RAIRO-Oper. Res. 57 (2023) 2025–2043
https://doi.org/10.1051/ro/2023059

RAIRO Operations Research
www.rairo-ro.org

A COMPARATIVE STUDY ON A FIRM'S OPTIMAL PRODUCTION DECISION CONSIDERING RISK AVERSION AND CARBON POLICIES

Qi Qi[1,*], Qingguo Bai[2], Jianteng Xu[2] and Qian Feng[3]

Abstract. To cope with global warming and achieve the goal of carbon emission reduction, various carbon policies have been implemented in many countries and regions. Some studies have indicated that with the popularity of low carbon, a small proportion of consumers show a green purchasing tendency. However, for firms, it is still difficult to predict the green purchase preferences of most consumers, and firms may face greater market demand uncertainty, which will bring profit risks. We assume that a firm's decision is subject to its behavioral considerations, that is, its risk attitude. Considering the risk preferences of decision-makers and carbon emission policies, this paper addresses decision optimization in operations management. Our purpose is to analyze the optimal production decisions of risk-averse firms under carbon cap and carbon tax policies. Adopting the CVaR (conditional value-at-risk) criterion, we construct an optimization model with the constraints of carbon policies and obtain the analytic formula of optimal solutions. Then, we compare the optimal operational decisions for risk-averse firms and risk-neutral firms and further explore the impact of risk aversion on production and the parameters of carbon policy. Finally, some numerical studies are presented to analyze the carbon policies and risk aversion on firms' operational decisions and give some managerial insights based on the analysis results.

Mathematics Subject Classification. 91A80.

Received October 18, 2022. Accepted April 23, 2023.

1. Introduction

The report of the United Nations Intergovernmental Panel on Climate Change (IPCC) in 2021 pointed out that the Earth's climate is changing, and the impact of human beings on the climate system is indisputable. Ed Hawkins, a professor at Reading University and one of the main authors of the report, said he hoped that readers would recognize from the report that climate change is undoubtedly caused by human activities (mainly fossil fuel burning and deforestation) and that greenhouse gas emissions need to be reduced immediately, rapidly and continuously to limit global temperature rise. Many countries and regions have set short-term and long-term emission reduction targets. For example, in September 2020, the Chinese government proposed achieving carbon peak and carbon neutral targets. In addition, carbon emission policies are being implemented, such as carbon tax, carbon cap and carbon cap-and-trade [1, 2, 4, 5]. In academic circles, many scholars have paid attention

Keywords. Risk aversion, Conditional value-at-risk, Carbon capacity, Carbon tax, Production decision.

[1] School of Logistics, Beijing Wuzi University, Beijing 101149, P.R. China.
[2] School of Management, Qufu Normal University, Rizhao, Shandong 276826, P.R. China.
[3] School of Economics and Management, Beihang University, Beijing 100191, P.R. China.
[*] Corresponding author: qiqi@bwu.edu.cn

to the optimization of the operation decisions of firms under carbon policy. They have analyzed the impact of carbon policies on the optimal decisions and profits of firms [3, 15, 22, 30, 48].

From the perspective of firms, many have also begun to adapt to the implementation of carbon emission policies. To actively implement carbon emission reduction actions, more than 200 firms around the world have signed the Climate Declaration to work together to achieve the goal of net zero carbon emissions. The 86 newly signed enterprises include P&G, HP, Salesforce, ASOS, Nespresso, etc. P&G has expanded the scale of renewable energy power procurement by 97% and generated 15 million tons of carbon emission reduction benefits through cold water washing. HP has also been committed to implementing the most comprehensive and positive climate action in the industry and is promoting emission reduction initiatives in the entire product and service portfolio, business scope and broader value chain. However, most firms are currently facing higher market demand uncertainty than in the past because they do not understand consumers' attitudes toward green products, which threatens their profits. Many firms are more risk averse and more cautious in making decisions [9, 10, 29, 31] . However, most of the research assumes that decision-makers are risk-neutral, and few studies have focused on the operational decision-making of risk-averse firms under the constraints of carbon policy.

Schweitzer and Cachon [38] showed that the final decision of a risk-averse firm always deviates from profit maximization. Some scholars have introduced the study of risk or loss avoidance to avoid risks or minimize losses. In the field of firm operations management and supply chain management, many scholars have studied the decision-making optimization process of risk-averse decision-makers [9, 28, 37, 42]. However, in a low-carbon environment, in addition to paying attention to the minimum loss, a firm's decision-makers must also consider the constraints of carbon emission policies. Influenced by decision-makers' risk aversion preference, it is difficult for firms to make decisions under carbon emission policies to meet both economic performance and environmental benefits. At present, few studies have focused on the decision-making optimization of risk-averse firms under carbon policies. When giving up the assumption of a perfectly rational person in economics (that is, only focus on the maximization of expected profits) and considering that decision-makers have a risk aversion preference, the following issues arise: (i) How can firms with risk aversion make decisions under carbon policies? (ii) How does risk aversion affect the performance of firms and the carbon emission reduction effect of carbon policies? (iii) How do different carbon policies affect the economic performance and environmental benefits of risk-averse firms?

To address the major issues mentioned above, we consider the operational decision-making of firms with different risk attitudes under carbon cap and carbon tax policies. Considering the risk aversion preference of firms, we use CVaR to construct the optimal decision model under two carbon policies and obtain the optimal production decision. Furthermore, we compare the performance of risk-neutral firms and risk-averse firms under two carbon policies. Finally, some numerical studies are conducted to present and verify the impact of risk aversion and carbon policies on the optimal operational decisions.

Our work has the following research contributions. First, we consider the risk-averse attitude of firms and use a downside risk measurement method, namely, CVaR, to measure firms' risk under carbon policies. Second, we analyze and compare the operational decisions for risk-averse firms and risk-neutral firms under carbon cap and carbon tax policies. Finally, we numerically investigate how risk aversion affects the parameters of carbon policy and optimal decisions and how different carbon policies affect the operational decisions of firms with different risk attitudes.

The remainder of this paper is arranged as follows. Section 2 reviews the relevant literature and shows the research gap. Section 3 interprets the considered problem and relevant notations. Sections 4 and 5 analyze the optimal decisions for risk-averse firms and risk-neutral firms under carbon cap and carbon tax policies, respectively. Section 6 contains some numerical studies to illustrate and complement the theoretical results. Conclusions are presented in the last section.

2. Literature review

Two streams of literature are related to this research: firms' operation management under carbon policies and the optimization of the operation decisions of risk-averse firms.

The first stream of research focuses on firms' operations management under carbon policies. With the increasing concern of the world on climate issues, green supply chain management has attracted more scholars' attention [35, 36]. At present, cap-and-trade policy is the most widely used in the world, and there are also many studies in the literature focused on relevant research [4, 7, 17, 20, 24, 40, 45]. Carbon tax and carbon cap are being implemented in some countries and regions and have attracted the attention of many scholars [1, 8, 18, 19]. Bai *et al.* [3] used the mean-variance model and explored the carbon emission reduction investment of a risk-averse firm under a carbon tax. Liu and Hu [26] considered consumer behavior and investigated the coordination of a green supply chain under a carbon tax. Yang and Chen [48] considered revenue-sharing and cost-sharing and investigated emission reduction decisions under a carbon tax. Ghosh *et al.* [22] studied optimal lot sizing under a carbon cap policy. Qi *et al.* [30] explored the pricing decision in a two-stage supply chain considering carbon emission capacity. Rami *et al.* [32] considered a sustainable dynamic lot sizing models under a carbon cap policy.

The abovementioned literature only considers firms' operation decisions under a single carbon policy. Some scholars have compared the impact of different carbon policies on firms' operation decisions [2, 25]. Bai *et al.* [5] investigated the robust decisions of sustainable manufacturing under carbon cap and carbon tax. Ghosh [21] explored the problem of production-inventory in a two-stage supply chain under carbon tax and cap-and-trade policies. Sun and Yang [41] considered the decisions between competitive manufacturers under carbon tax and cap-and-trade policies. Different from the above research methods, Das *et al.* [15] studied a location problem in a transportation system by using type-2 fuzzy logic under different carbon policies. There are also some studies that use heuristic methods to study green supply chain problems [11–14, 16, 39]. However, the abovementioned literature assumes that the decision-maker is a completely rational person, without considering the risk aversion attitude, except Bai *et al.* [3]. In contrast, in this paper, we introduce the risk attitudes of decision-makers and discuss the impact of different risk attitudes on firms' operation decisions and carbon emissions by using the CVaR criterion.

The second stream of research focuses on optimization of the operation decisions of risk-averse firms. It has been found that the operation decisions of firms in reality always deviate from those made when maximizing profit because most firms are risk averse [46]. Different from the goal of risk-neutral firms to maximize expected profit, the goal of risk-averse firms is to minimize risk; CVaR, which is a downside risk measurement, has attracted the attention of many scholars. Chen *et al.* [9] used CVaR to explore the order and pricing decisions of a risk-averse newsvendor. Wu *et al.* [43] considered CVaR and investigated the risk-averse newsvendor problem with random capacity. Xue *et al.* [47] studied inventory and hedging decisions by using CVaR. Dai and Meng [10] analyzed the order price and marketing under the CVaR criterion. Mohammad and Shokri [28] focused on the multiproduct selective newsvendor problem and explored the order quantity by using CVaR. Xu *et al.* [44] measured hedge against risks with CVaR and investigated the ordering decision of a loss-averse newsvendor. Schur *et al.* [37] studied a dynamic pricing problem by using CVaR and contrasted it with the risk-neutral case. Wang *et al.* [42] considered the CVaR measure and explored the inventory decisions of a risk-averse retailer when offering layaway. Qi *et al.* [31] studied a risk-averse firm's emission reduction and order quantity under car-and-trade policy.

