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OPTIMAL PRICING STRATEGY IN AN UNRELIABLE M/G/1 RETRIAL QUEUE WITH BERNOULLI PREVENTIVE MAINTENANCE

Fan Xu, Ruiling Tian* and Qi Shao

Abstract. This paper studies an unreliable M/G/1 retrial queue with Bernoulli preventive maintenance, in which the server may break down while providing service to customers. Before the start of each service, the server is subjected to preventive maintenance with a certain probability, which extends the lifetime of the server. If arriving customers find the server unavailable, they decide to balk or enter the orbit based on the linear reward-cost structure. First, the stationary distribution of system states and some important performance measures are derived. Then, we investigate the equilibrium joining strategies of customers in both cooperative and non-cooperative cases, respectively. Next, the optimal pricing strategy is considered from the perspective of the social planner. Finally, numerical examples are provided to illustrate the influence of system parameters on customer equilibrium joining probabilities and the optimal pricing strategy.

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1. Introduction

In recent years, considering customers' rational retrial behavior in queueing systems has become an important direction in the research of queueing theory. Specifically, if customers find the server unavailable upon arrival, they will enter a virtual orbit according to a certain probability rather than waiting in the queueing system. This retrial queue model was first proposed by Artalejo [2]. In the following decades, retrial queueing systems have been widely used in radio networks, computer networks and cloud computing data centers. Nazarov *et al.* [12] investigated diffusion limit for single-server retrial queues with renewal input and outgoing calls. Wang and Li [18] studied the application of retrial queueing systems with priority in radio networks with random access. Phung-Duc and Kawanishi [14] considered a multi-server retrial queue with setup times, which can be applied to data centers. Kim *et al.* [9] studied threshold control for a single-server retrial queue with batch arrivals and group services, and derived the stationary distribution.

It is well known that server breakdowns are usually unavoidable in practical applications. Once the server breaks down, it may cause economic losses and safety incidents. Therefore, it is crucial to study the unreliable retrial queue for practical applications. The reliability of the retrial queue with server breakdowns and repairs is analyzed by Wang *et al.* [19]. Melikov *et al.* [11] and Klimenok *et al.* [10] studied unreliable retrial queues with

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School of Science, Yanshan University, Hebei Qinhuangdao 066004, P.R. China.
*Corresponding author: tianrl@ysu.edu.cn

delayed feedback and with backup servers, respectively. Existing studies of unreliable retrial queues focused on corrective maintenance. However, in order to avoid the impact of server failure, it is necessary to carry out preventive maintenance on the server. Aissani *et al.* [1] analyzed a repairable retrial queue under postponed preventive actions and impatient customers, and obtained the joint probability distribution of the server state. Wang *et al.* [22] compared four unreliable retrial systems with preventive maintenance from a reliability perspective and provided observational insights into decision-making.

There is an important problem with preventive maintenance policy, namely, excessive preventive maintenance is likely to cause unnecessary waste of resources. To deal with this problem, many systems perform preventive maintenance according to their actual situation in a certain proportion. This proportion is called the system's inclination regarding preventive maintenance. For example, the aviation system has high requirements for system security. Therefore, preventive maintenance is necessary before each service. However, for service facilities such as elevators, it is not necessary to perform preventive maintenance at every service. Therefore, in view of the actual application, we propose a new preventive maintenance policy, called Bernoulli preventive maintenance.

The above literatures mainly focus on the performance analysis of retrial systems. However, the analysis of retrial queueing systems from an economic perspective is important for system management. Burnetas and Economou [3] analyzed equilibrium strategies in an M/M/1 queue with setup times. Equilibrium balking strategies in the observable repairable queue were derived by Economou and Kanta [5]. Economou and Kanta [6] explored a constant retrial queue from an economic viewpoint, and solved the social profit maximization problems for both the observable and unobservable cases. Wang and Zhang [17] discussed the equilibrium joining strategies in M/M/1 retrial queues under different observable cases. Based on [17], Zhang *et al.* [23] investigated equilibrium customer strategies in the single-server constant retrial queue with breakdowns. The social welfare maximization problem in a constant retrial queue with the N-policy was solved by Wang *et al.* [20]. Tian and Wang [15] investigated the pricing strategy in an M/M/1 queue with a single working vacation and multiple vacations. They derived the performance measures by using the quasi birth and death (QBD) process and matrix-geometric solution method. Wang *et al.* [21] studied customers' equilibrium strategies in a queue with working vacation and setup times. The customer's strategic behavior in constant retrial queues with working vacations was considered by Do *et al.* [4]. Gao *et al.* [7] derived pricing strategies for an unreliable retrial queue with limited idle period and single vacation. Panda and Goswami [13] explored the equilibrium problem of queues with negative customers and working vacations under four information levels, and obtained the equilibrium joining strategies of positive customers.

However, to our knowledge, there are no papers that provide an economic analysis of unreliable retrial queues with Bernoulli preventive maintenance, which motivates us to study such retrial queues from an economic perspective. A potential application is shown in Section 2. In summary, the main contributions of this paper are briefly listed as follows:

– A novel Bernoulli preventive maintenance policy is outlined from practical applications, which provides a more accurate description of of the repairable system.
– Different from [15], this paper investigates server breakdowns and Bernoulli preventive maintenance for retrial queues, in which both service time and repair time follow the general distribution. We adopt the supplementary variable method and probability generating functions to derive the performance measures.
– The closed-form solution of optimal joining strategies of customers in the non-cooperative and cooperative cases are obtained, respectively. Then, a pricing strategy is derived, which realizes the consistency between the individual equilibrium joining strategy and the socially optimal joining strategy. These results help decision-makers to better understand the difference in customers' joining strategies under different cases.

This paper is organized as follows. The model description and a practical example are provided in Section 2. In Section 3, we provide the steady-state analysis of our model. The equilibrium joining strategies of non-cooperative case and cooperative case are derived in Section 4. In Section 5, we derive the optimal pricing problem from the perspective of the social planner. Section 6 provides numerical examples. Finally, Section 7 concludes the paper with a brief summary.

