive tendency toward a certain manufacturer's product can be detrimental to the consumers of a society. Based on the results, to prevent market monopolization, retailers should control the level of utility ( $\theta_i$  and  $\theta_F$ ) and not allow it to exceed a certain level. As a result, a competitive environment between the manufacturers for the sale of the product prevails in the market. In other words, this practice prevents an aggressive selling price of a commodity relative to its value (the quality of that commodity) since it prevents market monopolization.

## 7. CONCLUSIONS

The competitive pricing problem in the SC plays an instrumental role in the pricing of manufacturers' commodities. The competitive environment governing the market forces manufacturers to price their products properly to maximize their market share and profit. Against this backdrop, we developed a bi-level model considering the competition between a domestic company and foreign manufacturer for selling their products to a retailer.

Taking the upper level of the model into consideration, with the participation of two different manufacturers (low-quality manufacturer and high-quality manufacturer), the domestic company seeks to maximize its total profit and compete with the foreign manufacturer through the pricing of its goods. Indeed, with suitable pricing strategies for low-quality and high-quality goods, it seeks to increase its market share and profit. Subsequently, considering the selling price of the domestic and foreign manufacturers' products as well as the needs of the market, the retailer at the lower level seeks to minimize the purchasing cost. In fact, the upper level and the lower level of the model are in conflict with each other. Hence, the proposed model acts as a Stackelberg game. The principal innovation of this study is employing a bi-level modeling proposal to capture the competition among the domestic company and the foreign manufacturer. To render the presented study more realistic, the concept of the utility function is also used to express the market share of each manufacturer.

To solve the proposed model, a new hybrid exact method based on the explicit enumeration method and Lambert-W function is used. Considering all response scenarios of the lower level, the solution space of the bi-level model is divided into four sub-single-level models. The optimal answer can always be obtained from one of the sub-problems. Then, to exactly solve this single-level model, the Lambert-W function is applied. To validate the presented model, sensitivity analysis is carried out on three main parameters of the model (the unit cost of producing one product, sensitivity parameter of the retailer to the price of the imported product,

and sensitivity parameter of the retailer to the product price of the domestic manufacturer). The analysis of the sensitivity of the model confirms the satisfactory performance of the model. Also, the results show that one alternative that the domestic company can increase its profit and competitive advantage is to produce two different types of quality (high and low) by its own manufacturers.

We studied the model for three different situations in the market. The results indicate that when the average income level of a community is low, customers' preference is to buy a low-quality product, and *vice versa*. Furthermore, when customers have patriotic sentiments toward a domestic product, the domestic manufacturer stands a better chance of selling its product than the foreign manufacturer. In fact, an interesting inference that can be drawn from these results is that the first step of any manufacturer or company before market entry is to determine the exact behavior of its prospective customers based on the competitive condition of the market. In other words, before entering a new market or pricing its commodities, a manufacturer or company must first study its customers' demand for such commodities and the competing manufacturers' commodities. This knowledge can go a long way toward gleaning actionable insight into the dynamics (in terms of demands and tastes) of a market where a company is dreaming of establishing a firm foothold and prices its commodities accordingly.

In this study, the utility function only depends on the selling price of commodities. Suggestions for future research include considering variables such as the product's quality and delivery time. In fact, instead of considering it as an exogenous factor, product quality should be considered as a decision variable in the problem. Furthermore, to render the situation more realistic, it should consider competition among domestic manufacturers, too. According to the proposed suggestion, concepts such as Nash equilibrium and contracts between domestic manufacturers could be added to the problem. In the other hand scraped, defective and reworkable items can be another topic for future research which is worth of consideration [66–68]. In addition, stochastic inventory systems with uncertain demand or period length could be worth of investigations [69, 70].