However, the abovementioned literature does not consider the impact of carbon emissions on the optimal operation decisions, except Qi *et al.* [31]. Our research is most closely related to that of Qi *et al.* [31]. In contrast, our research considers two carbon emission policies and analyzes the impact of different carbon policies on optimal decisions and carbon emissions. Moreover, we comprehensively consider the risk attitude and carbon policies and compare the optimal operation decisions for risk-neutral firms and risk-averse firms under two different carbon policies.

A comparison between previous related research and our work is presented in Table 1. Many studies have explored the optimization decisions of firms in supply chains under carbon policy, and some of them have also compared the impact of different carbon policies on firms' operation decisions and emission reduction effects. However, most of the existing studies have assumed that the decision-maker is a rational person, except for Bai *et al.* [4]. Our research has been further expanded by using the CVaR criterion, which is a downward risk measurement, to explore the optimal decisions of risk-averse firms. In addition, to achieve emission reduction goals, carbon tax and carbon cap are two policies widely used by an increasing number of countries and regions, including China, Canada, the European Union (EU), the International Maritime Organization and other international institutions and organizations. Therefore, different from the literature, our research compares the impact of different carbon policies on the operational decisions of risk-averse firms by using the CVaR criterion.

3. Model construction

3.1. Model hypothesis and parameters

This study considers two types of manufacturing firms, namely, risk-neutral firms and risk-averse firms, under the constraints of carbon cap and carbon tax policies. To analyze the operational decisions of firms, we propose the following hypothesis. The model parameters and decision variables are shown in Table 2.

Hypothesis 1. Firms produce certain types of products at cost c, and the production process will produce carbon emissions. If the firm produces q units of the product, then each unit emits carbon dioxide of e tons. Then, the total emission amount for producing q units is $E = eq$ tons. Similar assumptions can be found in [23, 27, 48].

Hypothesis 2. Considering the complexity and solvability of the model, the annual demand for the product D is stochastic and nonnegative, with a probability density function $f(\cdot)$ and a cumulative distribution function $F(\cdot)$.

Hypothesis 3. To make the model less complex and obtain a clear analytical solution to analyze the impact of carbon policies and risk aversion on the decision-making of firms, we refer to certain literature, such as [10, 44], and use uniform distribution to measure uncertain demand. In what follows, we use subscript "u" to represent the uniform distribution of the random demand.

Hypothesis 4. For the sake of simplicity, we also assume that the salvage value of the product is zero.

3.2. Model description

The firm's profit without carbon policies can be expressed as follows:

$$\pi(q) = p \cdot \min\{D, q\} - cq = (p - c)q - p(q - D)^+, \tag{1}$$

where the first item on the right-hand side is sales revenue minus production cost, and the second item represents the gap between production and actual demand. If the production is higher than actual demand, then $(q-D)^+ = q - D$ and $\pi(q) = pD - cq$, which means that production is too high, and the maximum sales can only be the actual demand; however, the firm has to pay the production cost of all the output. If the production is lower than the actual demand, then $(q - D)^+ = 0$ and $\pi(q) = (p - c)q$, which means that production is too low; furthermore, the maximum sales can only be the production, which cannot meet the actual demand. We use the subscripts "c" and "t" to represent the profit under the carbon cap policy and carbon tax policy, respectively, that is, π_c and π_t.

Considering the limitations of carbon policies, firms need to make optimal production decisions to achieve their goals. For risk-neutral firms, the goal is profit maximization, while for risk-averse firms, the goal is risk

TABLE 1. The comparison of previous related work with our research.

Works	Firms' operation management under single carbon policy	Firms' operation decisions under different carbon policies	Decision optimization of risk-averse firms	Decision-making of risk-averse firms using CVaR
Bai *et al.* [4] Chai *et al.* [7] Dong *et al.* [17] Fan *et al.* [20] Kumar *et al.* [24] Sun and Gao [40] Chelly *et al.* [8] Absi *et al.* [1] Xu *et al.* [45] Dou and Cao [18] Duan *et al.* [19] Ghosh *et al.* [22] Liu and Hu [26] Rami *et al.* [32] Qi *et al.* [30] Yang and Chen [48]	✓	✗	✗	✗
Bai *et al.* [3]	✓	✗	✓	✗
Bai and Chen [2] Bai *et al.* [5] Das *et al.* [15] Ghosh [21] Li *et al.* [25] Shaw *et al.* [39] Sun and Yang [41] Das *et al.* [13]	✗	✓	✗	✗
Chen *et al.* [9] Dai and Meng [10] Mohammad and Shokri [28] Schur *et al.* [37] Xue *et al.* [47] Xu *et al.* [46] Wu *et al.* [43] Xu *et al.* [44] Wang *et al.* [42]	✗	✗	✓	✓
Qi *et al.* [31]	✓	✗	✓	✓
Our research	✓	✓	✓	✓

minimization. Specifically, risk-neutral firms would like to maximize their expected profit, which can be expressed as follows:

$$\max \ \pi(q) = p \cdot \min\{D, q\} - cq = (p-c)q - p(q-D)^{+}. \tag{2}$$

For risk-averse firms, we use the downside risk measurement CVaR to investigate the optimal production decision under carbon policies. CVaR focuses on average profit maximization below a certain quantile level or loss minimization above a certain quantile level [43]. Specifically, for profit, it measures the average profit falling

Table 2. Model parameters and decision variables.

Model parameters	Description
D	Market demand, a random variable
c	Unit production cost of the product
p	Unit selling price of the product
e	Carbon emission of per unit product produced in the measure of tons
E	Total carbon emission from the production and $E = \text{eq}$, with q as the production quantity
e	Unit carbon emissions per unit product
π	Firm's expected profit
η	Risk-averse attitude, $\eta \in (0, 1]$
$\text{CVaR}_\eta(\pi)$	Conditional value of risk of profit at given risk attitude of η
Decision variable	
q	Production quantity

below the η-quantile level: $\text{CVaR}_\eta(\pi) = E[\pi|\pi \le v_\eta(\pi)]$, where we assume v_η as the η-quantile of random profit π as follows: $v_\eta(\pi) = \inf\{v|\Pr(\pi \le v) \ge \eta\}$, and v is a real number that is denoted as a target level of profit.

CVaR was first introduced by Rockafellar and Uryasev [33, 34] and has been widely used in financial risk management [44]. In recent years, many scholars in operations research have started to use CVaR to investigate the optimal operational decisions of risk-averse firms. Rockafellar and Uryasev [33] and Pflug [29] provided a general and analytically convenient definition [10]: $\text{CVaR}_\eta(\pi) = \max_{v \in R}\left\{v + \frac{1}{\eta}E[\min(\pi - v, 0)]\right\}$, where $\eta \in (0, 1]$ reflects the degree of risk aversion. The smaller η is, the more risk-averse the firm is. When $\eta = 1$, it means risk neutrality.

Therefore, the objective function of risk-averse firms can be expressed as follows:

$$\max_q \ \text{CVaR}_\eta(\pi) = \max_q \ \max_{v \in R}\left\{v + \frac{1}{\eta}E[\min\{\pi(q) - v, 0\}]\right\}. \tag{3}$$

4. Model analysis with carbon cap policy

In this section, we focus on the situation in which the firm is constrained by the carbon cap policy; that is, the firm's carbon emissions cannot exceed the emission capacity given by the government, or the firm will face a huge penalty. In this situation, we consider two kinds of firms according to different risk attitudes. The firm needs to determine the production decision to maximize profit for the risk-neutral firm or to minimize risk for the risk-averse firm.

4.1. The case with a risk-neutral firm

For the risk-neutral firm, the goal is to maximize the firm's own profit under the constraint of the carbon cap policy, and the optimization model can be expressed as follows:

$$\begin{cases} \max \ \pi_c(q) = (p - c)q - p(q - D)^+ \\ \text{s.t.} \qquad \text{eq} \le K_{\text{Cn}} \end{cases} \tag{4}$$

It is easy to verify that Model (1) is a convex programming problem, which implies that the optimal solution of Model (4) is unique. We can find the optimal production decision by constructing the Lagrange function as follows:

$$L_1(q, r_1) = \pi_c(q) + r_1(K_{\text{Cn}} - \text{eq}), \tag{5}$$

where r_1 is the Lagrange multiplier. To solve the optimization problem with inequality constraints, it is necessary to maximize the Lagrangian by satisfying KKT conditions to obtain feasible solutions under inequality constraints. KKT conditions refer to the process of solving all Lagrange multiplier problems that meet the conditions. KKT conditions mainly include (1) the necessary conditions for the Lagrangian to obtain feasible solutions, (2) the relaxation complementarity conditions, (3) the initial constraint conditions, and (4) the conditions for the Lagrange multiplier with inequality constraints, that is, multipliers greater than or equal to 0. By solving the KKT conditions, we can obtain the following results.