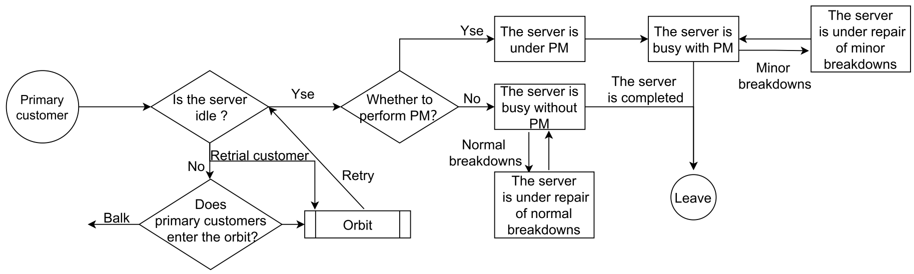

FIGURE 1. The operation mechanism of our model.

2. MODEL DESCRIPTION

2.1. Model description

We consider an unreliable M/G/1 retrial queueing system with Bernoulli preventive maintenance, in which the arriving customer only can observe whether the server is available, but can not observe the number of customers at the arrival instant. The primary customers arrive according to a Poisson process with parameter λ. The service time follows a general distribution with cumulative distribution function $B(x)$, probability density function $b(x)$, finite first two moments $b_1,\ b_2$, and the hazard rate function $\mu(x) = \frac{b(x)}{1-B(x)}$.

If the arriving customer finds that the server is free, the customer occupies the server immediately. Otherwise, the arriving customer must decide whether to enter the orbit (with probability q) or balk (with probability $1-q$). Then, each retrial customer repeats his demand when the server is idle until he/she receives the service. Assume that the inter-retrial time from each retrial customer follows an exponential distribution with parameter θ.

The system adopts the Bernoulli preventive maintenance policy. The proportion of services that have performed preventive maintenance among all services is represented by an exogenous parameter $\eta \in [0,1]$, which measures the inclination of the system for preventive maintenance. Therefore, the server performs preventive maintenance with probability η or directly starts service with probability $1-\eta$. Assume that the time of preventive maintenance follows an exponential distribution with parameter ξ.

This paper assumes that preventive maintenance is not perfect. The server is subject to breakdown while providing the service to customers. If the server does not obtain preventive maintenance, the server breaks down at an exponential rate β_1 (called a normal breakdown). After the server receives preventive maintenance, the server breaks down at an exponential rate β_2 (called a minor breakdown, and $\beta_2 < \beta_1$). If the server breaks down, the server is repaired immediately. The repair time of the normal breakdown follows a general distribution with cumulative distribution function $G(x)$, probability density function $g(x)$, finite first two moments $g_1,\ g_2$, and the hazard rate function $\gamma(x) = \frac{g(x)}{1-G(x)}$. The repair time of the minor breakdown follows a general distribution with cumulative distribution function $V(x)$, probability density function $\nu(x)$, finite first two moments $\nu_1,\ \nu_2$, and the hazard rate function $\omega(x) = \frac{\nu(x)}{1-V(x)}$. The service time is cumulative. When service is interrupted by breakdowns, the customer who is receiving service will wait for the server to complete the remainder of its service. As soon as the server is repaired, the server begins providing the remaining services to the customer. The operation mechanism of our model is shown in Figure 1.

Throughout the paper, we assumed that all random variables defined above are mutually independent, and define $F^*(s) = \int_0^\infty e^{-sx} dF(x)$ as the Laplace-Stieltjes transform (LST) of $F(x)$, and $\overline{F}(x) = 1 - F(x)$.

This paper considers the linear reward-cost structure, in which each customer receives a reward R after service completion, and has a waiting cost C per unit time due to the delay in the orbit. Under this structure, if a customer finds the server unavailable upon arrival, the customer needs to weigh the service reward against the waiting time cost to decide whether to join the orbit. Therefore, in the following, our interest is mainly focused on the joining strategy for customers who find the server unavailable.

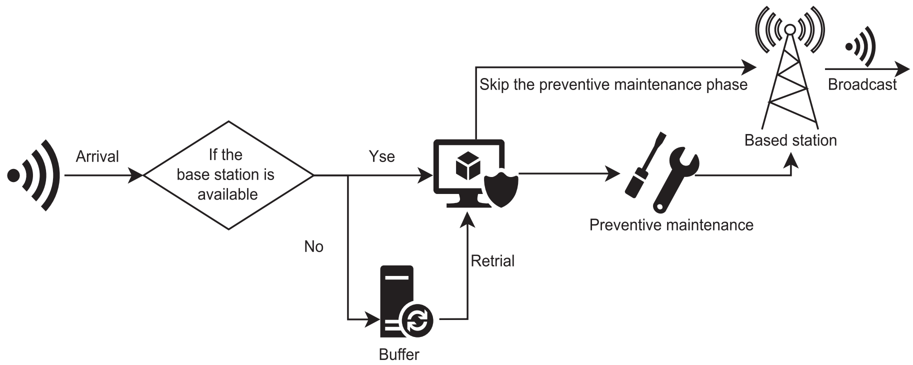

FIGURE 2. Schematic diagram of mobile communication systems.

2.2. Practical example

In mobile communication systems, the base station (server) is responsible for receiving user information and broadcasting it to aim stations. Therefore, the reliability of the base station is crucial for inter-station communication. Many communication companies perform different levels of preventive maintenance on base stations to ensure communication security, depending on the importance of the communication system in which the base station is located. For example, military communication systems tend to always perform preventive maintenance, while civilian communication systems may have slightly lower requirements. User (primary customer) generated information is propagated from the outside to the base station. Once the message is received, it is immediately transmitted to the destination station if the base station is not occupied. If the base station is unavailable, the message is stored in a buffer (orbit) and re-attempted after a period of time until the message is successfully delivered (retrial process). The framework diagram of mobile communication systems that considers Bernoulli preventive maintenance is shown in Figure 2.

3. Steady-state analysis

In this section, some important performance measures are derived to study the customer's equilibrium joining strategy.