## APPENDIX A.

*Proof of Proposition 1.* First, we calculate the objective function of the LLP for each response scenarios. That is as follows:

*For Strategy 1:*

$$\begin{aligned} W_1 &= p_1 \times M_1 \times d \times 0 + p_2 \times M_2 \times d \times 0 + (d - (M_1 \times d \times 0 + M_2 \times d \times 0)) \times p_F \\ &= d \times p_F. \end{aligned} \quad (\text{A.1})$$

*For Strategy 2:*

$$\begin{aligned} W_2 &= p_1 \times M_1 \times d \times 1 + p_2 \times M_2 \times d \times 0 + (d - (M_1 \times d \times 1 + M_2 \times d \times 0)) \times p_F \\ &= p_1 \times M_1 \times d + d \times p_F - M_1 \times d \times p_F = u_1 \times d \times (p_1 - p_F) + d \times p_F. \end{aligned} \quad (\text{A.2})$$

To simplify above formulation, let;

$$\varphi = M_1 \times d. \quad (\text{A.3})$$

It should be noted that  $\varphi$  is always positive.

$$\Rightarrow W_2 = \varphi \times (p_1 - p_F) + d \times p_F. \quad (\text{A.4})$$

For Strategy 3:

$$\begin{aligned} W_3 &= p_1 \times M_1 \times d \times 0 + p_2 \times M_2 \times d \times 1 + (d - (M_1 \times d \times 0 + M_2 \times d \times 1)) \times p_F \\ &= p_2 \times M_2 \times d + d \times p_F - M_2 \times d \times p_F = M_2 \times d \times (p_2 - p_F) + d \times p_F. \end{aligned} \quad (\text{A.5})$$

To simplify above formulation, let;

$$\varphi' = M_2 \times d. \quad (\text{A.6})$$

Note that  $\varphi'$  is always positive.

$$\Rightarrow W_3 = \varphi' \times (p_2 - p_F) + d \times p_F. \quad (\text{A.7})$$

For Strategy 4:

$$\begin{aligned} W_4 &= p_1 \times M_1 \times d \times 1 + p_2 \times M_2 \times d \times 1 + (d - (M_1 \times d \times 1 + M_2 \times d \times 1)) \times p_F \\ &= p_1 \times \varphi + p_2 \times \varphi' + (d - (\varphi + \varphi')) \times p_F \\ &= \varphi \times (p_1 - p_F) + \varphi' \times (p_2 - p_F) + d \times p_F. \end{aligned} \quad (\text{A.8})$$

Next, according to the obtained mathematical equations, the retailer chooses the strategy depending on its objective function, which must be less than the objective functions of the other strategies. In fact, the best strategy is determined by the LLP according to each decision space of the ULP. As a result, we have:

(i) For  $W_1$ , we have:

(a)  $W_1$  is smaller than  $W_2$  if:

$$d \times p_F \leq \varphi \times (p_1 - p_F) + d \times p_F \Rightarrow \varphi \times (p_1 - p_F) \geq 0 \Rightarrow p_1 \geq p_F. \quad (\text{A.9})$$

(b)  $W_1$  is smaller than  $W_3$  if:

$$d \times p_F \leq \varphi' \times (p_1 - p_F) + d \times p_F \Rightarrow \varphi' \times (p_2 - p_F) \geq 0 \Rightarrow p_2 \geq p_F. \quad (\text{A.10})$$

(c)  $W_1$  is smaller than  $W_4$  if:

$$\begin{aligned} d \times p_F &\leq \varphi \times (p_1 - p_F) + \varphi' \times (p_2 - p_F) + d \times p_F \\ &\Rightarrow \varphi \times (p_1 - p_F) + \varphi' \times (p_2 - p_F) \geq 0. \end{aligned} \quad (\text{A.11})$$

To establish this equation there are two modes:

If  $p_1 \geq p_F$  and  $p_2 \geq p_F$ , the above equation would always be correct. Otherwise, the following equation should be established:

$$\begin{aligned} &\Rightarrow \frac{\varphi \times p_1 + \varphi' \times p_2}{\varphi + \varphi'} \geq p_F \\ &\Rightarrow \frac{\varphi}{\varphi + \varphi'} \times p_1 + \frac{\varphi'}{\varphi + \varphi'} \times p_2 \geq p_F. \end{aligned} \quad (\text{A.12})$$