Proposition 1. *There exists a unique equilibrium solution. The optimal decision is* $q_{cn}^* = F^{-1}\left(\frac{p-c-er_1}{p}\right)$. *For a uniformly distributed demand with support over* $[0, U]$, *the optimal solution* $q_{cnu}^* = \frac{U(p-c-er_1)}{p}\cdot$ *When* $r_1 > 0$, *the results can be expressed as follows:* $r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}\cdot$

Proof. $\pi(q) = (p-c)q - p(q-D)^+ = (p-c)q - p\int_q^0 F(D)\mathrm{d}D$, and the Lagrange function is $L_1(q, r_1) = (p-c)q - p\int_q^0 F(D)\mathrm{d}D + r_1(K_{\mathrm{Cn}} - eq)$, then the KKT conditions are as follows:

$$\begin{cases} \frac{\partial L_1(q,r_1)}{\partial q} = 0 \\ r_1(K_{\mathrm{Cn}} - \mathrm{eq}) = 0 \\ K_{\mathrm{Cn}} - \mathrm{eq} \geq 0 \\ r_1 \geq 0 \end{cases}$$

Solving $\frac{\partial L_1(q,r_1)}{\partial q} = p - c - p \cdot F(q) - r_1 e = 0$, we can obtain $q_{cn}^* = F^{-1}\left(\frac{p-c-er_1}{p}\right)$. For a uniformly distributed demand with support over $[0, U]$, $F(q) = \frac{q}{U}$, and we have $q_{cnu}^* = \frac{U(p-c-er_1)}{p}\cdot$ When $r_1 > 0$, then $K_{\mathrm{Cn}} - eq = 0$. Substituting q_{cnu}^* into $K_{\mathrm{Cn}} - eq = 0$, we can obtain $r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}\cdot$ □

From the model setting, $r_1 = 0$ means no constraint and $r_1 > 0$ means that the constraint is working. Proposition 1 indicates that a fully used carbon emission capacity can result in a lower production quantity. Furthermore, substituting $r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}$ into $q_{cnu}^* = \frac{U(p-c-er_1)}{p}$, we can obtain $q_{cnu}^* = \frac{K_{\mathrm{Cn}}}{e}\cdot$ The result shows that under a strict carbon cap policy, the optimal production decision is only related to the carbon capacity allocated by the government and the carbon emissions per unit product produced by the firm.

Corollary 1. *The optimal decisions under the carbon cap policy satisfy the following:*

(1) *When* $r_1 = 0$, *the optimal decision exists only if the carbon capacity satisfies* $K_{\mathrm{Cn}} \geq \frac{Ue(p-c)}{p}$;
(2) *When* $r_1 > 0$, *the optimal decision exists only if the carbon capacity satisfies* $K_{\mathrm{Cn}} < \frac{Ue(p-c)}{p}\cdot$

Proof. When $r_1 = 0$, we have $K_{\mathrm{Cn}} - \mathrm{eq} \geq 0$, then we can obtain $K_{\mathrm{Cn}} \geq \frac{Ue(p-c)}{p}$; however, when $r_1 > 0$, we have $\frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2} > 0$, then we can obtain $K_{\mathrm{Cn}} < \frac{Ue(p-c)}{p}\cdot$ □

Corollary 1 indicates that to realize the restrictive effect of the carbon cap policy on the carbon emissions of the firm, there should be an upper limit to the emission capacity. Otherwise, a high capacity will cover the carbon emissions under the optimal decision of the firm and thus will not have a constraint effect on carbon emissions. In addition, combined with Proposition 1, we further verify that an effective carbon cap policy will reduce the firm's optimal production decision, which is consistent with the general understanding of people.

4.2. The case with a risk-averse firm

In this section, we discuss the risk-averse firm's optimal decision on production quantity under a carbon cap policy. For a risk-averse firm that pays more attention to risk, the goal is no longer to maximize expected profit but to minimize risks. Referring to certain literature [10, 42, 44], we use CVaR criteria to measure the firm's risk. According to the description of expression (3), the objective function for the risk-averse firm is as follows:

$$\max_q \ \mathrm{CVaR}_\eta(\pi_c) = \max_q \max_{v\in R}\left\{v + \frac{1}{\eta}E[\min\{\pi_c(q) - v, 0\}]\right\}, \tag{6}$$

where $\pi_c(q)$ represents the profit of a risk-averse firm with carbon cap policy, v is denoted as a target level of profit, and $\eta \in (0, 1]$ reflects the degree of risk aversion.

Considering the constraint of carbon emissions, the optimization model of the risk-averse firm is expressed as follows:

$$\begin{cases} \max_q \ \ \mathrm{CVaR}_\eta(\pi_c) = \max_q \ \max_{v\in R}\left\{v + \frac{1}{\eta}E[\min\{\pi_c(q) - v, 0\}]\right\} \\ \text{s.t.} \qquad\qquad \mathrm{eq} \le K_{\mathrm{Cr}} \end{cases} \tag{7}$$

Since we do not have a specific distribution for the demand, we now first discuss how the risk-averse firm sets its target profit by considering the following two cases:

(1) If $(p - c)q - v \le 0$, then we have the following:

$$\begin{aligned} \mathrm{CVaR}_\eta(\pi_c) &= \max_{v\in R}\left\{v + \frac{1}{\eta}E[(p-c)q - p(q-D)^+ - v]\right\} \\ &= v + \frac{p-c}{\eta}q - \frac{v}{\eta} - \frac{p}{\eta}\int_0^q (q-D)f(D)\,\mathrm{d}D \\ &= v + \frac{(p-c)q - v}{\eta} - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D \end{aligned}$$

To maximize $\mathrm{CVaR}_\eta(\pi_c)$, we set $v^* = (p - c)q$ as the optimal target profit v^* for a given risk attitude η. Then, we have $\mathrm{CVaR}_\eta(\pi_c) = (p - c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D$.

If $(p - c)q - v > 0$, then $\mathrm{CVaR}_\eta(\pi_c) = \max_{v\in R}\left\{v + \frac{1}{\eta}E[\min\{(p - c)q - p(q - D)^+ - v, 0\}]\right\}$.

From $(p - c)q - p(q - D)^+ - v < 0$, we have $q - D > 0$ and $D < \frac{cq+v}{p}$; then, we can obtain $\mathrm{CVaR}_\eta(\pi_c) = v + \frac{1}{\eta}\int_0^{\frac{cq+v}{p}} (pD - cq - v)dF(D)$.

Let $G(v) = v + \frac{1}{\eta}\int_0^{\frac{cq+v}{p}} (pD - cq - v)dF(D)$, and the first-order derivative is $\frac{\partial G(v)}{\partial v} = 1 - \frac{1}{\eta}F\left(\frac{cq+v}{p}\right)$. From $\frac{\partial G(v)}{\partial v} = 0$, we can obtain the optimal target profit $v^* = pF^(-1)(\eta) - cq$.

After the optimal target profit is set, the conditional value of the risk of profit is expressed as follows:

$$\mathrm{CVaR}_\eta(\pi_c) = pF^{-1}(\eta) - cq - \frac{p}{\eta}\int_0^{F^{-1}(\eta)} F(D)\,\mathrm{d}D.$$

Because $\mathrm{CVaR}_\eta(\pi_c)$ is a continuous risk measure, $(p - c)q - \frac{p}{\eta}\int_0^q F(D)\ \mathrm{d}D = pF^{-1}(\eta) - cq - \frac{p}{\eta}\int_0^{F^{-1}(\eta)} F(D)\,\mathrm{d}D$; we then have $q = F^{-1}(\eta)$. We now substitute the optimal target profit, $v^* = pF^{-1}(\eta) - cq$, into $(p-c)q-v > 0$, which yields $q > F^{-1}(\eta)$; thus, we have $\mathrm{CVaR}_\eta(\pi_c) = pF^{-1}(\eta) - cq - \frac{p}{\eta}\int_0^{F^{-1}(\eta)} F(D)\,\mathrm{d}D$ when $q > F^{-1}(\eta)$.

To further simplify $\mathrm{CVaR}_\eta(\pi_c)$, we obtain the following expressions:

$$\mathrm{CVaR}_\eta(\pi_c) = \begin{cases} (p-c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D, & q \le F^{-1}(\eta) \\ pF^{-1}(\eta) - cq - \frac{p}{\eta}\int_0^{F^{-1}(\eta)} F(D)\,\mathrm{d}D, & q > F^{-1}(\eta) \end{cases}$$

If $q > F^{-1}(\eta)$, then $\mathrm{CVaR}_\eta(\pi_c) = pF^{-1}(\eta) - cq - \frac{p}{\eta}\int_0^{F^{-1}(\eta)} F(D)\,\mathrm{d}D$. Note that at the optimal target, $\mathrm{CVaR}_\eta(\pi_c)$ must satisfy the condition of $\frac{\partial \mathrm{CVaR}_\eta(\pi_c)}{\partial q} = 0$. However, $\frac{\partial \mathrm{CVaR}_\eta(\pi_c)}{\partial q} = -c < 0$. Therefore, we need only to consider the first case of $q \le F^{-1}(\eta)$ as follows: If $q \le F^{-1}(\eta)$, $\mathrm{CVaR}_\eta(\pi_c) = (p-c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D$.

Next, we will investigate whether CVaR has an optimal solution for $q \le F^{-1}(\eta)$. From the above analysis, the CVaR function can be rewritten as follows:

$$\begin{cases} \max \ \ \mathrm{CVaR}_\eta(\pi_c) = (p-c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D \\ \text{s.t.} \qquad\qquad q \le F^{-1}(\eta) \end{cases} \tag{8}$$

Then, the optimization model of the risk-averse firm can be rewritten as follows:

$$\begin{cases} \max \ \ \mathrm{CVaR}_\eta(\pi_c) = (p-c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D \\ \text{s.t.} \qquad\qquad \mathrm{eq} \le K_{\mathrm{Cr}} \\ \qquad\qquad\qquad q \le F^{-1}(\eta) \end{cases} \tag{9}$$

It is easily verified that Model (9) is a convex programming problem, which implies that the optimal solution of Model (9) is unique. We can find the optimal production decision by constructing the Lagrange function as follows:

$$L_2(q, r_2, r_3) = \mathrm{CVaR}_\eta(\pi_c) + r_2(K_{\mathrm{Cr}} - \mathrm{eq}) + r_3(F^{-1}(\eta) - q), \tag{10}$$

where r_2 and r_3 are the Lagrange multipliers. By solving the KKT conditions, we can obtain the following results.