Define $N(t)$ and $I(t)$ as the number of retrial customers and the state of the server at time t, respectively, where

$$I(t) = \begin{cases} 0, \text{ if the server is idle,} \\ 1, \text{ if the server without preventive maintenance is busy,} \\ 2, \text{ if the server that gets preventive maintenance is busy,} \\ 3, \text{ if the server is under repair for normal breakdowns,} \\ 4, \text{ if the server is under repair for minor breakdowns,} \\ 5, \text{ if the server is under preventive maintenance.} \end{cases}$$

It is easy to see the stochastic process $\{N(t),\ t \geq 0\}$ is not a Markov process. Thus, let $S^1(t)$, $S^2(t)$, $R^1(t)$ and $R^2(t)$ be elapsed service time if the server without preventive maintenance, elapsed service time if the server gets preventive maintenance, elapsed repair time of normal breakdowns and elapsed repair time of minor breakdowns, respectively at time t. Next, define the following joint probabilities and probability densities:

$$\begin{aligned} P_{n,j}(t) &= P\{N(t) = n,\ I(t) = j\},\ n \geq 0, j = 0, 5, \\ P_{n,1}(t,x)dx &= P\{N(t) = n,\ I(t) = 1,\ x \leq S^1(t) < x + dx\},\ n \geq 0, \\ P_{n,2}(t,x)dx &= P\{N(t) = n,\ I(t) = 2,\ x \leq S^2(t) < x + dx\},\ n \geq 0, \\ P_{n,3}(t,x,y)dxdy &= P\{N(t) = n,\ I(t) = 3,\ x \leq S^1(t) < x + dx, \end{aligned}$$

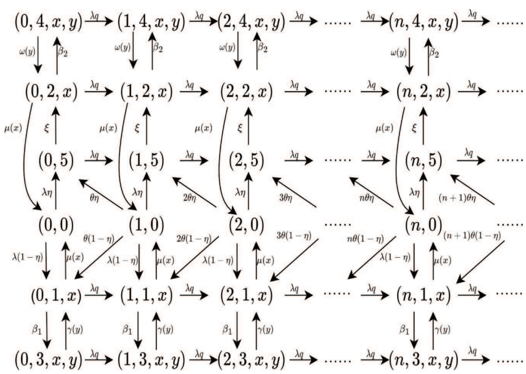

FIGURE 3. State transition diagram of the system.

$$y \le R^1(t) \le y + dy\},\ n \ge 0,$$
$$P_{n,4}(t,x,y)dxdy = P\{N(t) = n,\ I(t) = 4,\ x \le S^2(t) < x + dx,$$
$$y \le R^2(t) \le y + dy\},\ n \ge 0.$$

Therefore, the state of the system can be characterized by the Markov process $\{(N(t), I(t), S^1(t), S^2(t), R^1(t), R^2(t)) \ :\ t \ \ge\ 0\}$. Then, the state transition diagram of the system is shown in Figure 3. In addition, following the similar analysis in Wang and Li [16], we can obtain that the steady-state condition for the system is $\lambda q \left[\frac{\eta}{\xi} + \eta b_1 (1 + \nu_1\beta_2) + (1-\eta)b_1(1 + g_1\beta_1)\right] < 1$.

Define the following joint steady-state probabilities:

$$P_{n,j} = \lim_{t\to\infty} P_{n,j}(t),\ n \ge 0,\ j = 0, 5,$$
$$P_{n,j}(x) = \lim_{t\to\infty} P_{n,j}(t,x),\ n \ge 0,\ j = 1, 2,\ x \ge 0,$$
$$P_{n,j}(x,y) = \lim_{t\to\infty} P_{n,j}(t,x,y),\ n \ge 0,\ j = 3, 4,\ x \ge 0,\ y \ge 0.$$

Then, we have the following Champman-Kolmogorov forward equations under the steady-state condition:

$$(\lambda + n\theta) P_{n,0} = \int_0^\infty \mu(x) \left[P_{n,1}(x) + P_{n,2}(x)\right] dx,\ n \ge 0, \tag{1}$$

$$\left[\frac{d}{dx} + \lambda q + \beta_1 + \mu(x)\right] P_{n,1}(x) = \lambda q(1-\delta_{n,0})P_{n-1,1}(x) + \int_0^\infty P_{n,3}(x,y)\gamma(y)dy,\ n \ge 0, \tag{2}$$

$$\left[\frac{d}{dx} + \lambda q + \beta_2 + \mu(x)\right] P_{n,2}(x) = \lambda q(1-\delta_{n,0})P_{n-1,2}(x) + \int_0^\infty P_{n,4}(x,y)\omega(y)dy,\ n \ge 0, \tag{3}$$

$$\left[\frac{d}{dy} + \lambda q + \gamma(y)\right] P_{n,3}(x,y) = \lambda q(1-\delta_{n,0})P_{n-1,3}(x,y),\ n \ge 0, \tag{4}$$

$$\left[\frac{d}{dy} + \lambda q + \omega(y)\right] P_{n,4}(x,y) = \lambda q(1-\delta_{n,0})P_{n-1,4}(x,y),\ n \ge 0, \tag{5}$$

$$(\lambda q + \xi) P_{n,5} = \lambda\eta P_{n,0} + (n+1)\theta\eta P_{n+1,0} + \lambda q(1-\delta_{n,0})P_{n-1,5},\ n \ge 0, \tag{6}$$

where $\delta_{i,j}$ is the Kronecker delta function.

The boundary conditions are as follows:

$$P_{n,1}(0) = \lambda(1-\eta)P_{n,0} + (n+1)\theta(1-\eta)P_{n+1,0},\ n \geq 0, \tag{7}$$
$$P_{n,2}(0) = \xi P_{n,5},\ n \geq 0, \tag{8}$$
$$P_{n,3}(x,0) = \beta_1 P_{n,1}(x),\ n \geq 0, x > 0, \tag{9}$$
$$P_{n,4}(x,0) = \beta_2 P_{n,2}(x),\ n \geq 0, x > 0, \tag{10}$$

and the normalization condition is

$$\sum_{n=0}^{\infty}\left[P_{n,0} + P_{n,5} + \int_0^{\infty} P_{n,1}(x)dx + \int_0^{\infty} P_{n,2}(x)dx + \int_0^{\infty}\int_0^{\infty} P_{n,3}(x,y)dxdy + \int_0^{\infty}\int_0^{\infty} P_{n,4}(x,y)dxdy\right] = 1. \tag{11}$$

Define the following probability generating functions (P.G.Fs):

$$\pi_i(z) = \sum_{n=0}^{\infty} z^n P_{n,i},\ i = 0, 5,$$
$$\pi_i(z,x) = \sum_{n=0}^{\infty} z^n P_{n,1}(x),\ i = 1, 2,$$
$$\pi_i(z,x,y) = \sum_{n=0}^{\infty} z^n P_{n,2}(x,y),\ i = 3, 4.$$