To simplify the above formulation, let;

$$\alpha = \frac{\varphi}{\varphi + \varphi'}, \quad \beta = \frac{\varphi'}{\varphi + \varphi'}. \quad (\text{A.13})$$

Note that  $\alpha$  and  $\beta$  are always positive. Then, the equation (A.12) can be re-written as follows:

$$\alpha \times p_1 + \beta \times p_2 \geq p_F. \quad (\text{A.14})$$

Therefore:

$$\begin{cases} \text{(a) if } p_1 \geq p_F \Rightarrow W_1 \leq W_2 \\ \text{(b) if } p_2 \geq p_F \Rightarrow W_1 \leq W_3 \\ \text{(c) if } (p_1 \geq p_F \text{ and } p_2 \geq p_F) \text{ or } \alpha \times p_1 + \beta \times p_2 \geq p_F \Rightarrow W_1 \leq W_4 \\ \Rightarrow \text{ if } p_1 \geq p_F \text{ and } p_2 \geq p_F \Rightarrow W_1 \leq W_j \end{cases} \quad \forall j = 2, 3, 4. \quad (\text{A.15})$$

(ii) For  $W_2$ , we have:

(a)  $W_2$  is smaller than  $W_1$  if:

According to the mathematical equation between  $W_1$  and  $W_2$  obtained in Section i, we have:

$$\text{if } p_1 \leq p_F \Rightarrow W_2 \leq W_1. \quad (\text{A.16})$$

(b)  $W_2$  is smaller than  $W_3$  if:

$$\varphi \times (p_1 - p_F) + d \times p_F \leq \varphi' \times (p_2 - p_F) + d \times p_F \Rightarrow \varphi \times (p_1 - p_F) \leq \varphi' \times (p_2 - p_F). \quad (\text{A.17})$$

To establish equation (A.17) there are two modes:

If  $p_1 \leq p_F$  and  $p_2 \geq p_F$ , the above equation would be always correct. Otherwise, the following equation should be established:

$$\frac{p_1 - p_F}{p_2 - p_F} \leq \frac{\varphi'}{\varphi}. \quad (\text{A.18})$$

(c)  $W_2$  is smaller than  $W_4$  if:

$$\varphi \times (p_1 - p_F) + d \times p_F \leq \varphi \times (p_1 - p_F) + \varphi' \times (p_2 - p_F) + d \times p_F \Rightarrow \varphi' \times (p_2 - p_F) \geq 0. \quad (\text{A.19})$$

According to the above formulation if  $p_2 > p_F$ ,  $W_2$  would be always smaller than  $W_4$ .

Therefore:

$$\begin{cases} \text{(a) if } p_1 \leq p_F \Rightarrow W_2 \leq W_1 \\ \text{(b) if } (p_1 \leq p_F \text{ and } p_2 \geq p_F) \text{ or } \frac{p_1 - p_F}{p_2 - p_F} \leq \frac{\varphi'}{\varphi} \Rightarrow W_2 \leq W_3 \\ \text{(c) if } p_2 \geq p_F \Rightarrow W_2 \leq W_4 \\ \Rightarrow \text{ if } p_1 \leq p_F \leq p_2 \Rightarrow W_2 \leq W_j \end{cases} \quad \forall j = 1, 3, 4. \quad (\text{A.20})$$

(iii) For  $W_3$ , we have:

(a)  $W_3$  is smaller than  $W_1$  if:

According to the mathematical equation between  $W_1$  and  $W_3$  obtained in Section i, we have:

$$\text{if } p_2 \leq p_F \Rightarrow W_3 \leq W_1. \quad (\text{A.21})$$

(b)  $W_3$  is smaller than  $W_2$  if:

According to the mathematical equation between  $W_2$  and  $W_3$  calculated in Section ii, we have two mathematical equations:

If  $p_1 \geq p_F$  and  $p_2 \leq p_F$ ,  $W_3$  would be always smaller than  $W_2$ . Otherwise, the following equation should be established:

$$\frac{p_2 - p_F}{p_1 - p_F} \geq \frac{\varphi}{\varphi'}. \quad (\text{A.22})$$

(c)  $W_3$  is smaller than  $W_4$  if:

$$\varphi \times (p_2 - p_F) + d \times p_F \leq \varphi \times (p_1 - p_F) + \varphi' \times (p_2 - p_F) + d \times p_F \Rightarrow \varphi' \times (p_1 - p_F) \geq 0. \quad (\text{A.23})$$

According to the above formulation if  $p_1 \geq p_F$ ,  $W_3$  would be always smaller than  $W_4$ .