Proposition 2. *There exists a unique equilibrium solution. The optimal decision is* $q^*_{\mathrm{cr}} = F^{-1}\left(\frac{\eta(p-c-er_2)}{p}\right)$. *For a uniformly distributed demand with support over* $[0, U]$, *the optimal solution* $q^*_{cru} = \frac{\eta U(p-c-er_2)}{p}$. *In addition,* $r_2 \ge 0$ *and* $r_3 = 0$; *thus when* $r_2 > 0$, *the results can be expressed as* $r_2 = \frac{\eta U e(p-c) - pK_{\mathrm{Cr}}}{Ue^2}$.

Proof. The Lagrange function is $L_2(q, r_2, r_3) = (p-c)q - \frac{p}{\eta}\int_0^q F(D)\,\mathrm{d}D + r_2(K_{\mathrm{Cr}} - eq) + r_3(F^{-1}(\eta) - q)$, and the KKT conditions are as follows:

$$\begin{cases} \frac{\partial L_2(q, r_2, r_3)}{\partial q} = 0 \\ r_2(K_{\mathrm{Cr}} - \mathrm{eq}) \quad = 0 \\ r_3(F^{-1}(\eta) - q) \\ K_{\mathrm{Cr}} - \mathrm{eq} \ge 0 \\ F^{-1}(\eta) - q \ge 0 \\ r_2 \ge 0, \ \ r_3 \ge 0 \end{cases}$$

Solving $\frac{\partial L_2(q, r_2, r_3)}{\partial q} = p - c - \frac{p}{\eta}\cdot F(q) - r_2 e - r_3 = 0$, we can obtain $q^*_{\mathrm{cr}} = F^{-1}\left(\frac{\eta(p-c-er_2-r_3)}{p}\right)$. For a uniformly distributed demand with support over $[0, U]$, $F(q) = \frac{q}{U}$, and we have $q^*_{cru} = \frac{\eta U(p-c-er_2-r_3)}{p}$.

When $r_3 > 0$ and $r_2 = 0$, then $F^{-1}(\eta) - q = 0$; furthermore, we have $r_3 = -c$, which conflicts with $r_3 > 0$. When $r_3 > 0$ and $r_2 > 0$, then $K_{\mathrm{Cr}} - eq = 0$ and $F^{-1}(\eta) - q = 0$; furthermore, we have $r_2 + r_3 = -c$, which conflicts with $r_2 > 0$ and $r_3 > 0$. Therefore, $r_3 > 0$ does not exist; that is, $r_3 = 0$. Therefore, $F^{-1}(\eta) - q \ge 0$ must be satisfied. When $r_2 > 0$ and $r_3 = 0$, then $K_{\mathrm{Cr}} - eq = 0$; thus, substituting q^*_{cru} into $K_{\mathrm{Cr}} - eq = 0$, we can obtain $r_2 = \frac{\eta U e(p-c) - pK_{\mathrm{Cr}}}{Ue^2}$. □

Similar to the case with a risk-neutral firm, $r_2 = 0$ means no constraint, and $r_2 > 0$ means that the constraint is working. Proposition 2 indicates that a fully used carbon emission capacity can result in a lower production quantity. However, considering the firm's risk attitude, the result is different from the case with a risk-neutral firm. Substituting $r_2 = \frac{\eta U e(p-c) - pK_{\mathrm{Cr}}}{Ue^2}$ into $q^*_{cru} = \frac{\eta U(p-c-er_2)}{p}$, we can obtain $q^*_{cru} = \frac{\eta(1-\eta)U(p-c)}{p} + \frac{\eta K_{\mathrm{Cr}}}{e}$. The result shows that under a strict carbon cap policy, the optimal production decision is related not only to the carbon capacity allocated by the government and the carbon emissions per unit product produced by the firm but also to the risk aversion of the firm.

Exploring the relationship between risk aversion and the optimal production decision, the following corollaries can be obtained.

Corollary 2. *The optimal decisions under the carbon cap policy satisfy the following:*

(1) *When $r_2 = 0$, the optimal decision exists only if the carbon capacity satisfies $K_{\mathrm{Cr}} \geq \frac{\eta U e(p-c)}{p}$;*
(2) *When $r_2 > 0$, the optimal decision exists only if the carbon capacity satisfies $K_{\mathrm{Cr}} < \frac{\eta U e(p-c)}{p}$.*

Proof. When $r_2 = 0$, we have $K_{\mathrm{Cr}} - \mathrm{eq} \geq 0$; then we can obtain $K_{\mathrm{Cr}} \geq \frac{\eta U e(p-c)}{p}$. When $r_2 > 0$, we have $\frac{\eta U e(p-c) - pK_{\mathrm{Cr}}}{Ue^2} > 0$; then we can obtain $K_{\mathrm{Cr}} < \frac{\eta U e(p-c)}{p}$. □

Similar to Corollary 1, Corollary 2 also indicates that to realize the restrictive effect of the carbon cap policy on the carbon emissions of the firm, there should be an upper limit to the emission capacity. Otherwise, a high capacity will cover the carbon emissions under the optimal decision of the firm and thus will not have a constraint effect on carbon emissions. In addition, we also find that the upper limit is related to the firm's risk aversion. For a firm with a high degree of risk aversion, the upper limit will be smaller. This result indicates that for a risk-averse firm, the emission capacity allocated by the government should be lower to ensure the effectiveness of the carbon cap policy. This may be because the optimal production of a risk-averse firm is lower.

Corollary 3. *The change in the optimal production decision with the increase in risk aversion is as follows:*

(1) *When $r_2 = 0$, the optimal production decision decreases with increasing risk aversion.*
(2) *When $r_2 > 0$, the optimal production decision increases first and then decreases with increasing risk aversion. When $\eta = \frac{1}{2} + \frac{pK_{\mathrm{Cr}}}{2Ue(p-c)}$, the optimal production is highest.*

Proof. When $r_2 = 0$, $q^*_{cru} = \frac{\eta U(p-c)}{p}$; then, we have $\frac{\partial q^*_{cru}}{\partial \eta} = \frac{U(p-c)}{p} > 0$. When $r_2 > 0$, $q^*_{cru} = \frac{\eta(1-\eta)U(p-c)}{p} + \frac{\eta K_{\mathrm{Cr}}}{e}$, then $\frac{\partial q^*_{cru}}{\partial \eta} = \frac{(1-2\eta)U(p-c)}{p} + \frac{K_{\mathrm{Cr}}}{e}$ and $\frac{\partial^2 q^*_{cru}}{\partial \eta^2} = -2\frac{U(p-c)}{p} < 0$. When $\eta = 0$, then $\frac{\partial q^*_{cru}}{\partial \eta} > 0$, and when $\eta = 1$, then $\frac{\partial q^*_{cru}}{\partial \eta} < \frac{(\eta-1)U(p-c)}{p} < 0$. Let $\frac{\partial q^*_{cru}}{\partial \eta} = 0$; then, we can obtain $\eta = \frac{1}{2} + \frac{pK_{\mathrm{Cr}}}{2Ue(p-c)} < 1$. □

Corollary 3 indicates that the optimal production decision without the constraint of the carbon cap policy (that is, $r_2 = 0$) decreases with increasing risk aversion, which is consistent with existing research results on the impact of risk aversion on firms' optimal decisions. However, under the strict constraint of the carbon cap policy (that is, $r_2 > 0$), the firms' optimal production increases first and then decreases and achieves the highest production when $\eta = \frac{1}{2} + \frac{pK_{\mathrm{Cr}}}{2Ue(p-c)}$. The result shows that the carbon policy that strictly restricts the carbon emissions of firms has changed the impact of risk aversion on firms' decisions in general; that is, the risk aversion attitude does not have a negative impact on the decision-making of firms in all cases.

4.3. Comparison of firms' decisions under different risk attitudes

Under the constraint of the carbon cap policy, the optimal production of firms with different risk attitudes is summarized in Table 3. By comparing the optimal decisions under risk-neutral firms and risk-averse firms, the following corollaries can be obtained.

Corollary 4. *Under carbon cap policy, whether risk-neutral firms or risk-averse firms, optimal production will be affected by carbon emission constraints, and risk-averse firms will also be affected by risk-averse attitudes.*

Corollary 4 describes an obvious conclusion, which can be obtained under the general distribution of demand. However, the specific distribution function of demand needs to be defined if we want to further analyze the impact of carbon emission capacity and risk aversion on firms' optimal decisions. The following Corollaries 4 and 5 give the analysis results that the demand follows a uniform distribution.

TABLE 3. The optimal decisions under different risk attitudes.

	Risk attitude	Optimal production	Lagrange multiplier	Carbon capacity
General distribution	Risk-neutral	$q_{cn}^* = F^{-1}\left(\frac{p-c-er_1}{p}\right)$	–	–
	Risk-averse	$q_{\mathrm{cr}}^* = F^{-1}\left(\frac{\eta(p-c-er_2)}{p}\right)$	–	–
Uniform distribution	Risk-neutral	$q_{cnu}^* = \frac{U(p-c-er_1)}{p}$	$r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}$	$K_{\mathrm{Cn}} < \frac{Ue(p-c)}{p}$
	Risk-averse	$q_{cru}^* = \frac{\eta U(p-c-er_2)}{p}$	$r_2 = \frac{\eta Ue(p-c)-pK_{\mathrm{Cr}}}{Ue^2}$	$K_{\mathrm{Cn}} < \frac{\eta Ue(p-c)}{p}$

Corollary 5. *If the demand follows a uniform distribution with support over $[0, U]$, the upper limits are different for risk-neutral firms and risk-averse firms, and the upper limit of risk-averse firms is related to risk aversion.*

Corollary 5 shows that firms' risk attitudes will affect the upper limit of carbon emission capacity when the carbon cap policy is effectively implemented. The higher the degree of risk aversion is (η decreases), the smaller the upper limit is. These results mean that to make the carbon cap policy work, compared with risk-neutral firms, risk-averse firms should set a smaller carbon emission capacity.