Multiplying equations (1)–(10) by z^n and summing up to over n, we have

$$\lambda\pi_0(z) + z\theta\pi_0'(z) = \int_0^{\infty}\mu(x)\left[\pi_1(z,x) + \pi_2(z,x)\right]dx, \tag{12}$$
$$\left[\frac{\partial}{\partial x} + \lambda q + \beta_1 + \mu(x)\right]\pi_1(z,x) = \lambda qz\pi_1(z,x) + \int_0^{\infty}\pi_3(z,x,y)\gamma(y)dy, \tag{13}$$
$$\left[\frac{\partial}{\partial x} + \lambda q + \beta_2 + \mu(x)\right]\pi_2(z,x) = \lambda qz\pi_2(z,x) + \int_0^{\infty}\pi_4(z,x,y)\omega(y)dy, \tag{14}$$
$$\left[\frac{\partial}{\partial y} + \lambda q + \gamma(y)\right]\pi_3(z,x,y) = \lambda qz\pi_3(z,x,y), \tag{15}$$
$$\left[\frac{\partial}{\partial y} + \lambda q + \omega(y)\right]\pi_4(z,x,y) = \lambda qz\pi_4(z,x,y), \tag{16}$$
$$(\lambda q + \xi)\pi_5(z) = \lambda\eta\pi_0(z) + \theta\eta\pi_0'(z), \tag{17}$$
$$\pi_1(z,0) = \lambda(1-\eta)\pi_0(z) + \theta(1-\eta)\pi_0'(z), \tag{18}$$
$$\pi_2(z,0) = \xi\pi_5(z), \tag{19}$$
$$\pi_3(z,x,0) = \beta_1\pi_1(z,x), \tag{20}$$
$$\pi_4(z,x,0) = \beta_2\pi_2(z,x). \tag{21}$$

From equations (15), (16), (20) and (21), we can obtain

$$\pi_3(z,x,y) = \beta_1\pi_1(z,x)\overline{G}(y)\exp\{-a(z)y\}, \tag{22}$$
$$\pi_4(z,x,y) = \beta_2\pi_2(z,x)\overline{V}(y)\exp\{-a(z)y\}, \tag{23}$$

where $a(z) = \lambda q(1-z)$.

Substituting equations (22) and (23) into equations (13) and (14), respectively, we have the following equations

$$\pi_1(z,x) = \pi_1(z,0)\overline{B}(x)\exp\{-O_1(z)x\}, \tag{24}$$

$$\pi_2(z,x) = \pi_2(z,0)\overline{B}(x)\exp\{-O_2(z)x\}, \tag{25}$$

where $O_1(z) = a(z) + \beta_1\left[1 - G^*\big(a(z)\big)\right]$ and $O_2(z) = a(z) + \beta_2\left[1 - V^*\big(a(z)\big)\right]$.

According to equations (24), (25) and (17)–(19), we obtain

$$\pi_1(z,x) = \lambda(1-\eta)\frac{[a(z)+\xi](z-1)}{z[a(z)+\xi] - M(z)}\pi_0(z)\overline{B}(x)\exp\{-O_1(z)x\}, \tag{26}$$

$$\pi_2(z,x) = \frac{\lambda\eta\xi[a(z)+\xi](z-1)\pi_0(z)}{[a(z)+\xi]\{z[a(z)+\xi]-M(z)\}}\overline{B}(x)\exp\{-O_2(z)x\}, \tag{27}$$

$$\pi_5(z) = \frac{\lambda\eta(z-1)[a(z)+\xi]}{[a(z)+\xi]\{z[a(z)+\xi]-M(z)\}}\pi_0(z). \tag{28}$$

Combining equations (12), (26) and (27), we get

$$\pi_0'(z) = \frac{\lambda}{\theta}\left\{\frac{M(z) - [a(z)+\xi]}{z[a(z)+\xi]-M(z)}\right\}\pi_0(z), \tag{29}$$

where $M(z) = (1-\eta)[a(z)+\xi]B^*(O_1(Z)) + \eta\xi B^*(O_2(z))$.

Solving equation (29), we have

$$\pi_0(z) = \pi_0(1)\exp\left\{-\frac{\lambda}{\theta}\int_z^1 \frac{M(u)-[a(u)+\xi]}{u[a(u)+\xi]-M(u)}du\right\}. \tag{30}$$

Let $z = 1$ in equations (22), (23) and (26)–(28), we derive

$$\pi_5(1) = \frac{\lambda\eta}{\xi - \lambda q - M'(1)}\pi_0(1), \tag{31}$$

$$\pi_1(1,x) = \frac{\lambda\xi(1-\eta)}{\xi - \lambda q - M'(1)}\pi_0(1)\overline{B}(x), \tag{32}$$

$$\pi_2(1,x) = \frac{\lambda\xi\eta}{\xi - \lambda q - M'(1)}\pi_0(1)\overline{B}(x), \tag{33}$$

$$\pi_3(1,x,y) = \frac{\lambda\xi\beta_1(1-\eta)}{\xi - \lambda q - M'(1)}\pi_0(1)\overline{B}(x)\overline{G}(y), \tag{34}$$

$$\pi_4(1,x,y) = \frac{\lambda\xi\eta\beta_2}{\xi - \lambda q - M'(1)}\pi_0(1)\overline{B}(x)\overline{V}(x). \tag{35}$$

Then, let $c_0 = 1 + (1-\eta)\beta_1 g_1 + \eta\beta_2\nu_1$. According to equations (31)–(35) and the normalization condition, we obtain Theorem 3.1.

Theorem 3.1. *In the steady-state, we have*

(1) The probability that the server is idle

$$P_0 = \frac{\xi - \lambda q - M'(1)}{\xi + \lambda(1-q)(\eta + \xi b_1 c_0)}.$$

(2) The probability that the server without preventive maintenance is busy

$$P_1 = \frac{\lambda\xi b_1(1-\eta)}{\xi+\lambda(1-q)(\eta+\xi b_1 c_0)}.$$

(3) The probability that the server with preventive maintenance is busy

$$P_2 = \frac{\lambda\xi b_1\eta}{\xi+\lambda(1-q)(\eta+\xi b_1 c_0)}.$$

(4) The probability that the server is under repair for normal breakdowns

$$P_3 = \frac{\lambda\xi\beta_1 b_1 g_1(1-\eta)}{\xi+\lambda(1-q)(\eta+\xi b_1 c_0)}.$$

(5) The probability that the server is under repair for minor breakdowns

$$P_4 = \frac{\lambda\xi\beta_2 b_1\nu_1\eta}{\xi+\lambda(1-q)(\eta+\xi b_1 c_0)}.$$

(6) The probability that the server is under preventive maintenance

$$P_5 = \frac{\lambda\eta}{\xi+\lambda(1-q)(\eta+\xi b_1 c_0)}.$$

Based on the above analysis, we have the following Theorem 3.2.