TABLE A.1. Computational results for the problem 1.

Data set	$c_1$	$c_2$	$u_1$	$u_2$	$p_1$	$p_2$	Manufacturer L profit	Manufacturer H Profit
1	1	3	0.63833	0.23483	16.77	18.77	100 653	37 028
2	2	4	0.63292	0.23284	16.90	18.90	94 296	34 690
3	3	5	0.6269	0.23062	17.04	19.04	87 997	32 372
4	4	6	0.62016	0.22814	17.18	19.18	81 761	30 078
5	5	7	0.61258	0.22536	17.34	19.34	75 596	27 810
6	6	8	0.60401	0.2222	17.50	19.50	69 512	25 572

Therefore:

$$\begin{cases} \text{(a) if } p_2 \leq p_F \Rightarrow W_3 \leq W_1 \\ \text{(b) if } (p_2 \leq p_F \text{ and } p_1 \geq p_F) \text{ or } \frac{p_2 - p_F}{p_1 - p_F} \geq \frac{\varphi}{\varphi'} \Rightarrow W_3 \leq W_2 \\ \text{(c) if } p_1 \geq p_F \Rightarrow W_3 \leq W_4 \end{cases} \Rightarrow \text{if } p_2 \leq p_F \leq p_1 \Rightarrow W_3 \leq W_j \quad \forall j = 1, 2, 4. \quad (\text{A.24})$$

(iv) For  $W_4$ , we have:

(a)  $W_4$  is smaller than  $W_1$  if:

Based on relationship between  $W_1$  and  $W_4$ , as calculated in Section i, we have two mathematical equations:

If  $p_1 \leq p_F$  and  $p_2 \leq p_F$ ,  $W_4$  would be always smaller than  $W_1$ . Otherwise, the following equation should be established:

$$\alpha \times p_1 + \beta \times p_2 \leq p_F. \quad (\text{A.25})$$

(b)  $W_4$  is smaller than  $W_2$  if:

According the mathematical equation between  $W_2$  and  $W_4$  in Section ii, if  $p_2 \leq p_F$ ,  $W_2$  would be always smaller than  $W_4$ .

(c)  $W_4$  is smaller than  $W_3$  if:

According the mathematical equation between  $W_3$  and  $W_4$  in Section iii, if  $p_1 \leq p_F$ ,  $W_3$  would be always smaller than  $W_4$ . Therefore:

$$\begin{cases} \text{(a) if } (p_1 \leq p_F \text{ and } p_2 \leq p_F) \text{ or } (\alpha \times p_1 + \beta \times p_2 \leq p_F) \Rightarrow W_4 \leq W_1 \\ \text{(b) if } p_2 \leq p_F \Rightarrow W_4 \leq W_2 \\ \text{(c) if } p_1 \leq p_F \Rightarrow W_4 \leq W_3 \end{cases} \Rightarrow \text{if } p_1 \leq p_F \text{ and } p_2 \leq p_F \Rightarrow W_4 \leq W_j \quad \forall j = 1, 2, 3. \quad (\text{A.26})$$

□

*Proof of Proposition 2.* In order to prove Proposition 2, we have to show that the objective function  $Z_4$  (Eq. (19)) is equal or greater than the objective functions  $Z_1, Z_2$ , and  $Z_3$  (Eqs. (12), (13), and (16)). Indeed, we show that the difference between the objective function  $Z_4$  and other objective functions is always positive. Consequently, we have:

(1)  $Z_4$  vs.  $Z_1$ :

$$Z_4 - Z_1 = (p_1 - c_1) \times M_1 \times d + (p_2 - c_2) \times M_2 \times d - 0. \quad (\text{A.27})$$

Since the following equation is always positive:

$$= (p_1 - c_1) \times M_1 \times d + (p_2 - c_2) \times M_2 \times d \geq 0. \quad (\text{A.28})$$

TABLE A.2. Computational results for the problem 2.