Corollary 6. *If the demand follows a uniform distribution with support over $[0, U]$, $q_{cnu}^* = \frac{K_{\mathrm{Cn}}}{e}$ and $q_{cru}^* = \frac{\eta(1-\eta)U(p-c)}{p} + \frac{\eta K_{\mathrm{Cr}}}{e}$. Furthermore, $q_{cru}^* > q_{cnu}^*$ if $K_{\mathrm{Cr}} = K_{\mathrm{Cn}}$.*

Proof. Substituting r_1 and r_2 into q_{cnu}^* and q_{cru}^*, respectively, we can obtain $q_{cnu}^* = \frac{K_{\mathrm{Cn}}}{e}$ and $q_{cru}^* = \frac{\eta(1-\eta)U(p-c)}{p} + \frac{\eta K_{\mathrm{Cr}}}{e}$, respectively. $q_{cru}^* - q_{cnu}^* = \frac{\eta(1-\eta)U(p-c)}{p} + \eta\frac{K_{\mathrm{Cr}}}{e} - \frac{K_{\mathrm{Cn}}}{e}$, when $K_{\mathrm{Cr}} = K_{\mathrm{Cn}}$, $q_{cru}^* - q_{cnu}^* = (1-\eta)\left(\frac{\eta Ue(p-c)-pK_C}{pe}\right)$. According to $r_2 = \frac{\eta Ue(p-c)-pK_{\mathrm{Cr}}}{Ue^2} > 0$, we have $\eta Ue(p-c) - pK_{\mathrm{Cr}}$; then, we can obtain $q_{cru}^* - q_{cnu}^* > 0$. □

Corollary 6 shows that the optimal production of risk-neutral firms is only related to carbon emission capacity and carbon emission per unit product, while the optimal production of risk-averse firms is also affected by many factors, including market demand, selling price, product costs and risk aversion degree. In addition, the optimal production of risk-averse firms is higher than that of risk-neutral firms when their emission capacities are the same. These results are interesting because the usual research conclusion is that the decisions of risk-averse firms are lower than those of risk-neutral firms due to the caution of decision-makers; however, due to the setting of carbon emission capacity, the optimal decision of risk-averse firms is higher.

5. Model analysis with carbon tax policy

In this section, we focus on the situation in which firms need to pay tax to the government according to their carbon emissions. Referencing certain extant literature [6], we assume that the carbon tax has a linear relationship with the amount of carbon emissions generated from production. Let t denote the carbon tax on the emission per ton and then the total amount of carbon tax that the firm needs to pay to the government is teq. Similar to Section 4, we also consider two kinds of firms according to different risk attitudes in this situation. The firm still needs to determine the production decision to maximize profit for the risk-neutral firm or to minimize risk for the risk-averse firm.

5.1. The case with a risk-neutral firm

For the risk-neutral firm, the goal is to maximize the firm's own profit under carbon tax policy, and the profit function of the firm is as follows:

$$\pi_t(q) = p \cdot \min\{D, q\} - cq - teq, \tag{11}$$

and the optimal model can be expressed as follows:

$$\max_{q} \pi_t(q) = \max_{q}\{p \cdot \min\{D, q\} - cq - teq\}. \tag{12}$$

By solving Model (12), we can obtain the following Proposition 3.

Proposition 3. *There exists a unique equilibrium solution. The optimal decision is* $q^*_{tn} = F^{-1}\left(\frac{p-c-te}{p}\right)$. *For a uniformly distributed demand with support over* $[0, U]$, *the optimal solution is as follows:* $q^*_{tnu} = \frac{U(p-c-te)}{p}$.

Proof. According to $\frac{\partial^2 \pi_t(q)}{\partial q^2} < 0$, we have that the function of $\pi_t(q)$ is concave; thus, if we let $\frac{\partial \pi_t(q)}{\partial q} = 0$, then we can obtain the optimal response function of the firm's production quantity $q^*_{tn} = F^{-1}\left(\frac{p-c-te}{p}\right)$. When the demand follows a uniform distribution, we can obtain $q^*_{tnu} = \frac{U(p-c-te)}{p}$. □

Proposition 3 indicates that the optimal production is proportional to the maximum demand (U) and the unit profit ($p - c$) but negatively correlated with the tax rate of t. This means that as the carbon tax and emissions per unit product increase, the optimal production decreases.

Corollary 7. $\frac{\partial q^*_{tnu}}{\partial t} < 0$.

Corollary 7 shows that the optimal production is negatively correlated with the carbon tax. These results are easy to understand. The higher the carbon tax is, the lower the production of firms to pay less carbon tax is. The lower the production of firms is, the lower the carbon emissions are; that is, the carbon tax policy plays a role in reducing carbon emissions.

5.2. The case with a risk-averse firm

In this section, we discuss the risk-averse firm's optimal decision on production quantity under carbon tax policy. Similar to Section 4.2, we also use CVaR criteria to measure the firm's risk, and the objective function for the risk-averse firm is as follows:

$$\max_q \quad \text{CVaR}_\eta(\pi_t) = \max_q \underset{v\in R}{max}\left\{v + \frac{1}{\eta}E[\min\{\pi_t(q) - v, 0\}]\right\}, \tag{13}$$

According to the analysis of Section 4.2, we can obtain the following results of optimal production under carbon tax policy.

Proposition 4. *There exists a unique equilibrium solution. The optimal decision is* $q^*_{tr} = F^{-1}\left(\frac{\eta(p-c-te)}{p}\right)$. *For a uniformly distributed demand with support over* $[0, U]$, *the optimal solution is as follows:* $q^*_{tru} = \frac{\eta U(p-c-te)}{p}$.

Proof. According to $\frac{\partial^2 \text{CVaR}_\eta(\pi_t)}{\partial q^2} < 0$, we have that the function of $\text{CVaR}_\eta(\pi_t)$ is concave; thus, if we let $\frac{\partial \text{CVaR}_\eta(\pi_t)}{\partial q} = 0$, then we can obtain the optimal response function of the firm's production quantity $q^*_{tr} = F^{-1}\left(\frac{\eta(p-c-te)}{p}\right)$. Therefore, when the demand follows a uniform distribution, we can obtain $q^*_{tru} = \frac{\eta U(p-c-te)}{p}$. □

From Propositions 3 to 4, we find that the difference between the optimal production of risk-neutral firms and risk-averse firms is that the optimal production for the risk-averse firm is related to the risk attitude, while that of the risk-neutral firm is not. Comparing the optimal production in the two cases, we can obtain Corollary 8.

Corollary 8. $q^*_{tru} \leq q^*_{tnu}$ *and* $t < \frac{p-c}{e}$.

TABLE 4. The optimal decision under different carbon policies.

		Risk-neutral firm		Risk-averse firm	
		Optimal production	Lagrange multiplier	Optimal production	Lagrange multiplier
Carbon cap	General distribution	$q^*_{cn} = F^{-1}\left(\frac{p-c-er_1}{p}\right)$	–	$q^*_{cr} = F^{-1}\left(\frac{\eta(p-c-er_2)}{p}\right)$	–
	Uniform distribution	$q^*_{cnu} = \frac{U(p-c-er_1)}{p}$	$r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}$	$q^*_{cru} = \frac{\eta U(p-c-er_2)}{p}$	$r_2 = \frac{\eta Ue(p-c)-pK_{\mathrm{Cr}}}{Ue^2}$
Carbon tax	General distribution	$q^*_{tn} = F^{-1}\left(\frac{p-c-te}{p}\right)$	–	$q^*_{tr} = F^{-1}\left(\frac{\eta(p-c-te)}{p}\right)$	–
	Uniform distribution	$q^*_{tnu} = \frac{U(p-c-te)}{p}$	–	$q^*_{tru} = \frac{\eta U(p-c-te)}{p}$	–

Proof. If $q^*_{tr} = \frac{\eta U(p-c-te)}{p} > 0$, then we have $t < \frac{p-c}{e}$. Moreover, $q^*_{tru} - q^*_{tnu} = \frac{U(p-c-te)}{p} - \frac{\eta U(p-c-te)}{p} = (1-\eta)\frac{U(p-c-te)}{p} \geq 0.$ □

Compared with Corollary 6, we find that the influence of risk aversion on optimal production is different under different carbon policies. Considering the carbon cap policy, the optimal production of risk-averse firms may be higher than that of risk-neutral firms; however, considering the carbo tax policy, the optimal production of risk-averse firms is lower than that of risk-neutral firms. In addition, there must be an upper limit of the carbon tax to ensure the normal operation of firms.

5.3. Comparison of carbon cap policy and carbon tax policy

Under two different carbon policies, the optimal production of firms with different risk attitudes is summarized in Table 4. By comparing the optimal decisions under the carbon cap policy and carbon tax policy, the following corollaries can be obtained.