Theorem 3.2. *In the steady-state, for an unobservable M/G/1 unreliable retrial queue with Bernoulli preventive maintenance, we derive the following results:*

(1) The mean queue length in orbit is given by

$$N_o = \frac{2\lambda qc_1(\lambda c_1+\theta)+\lambda^3q^2\theta c_1k_2}{2\theta(\xi-\lambda qc_1)\left[\xi+\lambda(1-q)c_1\right]} + \frac{\lambda^2 q(2c_1+\theta k_2)}{2\theta\left[\xi+\lambda(1-q)c_1\right]}. \tag{36}$$

(2) The expected waiting time of retrial customers is given by

$$W = \frac{2c_1(\lambda c_1+\theta)+\lambda q\theta c_1c_2+(\xi-\lambda qc_1)(2c_1+\theta c_2)}{2\theta c_1(\xi-\lambda qc_1)}, \tag{37}$$

where $c_1=\eta+\xi b_1c_0$ *and* $c_2=\xi b_2\left[(1-\eta)(1+\beta_1g_1)^2+\eta(1+\beta_2\nu_1)^2\right]+\xi b_1[(1-\eta)\beta_1g_2+\eta\beta_2\nu_2]-2b_1(1-\eta)(1+\beta_1g_1)$.

Proof. First, let $\pi_i(z)=\int_0^\infty \pi_i(z,x)dx, i=1,2$. From equations (26) and (27), we can obtain $\pi_1(z)$ and $\pi_2(z)$. Similarly, let $\pi_i(z)=\int_0^\infty\int_0^\infty \pi_i(z,x,y)dxdy$, $i=3,4$, we can obtain $\pi_3(z)$ and $\pi_4(z)$.

Thus, the generating function of N_o is $\pi(z)=\sum_{i=0}^{5}\pi_i(z)$, and the mean queue length in orbit is given by $N_o=\frac{\partial\pi(z)}{\partial z}|_{z=1}$. After some algebra calculations, we derive equation (36). Then, using Little's law $W=\frac{N_o}{\lambda_o}$, the expected waiting time of retrial customers is derived, where $\lambda_o=\lambda q\sum_{i=1}^{5}P_i$, which is total arrival rate for retrial customers. □

4. Equilibrium joining strategies

In this section, we investigate the customer equilibrium joining strategies in the non-cooperative and cooperative cases, respectively.

4.1. Equilibrium joining strategy in non-cooperative case

We consider all customers are selfish, *i.e.*, customers are only interested in maximizing their expected net benefit. Customers must decide whether to join the orbit or balk based on the reward-cost structure if the server is unavailable upon their arrival. Specifically, customers are more willing to join the orbit when their reward is greater than the expected waiting cost, and are indifferent to joining the orbit and balking when their reward is equal to the expected waiting cost, and they choose to balk when their reward is less than the expected waiting cost. Each customer's decision has an influence on the decisions of other customers and the system's performance.

Since customers are indistinguishable from each other, we can model the system as a symmetric game among customers. Denote the common strategic set and utility function by X and U, respectively. Specifically, let $U(x, q)$ be the utility of a tagged customer, where x represents the mixed strategy of a tagged customer, and q represents the common mixed strategy of the other customers. A strategy q_e is a (symmetric Nash) equilibrium if $U(q_e, q_e) \geq U(x, q_e)$, for every $x \in X$. This means that it is the best response against itself, *i.e.*, if all customers follow it, no one can benefit by changing it. According to Section 1.2 in [8], transition probabilities are induced by the common strategy selected by all (based on the fact that all use strategy q), which means that the strategy x of the tagged customer does not affect his utility. Therefore, we next focus on Nash equilibrium under non-cooperative conditions, *i.e.*, all customers follow strategy q.

According to the previously mentioned reward-cost structure, when all arriving customers follow the mixed strategy "enter the orbit with probability q if the server is unavailable", the expected net utility of an arriving tagged customer that finds the server unavailable and decides to enter the orbit is

$$\begin{aligned} U(q) &= R - CW \\ &= R - \frac{C\left[2c_1(\lambda c_1 + \theta) + \lambda q\theta c_1 c_2 + (\xi - \lambda q c_1)(2c_1 + \theta c_2)\right]}{2\theta c_1(\xi - \lambda q c_1)}. \end{aligned} \tag{38}$$

Then, we derive Theorem 4.1.

Theorem 4.1. *In the steady-state, for the unreliable M/G/1 retrial queue with Bernoulli preventive maintenance, the customer's symmetric Nash equilibrium strategy is as follows:*

$$q_e = \begin{cases} 0, & \frac{R}{C} \leq W_1, \\ q_e^*, & W_1 < \frac{R}{C} \leq W_2, \\ 1, & W_2 < \frac{R}{C}. \end{cases}$$

where

$$\begin{aligned} W_1 &= \frac{1}{\theta} + \frac{1}{\xi} + \frac{c_2}{2c_1} + \frac{\lambda c_1}{\theta\xi}, \\ W_2 &= \frac{1}{\theta} + \frac{c_2}{2c_1} + \frac{\lambda c_1 + \theta}{\theta(\xi - \lambda c_1)} + \frac{\lambda c_2}{2(\xi - \lambda c_1)}, \\ q_e^* &= \frac{2\theta\xi R c_1 - C\left[2c_1(\lambda c_1 + \theta) + \xi(2c_1 + \theta c_2)\right]}{2\lambda c_1^2(\theta R - C)}. \end{aligned}$$

Proof. It is easy to see that $U(q)$ is strictly decreasing with respect to $q \in [0, 1]$. Therefore, the unique maximum $U(0)$ and minimum $U(1)$ of $U(q)$ for $q \in [0, 1]$ are obtained, respectively, as follows

$$\begin{aligned} U(0) &= R - CW_1, \\ U(1) &= R - CW_2. \end{aligned}$$

Therefore, we have the following three cases. □

Case 1. $\frac{R}{C} \leq W_1 \Leftrightarrow U(0) \leq 0$.
In this case, $U(q)$ is non-positive for every q, which means that even if no other customer joins, the expected utility of a customer who joins is non-positive. Thus, the best response of the tagged customer is to balk, *i.e.*, $q_e = 0$.