Data set	$c_1$	$c_2$	$u_1$	$u_2$	$p_1$	$p_2$	Manufacturer L profit	Manufacturer H Profit
1	1	3	0.96532	0.021464	19.03	19.03	174 051	3441
2	2	4	0.96528	0.019807	19.43	19.43	168 267	3057
3	3	5	0.96490	0.018219	19.85	19.85	162 564	2705
4	4	6	0.96466	0.017668	20	20	15 4346	2473
5	5	7	0.96466	0.017668	20	20	144 699	2297
6	6	8	0.96466	0.017668	20	20	135 053	2120

TABLE A.3. Computational results for the problem 3.

Data set	$c_1$	$c_2$	$u_1$	$u_2$	$p_1$	$p_2$	Manufacturer L profit	Manufacturer H Profit
1	1	3	0.07064	0.91264	17.11	20	11 385	155 150
2	2	4	0.062112	0.92102	17.39	20	9561	147 363
3	3	5	0.053984	0.929	17.69	20	7931	139 350
4	4	6	0.04637	0.93648	18.01	20	6497	131 107
5	5	7	0.039362	0.94336	18.35	20	5256	127 893
6	6	8	0.033025	0.94958	18.71	20	4199	113 949

TABLE A.4. Computational results for the problem 4.

Data set	$c_1$	$c_2$	$u_1$	$u_2$	$p_1$	$p_2$	Manufacturer L profit	Manufacturer H Profit
1	1	3	0.49546	0.49546	20	20	94 138	84 229
2	2	4	0.49546	0.49546	20	20	89 183	79 274
3	3	5	0.49546	0.49546	20	20	84 228	74 319
4	4	6	0.49546	0.49546	20	20	79 274	69 365
5	5	7	0.49546	0.49546	20	20	74 319	64 410
6	6	8	0.49546	0.49546	20	20	69 365	59 456

TABLE A.5. Computational results for the problem 5.

Data set	$c_1$	$c_2$	$u_1$	$u_2$	$p_1$	$p_2$	Manufacturer L profit	Manufacturer H Profit
1	1	3	0.69703	0.17189	11.90	13.90	75 966	18 733
2	2	4	0.68799	0.16966	12.03	14.03	69 040	17 025
3	3	5	0.6774	0.16705	12.18	14.18	62 212	15 341
4	4	6	0.66489	0.16396	12.34	14.34	55 499	13 686
5	5	7	0.64994	0.16027	12.52	14.52	48 922	12 064
6	6	8	0.63185	0.15581	12.73	14.73	42 510	10 483

We have:

$$Z_4 - Z_1 \geq 0 \Rightarrow Z_4 \geq Z_1. \quad (\text{A.29})$$

(2)  $Z_4$  vs.  $Z_2$ :

$$Z_4 - Z_1 = (p_1 - c_1) \times M_1 \times d + (p_2 - c_2) \times M_2 \times d - (p_1 - c_1) \times M_1 \times d. \quad (\text{A.30})$$

Since the following equation is always positive:

$$= (p_2 - c_2) \times M_2 \times d \geq 0. \quad (\text{A.31})$$

We have:

$$Z_4 - Z_2 \geq 0 \Rightarrow Z_4 \geq Z_2. \quad (\text{A.32})$$

(3)  $Z_4$  vs.  $Z_3$ :

$$Z_4 - Z_3 = (p_1 - c_1) \times M_1 \times d + (p_2 - c_2) \times M_2 \times d - (p_2 - c_2) \times M_2 \times d. \quad (\text{A.33})$$

Since the following equation is always positive:

$$= (p_1 - c_1) \times M_1 \times d \geq 0. \quad (\text{A.34})$$

We have:

$$Z_4 - Z_3 \geq 0 \Rightarrow Z_4 \geq Z_3. \quad (\text{A.35})$$

□

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