Corollary 9. *For risk-neutral firms, the optimal production under two carbon policies satisfies the following conditions:*

$$\begin{cases} q^*_{cnu} > q^*_{tnu}, & \text{if } K_{\mathrm{Cn}} > \frac{Ue(p-c-te)}{p}, \\ q^*_{cnu} = q^*_{tnu}, & \text{if } K_{\mathrm{Cn}} = \frac{Ue(p-c-te)}{p}, \\ q^*_{cnu} < q^*_{tnu}, & \text{if } K_{\mathrm{Cn}} < \frac{Ue(p-c-te)}{p}. \end{cases}$$

Proof. Under the carbon cap policy, the optimal production is $q^*_{cnu} = \frac{U(p-c-er_1)}{p}$, and the Lagrange multiplier is $r_1 = \frac{Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}$. Substituting r_1 into q^*_{cnu}, we have $q^*_{cnu} = \frac{K_{\mathrm{Cn}}}{e}$. Under a carbon tax policy, the optimal production is $q^*_{tnu} = \frac{U(p-c-te)}{p}$. From $q^*_{cnu} - q^*_{tnu} = \frac{pK_{\mathrm{Cn}}-Ue(p-c-te)}{pe}$, we can obtain that $q^*_{cnu} > q^*_{tnu}$ if $K_{\mathrm{Cn}} > \frac{Ue(p-c-te)}{p}$, $q^*_{cnu} = q^*_{tnu}$ if $K_{\mathrm{Cn}} = \frac{Ue(p-c-te)}{p}$, and $q^*_{cnu} < q^*_{tnu}$ if $K_{\mathrm{Cn}} < \frac{Ue(p-c-te)}{p}$. □

From Corollary 9, we find that the optimal production of risk-neutral firms under different carbon policies is different unless the carbon emission capacity and carbon tax satisfy $K_{\mathrm{Cn}} = \frac{Ue(p-c-te)}{p}$. Otherwise, when the emission capacity is high and the carbon tax is also high (that is, $K_{\mathrm{Cn}} > \frac{Ue(p-c-te)}{p}$), the optimal production with the carbon cap policy is higher than that of the carbon tax policy ($q^*_{cnu} > q^*_{tnu}$). The carbon tax policy will impose a heavier burden on firms. When the emission capacity is small and the carbon tax is also small (that is, $K_{\mathrm{Cn}} < \frac{Ue(p-c-te)}{p}$, the optimal production with the carbon cap policy is higher than that of the carbon tax policy ($q^*_{cnu} < q^*_{tnu}$). The carbon cap policy will impose a heavier burden on firms.

Corollary 10. *For risk-averse firms, the optimal production under two carbon policies satisfies the following conditions: for risk-neutral firms, the optimal production under two carbon policies satisfies the following conditions:*

$$\begin{cases} q^*_{cru} > q^*_{tru}, & \text{if } K_{\mathrm{Cr}} > \frac{Ue(\eta(p-c)-te)}{p}, \\ q^*_{cru} = q^*_{tru}, & \text{if } K_{\mathrm{Cr}} = \frac{Ue(\eta(p-c)-te)}{p}, \\ q^*_{cru} < q^*_{tru}, & \text{if } K_{\mathrm{Cr}} < \frac{Ue(\eta(p-c)-te)}{p}. \end{cases}$$

Proof. Under the carbon cap policy, the optimal production is $q^*_{cru} = \frac{\eta U(p-c-er_2)}{p}$, and the Lagrange multiplier is $r_2 = \frac{\eta Ue(p-c)-pK_{\mathrm{Cn}}}{Ue^2}$. Substituting r_2 into q^*_{cru}, we have $q^*_{cru} = \frac{\eta(1-\eta)U(p-c)}{p} + \frac{\eta K_{\mathrm{Cn}}}{e}$. Under a carbon tax policy, the optimal production is $q^*_{tru} = \frac{\eta U(p-c-te)}{p}$. From $q^*_{cru} - q^*_{tru} = \frac{\eta}{pe}[pK_{\mathrm{Cr}} - Ue(\eta(p-c)-te)]$, we can obtain that $q^*_{cru} > q^*_{tru}$ if $K_{\mathrm{Cr}} > \frac{Ue(\eta(p-c)-te)}{p}$, $q^*_{cru} = q^*_{tru}$ if $K_{\mathrm{Cr}} = \frac{Ue(\eta(p-c)-te)}{p}$, and $q^*_{cru} < q^*_{tru}$ if $K_{\mathrm{Cr}} < \frac{Ue(\eta(p-c)-te)}{p}$. □

From Corollary 10 we can also find that the optimal production of risk-averse firms under different carbon policies is different unless the carbon emission capacity and carbon tax satisfy $K_{\mathrm{Cr}} = \frac{Ue(\eta(p-c)-te)}{p}$. Compared with Corollary 9, the results show that there is an additional influence of the risk aversion degree for risk-averse firms. Furthermore, the impact of risk aversion is only shown in the unit income and will not affect the cost of the carbon tax. In contrast with risk-neutral firms, for a given carbon tax, the emission capacity that makes the optimal production of risk-averse firms under two carbon policies equal will be smaller (that is, $K_{\mathrm{Cr}} < K_{\mathrm{Cn}}$). Therefore, if the government sets emission capacity without considering firms' risk aversion attitude, that is, implementing the same capacity to control carbon emissions, then it is more likely that the optimal production under a carbon cap policy is higher than that under a carbon tax policy.

6. Numerical analysis

In this section, we conduct a numerical experiment to verify the theoretical results obtained in the proposed models. We analyze the impact of risk aversion η and demand U on a firm's operational decision and profit, the impact of risk aversion and demand on carbon capacity K, and the impact of two different carbon policies, namely, carbon capacity K and carbon tax t, on optimal production and profit. On the basis of considering the actual situation and literature, we select the numerical data of the model parameters. Specifically, we set the parameters of numerical examples with reference to [5, 40, 45] and the parameters are chosen as $p = 40,\ c = 10,\ e = 1,\ t = 20,\ K = 1200,\ U = 3000$, and $\eta = (0, 1]$.

Under two different carbon policies, namely, carbon cap and carbon tax, the impact of risk aversion and demand on firms' optimal production and profit is shown in Figures 1 and 2.

Figure 1 shows the impact of risk aversion on firms' optimal production and profit under carbon cap and carbon tax policies. It can be seen that the decision-making of the risk-neutral firm is not affected by the degree of risk aversion (see two horizontal lines), while that of the risk-averse firm is affected (see the green line and the blue line). Under a carbon cap policy, the optimal production of a risk-averse firm (see the green line) increases first and then decreases with risk aversion (that is, η decreases). Moreover, the optimal production of risk-averse firms may be higher than that of risk-neutral firms under some conditions (see the green line and the magenta line). Under a carbon tax policy, the stronger the risk aversion is, the smaller the optimal production is (see the blue line). From the perspective of firms' profit, risk aversion has a negative influence under a carbon tax; that is, the profit of risk-averse firms is lower than that of risk-neutral firms, which is consistent with existing studies. However, under a carbon cap policy, the profit of a risk-averse firm may be higher than that of a risk-neutral firm in some cases. This result is interesting and indicates that the carbon cap policy would change the negative influence of risk aversion on firms' operational decisions.

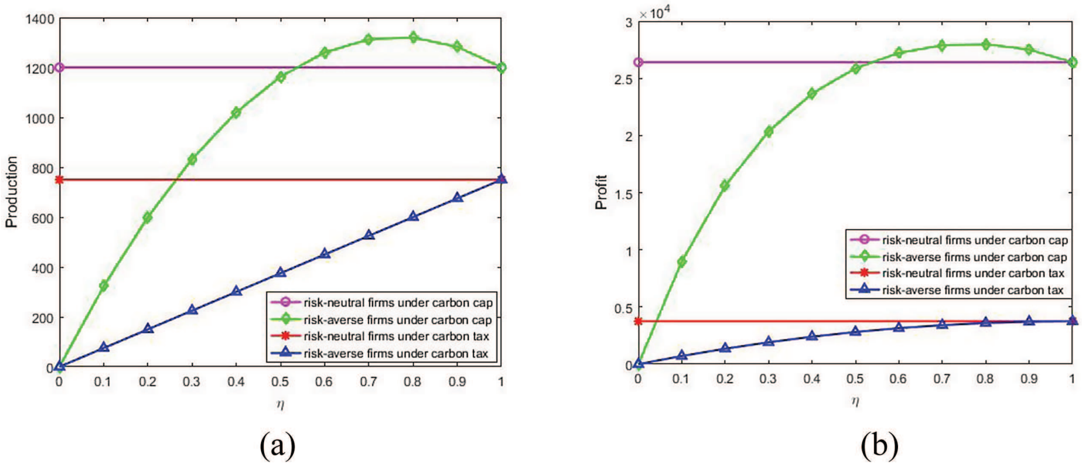


FIGURE 1. The impact of risk aversion η on a firm's operational decisions. (a) The impact of η on production and (b) the impact of η on profit.

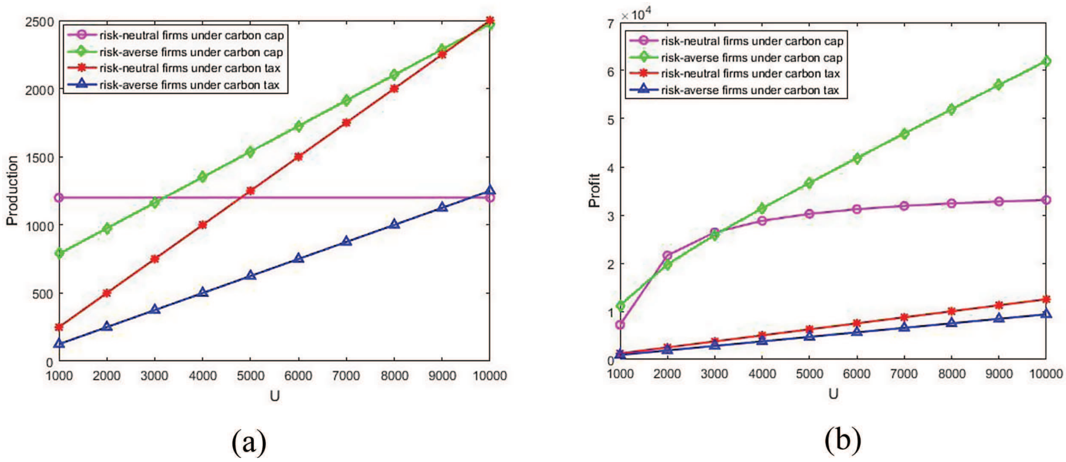
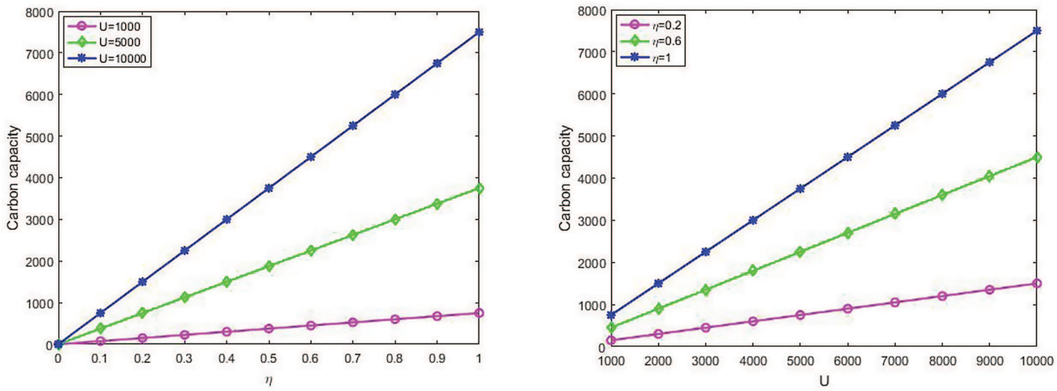


FIGURE 2. The impact of demand U on a firm's operational decisions. (a) The impact of demand U on production and (b) the impact of demand U on profit.