Case 2. $W_1 < \frac{R}{C} \leq W_2 \Leftrightarrow U(1) \leq 0 < U(0)$.
In this case, there exists a unique q_e^* satisfying $U(q_e^*) = 0$. Hence, $q_e = q_e^*$.

Case 3. $W_2 < \frac{R}{C} \Leftrightarrow U(1) > 0$.
In this case, $U(q)$ is positive for every q. Thus, regardless of the action of others, the customer has a positive expected utility if the customer decides to enter into the orbit. Thus, $q_e = 1$.

Remark 4.2. The focus of Theorem 4.1 is on identifying a symmetric Nash equilibrium as the solution. The symmetric equilibrium strategy is the best response to itself.

4.2. Equilibrium joining strategy in cooperative case

In this subsection, we turn our research attention to analyzing the equilibrium joining strategy from a social perspective in order to maximize social welfare. In contrast to the non-cooperative case, the social planner encourages customers to collaborate with each other to enter the orbit using the socially optimal joining probability.

When all customers who find the server unavailable upon arrival enter the orbit with probability q, the social net welfare per unit time is as follows:

$$S(q) = \lambda^* R - CN_o$$

where λ^* is the effective arrival rate for customers. According to PASTA property, we have

$$\lambda^* = \lambda P_0 + \lambda q \sum_{i=1}^{5} P_i = \frac{\lambda\xi}{\xi + \lambda(1-q)c_1}.$$

Thus, we obtain

$$\begin{aligned} S(q) = & \frac{2C\lambda^3 c_1^2 q^2 - \lambda^2 \left\{2\theta\xi c_1 R + C\left[2c_1(\xi + \lambda c_1 + \theta) + \theta\xi c_2\right]\right\} q}{2\theta(\xi - \lambda q c_1)\left[\xi + \lambda(1-q)c_1\right]} \\ & + \frac{\xi^2 \lambda R}{(\xi - \lambda q c_1)\left[\xi + \lambda(1-q)c_1\right]}. \end{aligned}$$

Now, we can obtain the socially optimal joining probability q_s, and the result is as follows in Theorem 4.3.

Theorem 4.3. *In the steady-state, for the unreliable M/G/1 retrial queue with Bernoulli preventive maintenance, a unique mixed strategy 'join orbit with probability q_s while observing an unavailable server that maximizes the social benefit per time unit exists and is given by*

$$q_s = \begin{cases} 0, & \frac{R}{C} \leq S_1, \\ \frac{\xi - x_2}{\lambda c_1}, & S_1 < \frac{R}{C} \leq S_2, \\ 1, & S_2 < \frac{R}{C}. \end{cases}$$

where

$$\begin{aligned} x_2 &= \frac{-A_1 - \sqrt{A_1^2 - \lambda c_1 A_0 A_1}}{A_0}, \\ A_0 &= 2c_1\xi C - 2c_1\theta C - c_2\theta\xi C - 2\xi\theta c_1 R, \\ A_1 &= 2c_1^2\lambda\xi C + 2c_1\theta\xi C + \xi^2 c_2\theta C, \end{aligned}$$

$$S_1 = \frac{2c_1\xi - \theta(2c_1 + \xi c_2)}{2\xi\theta c_1} + \frac{(2\xi + \lambda c_1)(2\lambda c_1^2 + 2\theta c_1 + \theta\xi c_2)}{2c_1\theta\xi^2},$$
$$S_2 = \frac{2c_1\xi - \theta(2c_1 + \xi c_2)}{2\xi\theta c_1} + \frac{(2\xi - \lambda c_1)(2\lambda c_1^2 + 2\theta c_1 + \xi\theta c_2)}{2\theta c_1(\xi - \lambda c_1)^2}.$$

Proof. Let $x = \xi - \lambda c_1 q$, then $S(q)$ is rewritten as a function of x

$$f(x) = \frac{\lambda\xi R}{x + \lambda c_1} - C\left[\frac{\lambda(\xi - x)}{\theta x} + \frac{\lambda(\xi - x)}{x(x + \lambda c_1)} + \frac{\lambda\xi c_2(\xi - x)}{2xc_1(x + \lambda c_1)}\right].$$

Therefore, the maximization problem of social welfare can be solved by analyzing the problem of maximizing $f(x)$ for $x \in [\xi - \lambda k_0, \xi]$. Then, taking the first order derivative of $f(x)$ with respect to x, we have

$$f'(x) = \frac{\lambda}{2c_1\theta x^2(x + \lambda c_1 x)^2}\left(A_0x^2 + 2A_1x + \lambda c_1 A_1\right).$$

□

We can find that $f'(x) = 0 \Leftrightarrow A_0x^2 + 2A_1x + \lambda c_1A_1 = 0$. Then, we discuss two cases $A_0 \geq 0$ and $A_0 < 0$, respectively.

Case 1. $A_0 \geq 0 \Leftrightarrow \frac{R}{C} \leq \frac{1}{\theta} - \frac{1}{\xi} - \frac{c_2}{2c_1}$.
In this case, $f'(x) > 0$ for $x \in [\xi - \lambda c_1, \xi]$. Therefore, $f(x)$ is strictly monotonically increasing with respect to x and reaches a unique maximum at $x = \xi$, *i.e.*, $q_s = 0$.