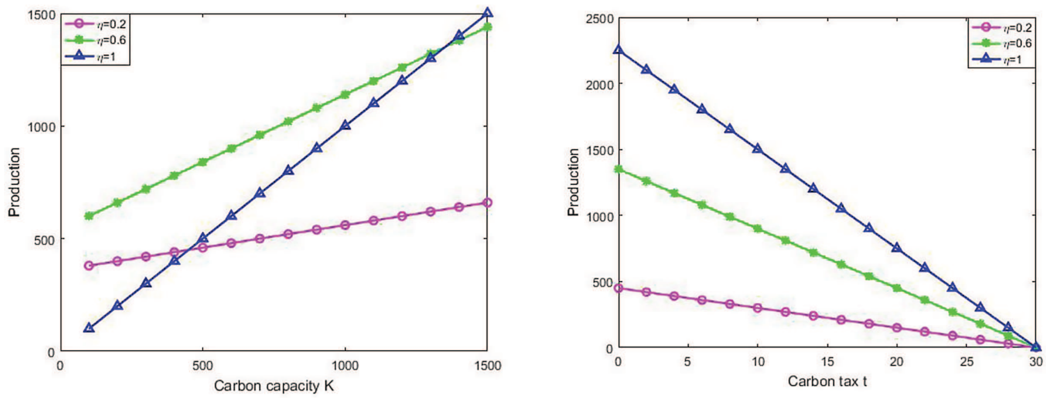
Figure 2 shows the impact of demand on firms' optimal production and profit under carbon cap and carbon tax. Setting $\eta = 0.5$, U changes from 1000 to 10 000 in steps of 1000. It can be shown that the optimal production increases with demand except in the case of risk-neutral firms under the carbon cap policy. When $U > 3000$, the optimal production of risk-averse firms is higher than that of risk-neutral firms, which verifies the conclusion of Figure 1(a). From the perspective of profit, the decreased demand brings increased profit for different carbon policies and different risk attitudes. Moreover, when demand is within a certain range, the profit of risk-averse firms is higher than that of risk-neutral firms.

Figure 3 shows the impact of risk aversion η and demand U on the upper limit of carbon emission capacity. Figure 3(a) considers three different demands, that is, $U = 1000,\ 5000,\ 10\,000$, and we find that higher demand and less risk aversion bring a higher upper limit of carbon emission capacity. Figure 3(b) considers three different risk aversion levels, namely, $\eta = 0.2,\ 0.6$, and 1; we can also find that higher demand and less risk aversion



(a) (b)

FIGURE 3. The impact of risk aversion η and demand U on carbon capacity. (a) The impact of η on carbon capacity and (b) the impact of U on carbon capacity.



(a) (b)

FIGURE 4. The impact of parameters of carbon policies on production. (a) The impact of carbon capacity on production and (b) the impact of carbon tax on production.

bring a higher upper limit of carbon emission capacity. The results show that the upper limit of carbon emission capacity is positively related to risk aversion and demand.

Furthermore, we analyze the impact of two different carbon policies, carbon cap and carbon tax, on the optimal production and profit for risk-neutral firms and risk-averse firms, which is shown in Figures 4 and 5.

Figure 4 shows the impact of carbon cap and carbon tax on the optimal production for risk-neutral firms and risk-averse firms. Setting $\eta = 0.2$, 0.6, and 1, K changes from 0 to 1500 in steps of 100, and t changes from 0 to 30 in steps of 3. Under a carbon tax policy, no matter how much risk aversion there is, the optimal production decreases with the carbon tax. Under the carbon cap policy, the optimal production decreases with the carbon capacity. Furthermore, production with risk aversion of 0.6 is higher than that with risk aversion of 0.2, which is consistent with the comment result that risk aversion has a negative influence. However, for risk-neutral firms ($\eta = 1$), this result is tenable. From Figure 4(a), we find that the production of risk-neutral firms may be lower or higher than that of risk-averse firms with the change in carbon emission capacity. This is because the

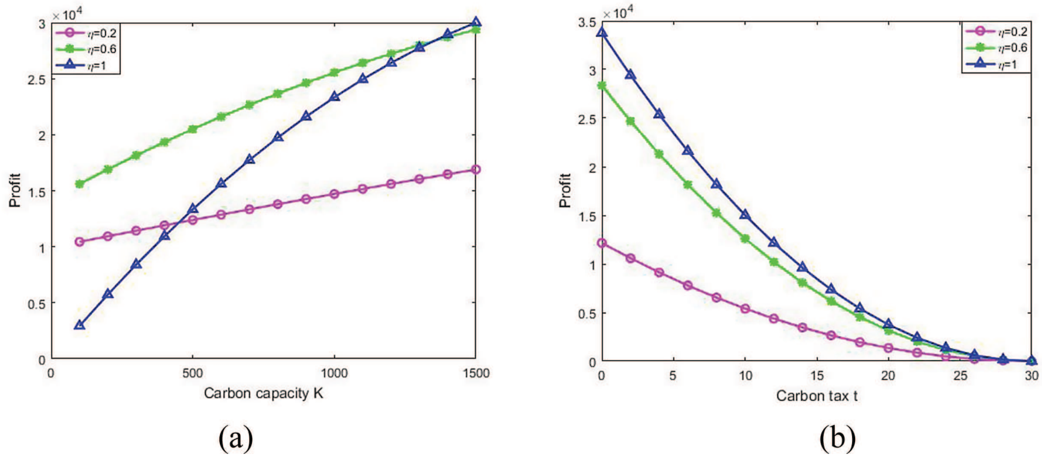


FIGURE 5. The impact of parameters of carbon policies on profit. (a) The impact of carbon capacity on profit and (b) the impact of carbon tax on profit.

production of risk-neutral firms is only related to carbon capacity and emissions per unit of product, which is shown in Proposition 1.

Figure 5 shows the impact of carbon cap and carbon tax on the optimal profit for risk-neutral firms and risk-averse firms. Consistent with Figure 4, we set $\eta = 0.2$, 0.6, and 1, and K changes from 0 to 1500 in steps of 100, while t changes from 0 to 30 in steps of 3. The profit results are basically consistent with the information shown in Figure 4. The optimal profit decreases with the carbon tax under the carbon tax policy, and the optimal profit increases with the carbon capacity under the carbon cap policy. Considering risk aversion, a higher degree of risk aversion leads to less profit except for risk-neutral firms under a strict carbon cap policy.

7. CONCLUSIONS AND LIMITATIONS OF THE RESEARCH

Considering the risk preference of decision-makers, this study incorporates different risk attitudes into firms' decision-making process under the constraints of carbon cap and carbon tax policies. First, we construct the decision optimization model under two carbon policies based on the CVaR criterion to measure firms' risk aversion. The optimal decisions of firms can be obtained by using KKT conditions and convex optimization theory. Then, we compare the optimal operational decisions for risk-neutral firms and risk-averse firms with different carbon policies. Finally, numerical studies are carried out to verify the impact of risk aversion and carbon policies on firms' operational decisions and profit. The results show that the optimal production of risk-averse firms is lower than that of risk-neutral firms under the carbo tax policy. A carbon cap policy would change the influence of risk aversion on firms' operational decisions, and the optimal production of risk-averse firms may be higher than that of risk-neutral firms. This also means that risk aversion may not only have a negative impact on firms' production decisions. In addition, when the carbon capacity and carbon tax satisfy some conditions, the optimal decisions of production are consistent under the two carbon policies. This shows that different carbon policies may also have the same impact on firms, which will depend on the setting of carbon policy parameters.

However, our paper has some limitations. This study assumes that the demand function follows a uniform distribution. Further research can be conducted from other distribution functions or unknown demand functions in the future. In addition, the model proposed in this study only investigates the optimization of firms' production decisions under carbon policies and does not consider active emission reduction actions, such as investing in cleaner technologies and equipment. Thus, the joint decision optimization of a firm's production and emission reduction is an interesting research issue.

Acknowledgements. This study is financially supported by the Beijing Social Science Foundation (No. 22JCC113), the National Natural Science Foundation of China (Grant No. 72271141), the Youth Innovation Science and Technology Support Program of Shandong Province Higher Education (Grant No. 2021RW024), and the Special funds for Taishan Scholars, Shandong (Grant No. tsqn202103063).