Case 2. $A_0 < 0 \Leftrightarrow \frac{R}{C} > \frac{1}{\theta} - \frac{1}{\xi} - \frac{c_2}{2c_1}$.
In this case, $f(x)$ is not always a monotone function with respect to x in $x \in [\xi - \lambda c_1, \xi]$. Then, we obtain the following two roots by solving $f'(x) = 0$.

$$x_1 = \frac{-A_1 + \sqrt{A_1^2 - \lambda c_1 A_0 A_1}}{A_0} < 0,$$
$$x_2 = \frac{-A_1 - \sqrt{A_1^2 - \lambda c_1 A_0 A_1}}{A_0} > 0.$$

Hence, $f'(x) < 0$ in $(-\infty, x_1)$ and $(x_2, +\infty)$, and $f'(x) > 0$ in (x_1, x_2), respectively. Then, we analyze the following three subcases:

Case 2a. $x_2 \geq \xi \Leftrightarrow \frac{R}{C} \leq S_1$.
In this subcase, $f'(x) > 0$ for $x \in [\xi - \lambda c_1, \xi]$. This means that $f(x)$ is monotone increasing in the interval $[\xi - \lambda c_1, \xi]$. Thus, the unique maximum is achieved at $x = \xi$, *i.e.*, $q_s = 0$.

Case 2b. $\xi - \lambda c_1 \leq x_2 < \xi \Leftrightarrow S_1 < \frac{R}{C} \leq S_2$.
In this subcase, we easily to see that $f'(x) > 0$ for $x \in [\xi - \lambda c_1, x_2]$, and $f'(x) < 0$ for $x \in [x_2, \xi]$. This means that $f(x)$ is monotone increasing in the interval $[\xi - \lambda c_1, x_2]$, and is monotone decreasing in the interval $[x_2, \xi]$. Thus, the unique maximum is achieved at $x = x_2$, *i.e.*, $q_s = \frac{\xi - x_2}{\lambda c_1}$.

Case 2c. $x_2 < \xi - \lambda c_1 \Leftrightarrow S_2 < \frac{R}{C}$.
In this subcase, $f'(x) < 0$ for $x \in [\xi - \lambda c_1, \xi]$. This means that $f(x)$ is monotone decreasing in the interval $[\xi - \lambda c_1, \xi]$. Thus, the unique maximum is achieved at $x = \xi - \lambda c_1$, *i.e.*, $q_s = 1$.

Based on the above analysis, we complete the proof of Theorem 4.3.

Theorem 4.4. *For the unreliable M/G/1 retrial queue with Bernoulli preventive maintenance, the equilibrium joining probability q_e is not smaller than the socially optimal joining probability q_s, i.e., $q_e \geq q_s$.*

Proof. According to Theorems 4.1 and 4.3, we can know that $W_1 < S_1$ and $W_2 < S_2$. Thus, we only need to consider the relationship between S_1 and W_2. Then, we consider the following cases:

Case 1. $\frac{R}{C} \leq W_1$. In this case, $q_e = q_s = 0$.

If $S_1 < W_2$. In this case, we have the following three subcases:

Case 2a. $W_1 < \frac{R}{C} \leq S_1$. In this case, $q_e = q_e^* > q_s = 0$.

Case 2b. $S_1 < \frac{R}{C} \leq W_2$. In this case, $q_e = q_e^*$ and $q_s = \frac{\xi - x_2}{\lambda c_1}$. After some complex algebraic calculations, we have $q_e > q_s$.

Case 2c. $W_2 < \frac{R}{C} \leq S_2$. In this case, $q_e = 1 > q_s = \frac{\xi - x_2}{\lambda c_1}$.

If $W_2 < S_1$. In this case, we have the following three subcases:

Case 3a. $W_1 < \frac{R}{C} \leq W_2$. In this case, $q_e = q_e^* > q_s = 0$.

Case 3b. $W_2 < \frac{R}{C} \leq S_1$. In this case, $q_e = 1 > q_s = 0$.

Case 3c. $S_1 < \frac{R}{C} \leq S_2$. In this case, $q_e = 1 > q_s = \frac{\xi - x_2}{\lambda c_1}$.

Case 4. $\frac{R}{C} > S_2$. In this case, $q_e = q_s = 1$.

From the above derived results, we can obtain $q_e > q_s$. □

5. Optimal pricing analysis

According to Theorem 4.4, we can easily find that the equilibrium joining strategy in the non-cooperative case is different from the socially optimal joining strategy in the cooperative case. In other words, customers who are unwilling to cooperate will excessively use the system, which leads to the fact that social welfare usually can not be maximized. However, in practical applications, social planners usually hope that the individual's equilibrium joining strategy is consistent with the socially optimal strategy. Therefore, the social planner can discourage the willingness that selfish customers to enter the orbit by charging an admission fee p $(0 \leq p < R)$ to customers who decide to enter the orbit. Then, the reward of customers in the non-cooperative case is decreased to $R - p$, which reduces the willingness that selfish customers enter the orbit. Therefore, the equilibrium joining probability $q_e(p)$ changes to

$$q_e(p) = \begin{cases} 0, & \frac{R-p}{C} \leq W_1, \\ q_e^*(p), & W_1 < \frac{R-p}{C} \leq W_2, \\ 1, & W_2 < \frac{R-p}{C}. \end{cases}$$

where

$$q_e^*(p) = \frac{2\theta\xi(R-p)c_1 - C\left[2c_1(\lambda c_1 + \theta) + \xi(2c_1 + \theta c_2)\right]}{2\lambda c_1^2(\theta(R-p) - C)}.$$

On the other hand, the social welfare per unit time is not changed due to the admission fee being transferred from one social group (customers) to another (system managers). Then, the corresponding socially optimal joining probability q_s is also unchanged.

Based on the above analysis, we obtain Theorem 5.1.

Theorem 5.1. *In the steady-state, for the unreliable M/G/1 retrial queue with Bernoulli preventive maintenance, the price that makes $q_e = q_s$ exists and is given by*

$$p_e = \begin{cases} C(S_1 - W_1), & W_1 < \frac{R}{C} \leq S_1, \\ p_2, & S_1 < \frac{R}{C} \leq S_2, \\ 0, & \textit{otherwise}, \end{cases}$$

where

$$p_2 = \frac{2c_1C(\xi - x_2) + 2c_1\theta R x_2 - C\left[2c_1(\lambda c_1 + \theta) + \xi(2c_1 + \theta c_2)\right]}{2c_1 x_2 \theta}.$$

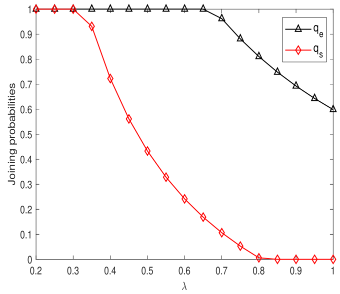

FIGURE 4. Joining probabilities *vs.* λ when $\theta = 1,\ \eta = 0.3,\ \xi = 1.2,\ b_1 = 0.3,\ b_2 = 0.7,\ \beta_1 = 0.6,\ \beta_2 = 0.4,\ g_1 = 1,\ g_2 = 0.8,\ \nu_1 = 0.5,\ \nu_2 = 0.8,\ C = 1,\ R = 5.$