References

[1] N. Absi, S. Dauzère-Pérès, S. Kedad-Sidhoum, B. Penz and C. Rapine, Lot sizing with carbon emission constraints. *Eur. J. Oper. Res.* **227** (2013) 55–61.
[2] Q. Bai and M. Chen, The distributionally robust newsvendor problem with dual sourcing under carbon tax and cap-and-trade regulations. *Comput. Ind. Eng.* **98** (2016) 260–274.
[3] Q. Bai, J. Xu and S.C. Satyaveer, Effects of sustainability investment and risk aversion on a two-stage supply chain coordination under a carbon tax policy. *Comput. Ind. Eng.* **142** (2020) 106–324.
[4] Q. Bai, M. Chen, Y. Nikolaidis and J. Xu, Improving sustainability and social responsibility of a two-tier supply chain investing in emission reduction technology. *Appl. Math. Model.* **95** (2021) 688–714.
[5] Q. Bai, J. Xu, Y. Gong and S.S. Chauhan, Robust decisions for regulated sustainable manufacturing with partial demand information: mandatory emission capacity versus emission tax. *Eur. J. Oper. Res.* **298** (2022) 874–893.
[6] S. Benjaafar, Y. Li and M. Daskin, Carbon footprint and the management of supply chains: insights from simple models. *IEEE Trans. Autom. Sci. Eng.* **10** (2013) 99–116.
[7] Q. Chai, Z. Xiao, K.H. Lai and G. Zhou, Can carbon cap and trade mechanism be beneficial for remanufacturing? *Int. J. Prod. Econ.* **203** (2018) 311–321.
[8] A. Chelly, I. Nouira, A.B. Hadj-Alouane and Y. Frein, A comparative study of progressive carbon taxation strategies: impact on firms' economic and environmental performances. *Int. J. Prod. Res.* **60** (2021) 3476–3500.
[9] Y. Chen, M. Xu and Z.G. Zhang, A risk-averse newsvendor model under the CVaR criterion. *Oper. Res.* **57** (2009) 1040–1044.
[10] J. Dai and W. Meng, A risk-averse newsvendor model under marketing-dependency and price-dependency. *Int. J. Prod. Econ.* **160** (2015) 220–229.
[11] S.K. Das and S.K. Roy, Effect of variable carbon emission in a multi-objective transportation-p facility location problem under neutrosophic environment. *Comput. Ind. Eng.* **132** (2019) 311–324.
[12] S.K. Das and S.K. Roy, An approximation approach for fixed-charge transportation-p-facility location problem, in LSCM 2020: Logistics and Supply Chain Management. Springer International Publishing, Cham (2021) 219–237.
[13] S.K. Das, S.K. Roy and G.W. Weber, An exact and a heuristic approach for the transportation-p-facility location problem. *Comput. Manag. Sci.* **17** (2020) 389–407.
[14] S.K. Das, S.K. Roy and G.W. Weber, Heuristic approaches for solid transportation-p-facility location problem. *Cent. Eur. J. Oper. Res.* **28** (2020) 939–961.
[15] S.K. Das, S.K. Roy and G.W. Weber, Application of type-2 fuzzy logic to a multi objective green solid transportation-location problem with dwell time under carbon tax, cap, and offset policy: fuzzy versus nonfuzzy techniques. *IEEE Trans. Fuzzy Syst.* **28** (2020) 2711–2725.
[16] S.K. Das, M. Pervin, S.K. Roy and G.W. Weber, Multi-objective solid transportation-location problem with variable carbon emission in inventory management: a hybrid approach. *Ann. Oper. Res.* **324** (2023) 283-309.
[17] C. Dong, B. Shen, P.S. Chow, L. Yang and C.T. Ng, Sustainability investment under cap-and-trade regulation. *Ann. Oper. Res.* **240** (2016) 509–531.
[18] G. Dou and K. Cao, A joint analysis of environmental and economic performances of closed-loop supply chains under carbon tax regulation. *Comput. Ind. Eng.* **146** (2020) 106624.
[19] G. Duan, F. Nur, M. Alizadeh, L. Chen, M. Marufuzzaman and J. Ma, Vessel routing and optimization for marine debris collection with consideration of carbon cap. *J. Clean. Prod.* **263** (2020) 121399.
[20] Y.Y. Fan, M. Wang and L.D. Zhao, Production-inventory and emission reduction investment decision under carbon cap-and-trade policy. *RAIRO: OR* **52** (2018) 1043–1067.
[21] A. Ghosh, Optimisation of a production-inventory model under two different carbon policies and proposal of a hybrid carbon policy under random demand. *Int. J. Sustain. Eng.* **14** (2021) 280–292.
[22] A. Ghosh, J.K. JhA and S.P. Sarmah, Optimal lot-sizing under strict carbon cap policy considering stochastic demand. *Appl. Math. Model.* **44** (2017) 688–704.
[23] D. Krass, T. Nedorezov and A. Ovchinnikov, Environmental taxes and the choice of green technology. *Prod. Oper. Manag.* **22** (2013) 1035–1055.
[24] S. Kumar, B. Sarkar and A. Kumar, Fuzzy reverse logistics inventory model of smart items with two warehouses of a retailer considering carbon emissions. *RAIRO: OR* **55** (2021) 2285–2307.
[25] J. Li, Q. Su and L. Ma, Production and transportation outsourcing decisions in the supply chain under single and multiple carbon policies. *J. Clean. Prod.* **141** (2017) 1109–1122.

[26] J. Liu and C. Hu, Study on green supply chain cooperation and carbon tax policy considering consumer's behavior. *Math. Prob. Eng.* **2020** (2020) 1–17.

[27] X. Meng, Z. Yao, J. Nie, Y. Zhao and Z. Li, Low-carbon product selection with carbon tax and competition: effects of the power structure. *Int. J. Prod. Econ.* **200** (2018) 224–230.

[28] A.M.A. Mohammad and Z.S. Shokri, Risk-averse multi-product selective newsvendor problem with different market entry scenarios under CVaR criterion. *Comput. Ind. Eng.* **103** (2017) 250–261.

[29] G.C. Pflug, A value-of-information approach to measuring risk in multi-period economic activity. *J. Bank. Financ.* **26** (2006) 1443–1471.

[30] Q. Qi, J. Wang and Q. Bai, Pricing decision of a two-echelon supply chain with one supplier and two retailers under a carbon cap regulation. *J. Clean. Prod.* **151** (2017) 286–302.

[31] Q. Qi, R. Zhang and Q. Bai, Joint decisions on emission reduction and order quantity by a risk-averse firm under cap-and-trade regulation. *Comput. Ind. Eng.* **162** (2021) 107783.

[32] A. Rami, M. Hariga and A. Shamayleh, Sustainable dynamic lot sizing models for cold products under carbon cap policy. *Comput. Ind. Eng.* **149** (2020) 106800.

[33] R.T. Rockafellar and S. Uryasev, Optimization of conditional value-at-risk. *J. Risk* **2** (2000) 21–42.

[34] R.T. Rockafellar and S. Uryasev, Conditional value-at-risk for general loss distributions. *J. Bank. Financ.* **26** (2002) 1443–1471.

[35] M.D. Roy and S.S. Sana, Multi-echelon green supply chain model with random defectives, remanufacturing and rework under setup cost reduction and variable transportation cost. *Sādhanā* **46** (2021) 211.

[36] M.D. Roy and S.S. Sana, The return and remanufacturing rates in a multi-stage green supply chain system with rework, variable shipping cost and ordering cost reduction. *Int. J. Syst. Sci. Oper. Logist.* **10** (2023) 2113174

[37] R. Schur, J. Gönsch and M. Hassler, Time-consistent, risk-averse dynamic pricing. *Eur. J. Oper. Res.* **277** (2019) 587–603.

[38] M.E. Schweitzer and G.P. Cachon, Decision bias in the newsvendor problem with a known demand distribution: experimental evidence. *Manag. Sci.* **46** (2000) 404–420.

[39] L. Shaw, S.K. Das and S.K. Roy, Location-allocation problem for resource distribution under uncertainty in disaster relief operations. *Socio-Econ. Plan. Sci.* **82** (2022) 101232.

[40] H. Sun and J.K. Gao, Research on the carbon emission regulation and optimal state of market structure: based on the perspective of evolutionary game of different stages. *RAIRO: OR* **56** (2022) 2351–2366.

[41] H. Sun and J. Yang, Optimal decisions for competitive manufacturers under carbon tax and cap-and-trade policies. *Comput. Ind. Eng.* **156** (2021) 107244.

[42] D. Wang, S. Dimitrov and L. Jian, Optimal inventory decisions for a risk-averse retailer when offering layaway. *Eur. J. Oper. Res.* **284** (2020) 108–120.

[43] M. Wu, S. Zhu and R.H. Teunter, The risk-averse newsvendor problem with random capacity. *Eur. J. Oper. Res.* **231** (2013) 328–336.

[44] X. Xu, C.C. Chan and A. Langevin, Coping with risk management and fill rate in the loss-averse newsvendor model. *Int. J. Prod. Econ.* **195** (2018) 296–310.

[45] L. Xu, C. Wang, Z. Miao and J. Chen, Governmental subsidy policies and supply chain decisions with carbon emission limit and consumer's environmental awareness. *RAIRO: OR* **53** (2019) 1675–1689.

[46] X. Xu, Z. Meng, R. Shen, M. Jiang and P. Ji, Optimal decisions for the loss-averse newsvendor problem under CVaR. *Int. J .Prod. Econ.* **164** (2015) 146–159.

[47] W. Xue, L. Ma and H. Shen, Optimal inventory and hedging decisions with CVaR consideration. *Int. J. Prod. Econ.* **162** (2015) 70–82.

[48] H. Yang and W. Chen, Retailer-driven carbon emission abatement with consumer environmental awareness and carbon tax: revenue-sharing versus Cost-sharing. *Omega* (2017) 1–13.