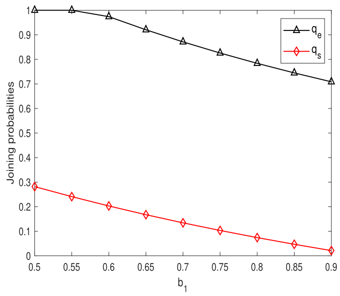

FIGURE 5. Joining probabilities *vs.* b_1 when $\lambda = 0.5,\ \theta = 1,\ \eta = 0.5,\ \xi = 1.3,\ b_2 = 0.6,\ \beta_1 = 0.7,\ \beta_2 = 0.3,\ g_1 = 0.9,\ g_2 = 0.8,\ \nu_1 = 0.4,\ \nu_2 = 0.7,\ C = 1,\ R = 5.$

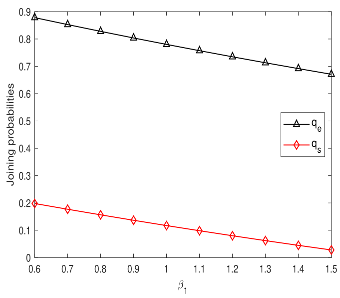

FIGURE 6. Joining probabilities *vs.* β_1 when $\lambda = 0.6,\ \theta = 1.5,\ \eta = 0.4,\ \xi = 0.9,\ b_1 = 0.5,\ b_2 = 0.6,\ \beta_2 = 0.5,\ g_1 = 0.7,\ g_2 = 0.9,\ \nu_1 = 0.6,\ \nu_2 = 0.8,\ C = 1,\ R = 5.$

Proof. According to Theorem 4.4, we consider the following four cases:

Case 1. $W_1 < \frac{R}{C} \le S_1$.

In this case, $q_s = 0$. If we set $p = C(S_1 - W_1)$, there is $\frac{R-p}{C} \le W_1$, which means that the equilibrium joining probability $q_e(p) = 0$. Thus, $p_e = C(S_1 - W_1)$ makes the equilibrium joining probability consistent with the socially optimal joining probability. At this point, social welfare is maximize.

Case 2. $S_1 < \frac{R}{C} \le S_2$.

In this case, $q_s = \frac{\xi - x_2}{\lambda c_1}$. If we let $p = p_2$, this results in $q_e(p) = q_s$. According to $W_1 < \frac{R}{C} \le S_1$ and $\xi - \lambda c_1 \le x_2 < \xi$, we have that $W_1 < \frac{R-p}{C} \le W_2$ holds. Thus, $p_e = p_2$ in this case.

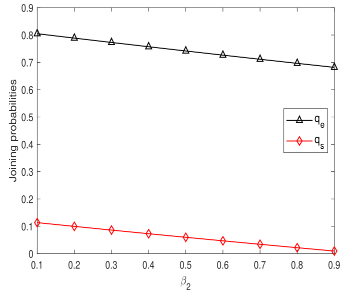

FIGURE 7. Joining probabilities *vs.* β_2 when $\lambda = 0.6,\ \theta = 1.3,\ \eta = 0.4,\ \xi = 0.9,\ b_1 = 0.5,\ b_2 = 0.6,\ \beta_1 = 1,\ g_1 = 0.7,\ g_2 = 0.9,\ \nu_1 = 0.6,\ \nu_2 = 0.8,\ C = 1,\ R = 5.$

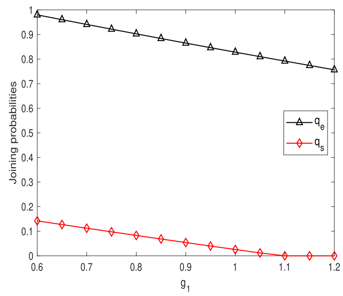

FIGURE 8. Joining probabilities *vs.* g_1 when $\lambda = 0.5,\ \theta = 1,\ \eta = 0.4,\ \xi = 0.9,\ b_1 = 0.4,\ b_2 = 0.6,\ \beta_1 = 1,\ \beta_2 = 0.5,\ g_2 = 1,\ \nu_1 = 0.5,\ \nu_2 = 0.7,\ C = 1,\ R = 5.$

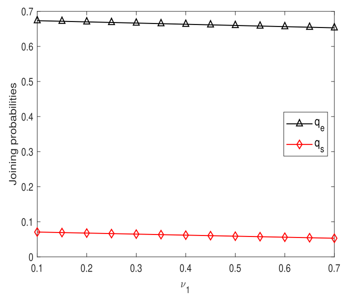

FIGURE 9. Joining probabilities *vs.* ν_1 when $\lambda = 1,\ \theta = 1.3,\ \eta = 0.2,\ \xi = 1.2,\ b_1 = 0.5,\ b_2 = 0.4,\ \beta_1 = 0.7,\ \beta_2 = 0.3,\ g_1 = 0.9,\ g_2 = 0.9,\ \nu_2 = 0.5,\ C = 1,\ R = 5.$

Case 3. $\frac{R}{C} \leq W_1$.
In this case, $q_e = q_s = 0$. Therefore, social welfare can be maximized without charging an admission fee.

Case 4. $S_2 < \frac{R}{C}$.
In this case, $q_e = q_s = 1$. Therefore, social welfare is already at its maximum when no admission fee is charged.

$\square$

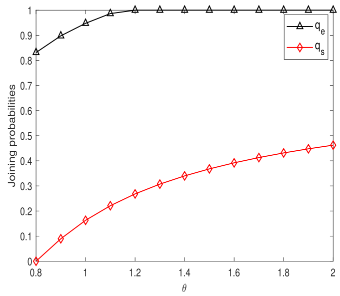

FIGURE 10. Joining probabilities *vs.* θ when $\lambda = 0.5,\ \eta = 0.6,\ \xi = 1.3,\ b_1 = 0.5,\ b_2 = 0.6,\ \beta_1 = 0.8,\ \beta_2 = 0.4,\ g_1 = 1,\ g_2 = 0.9,\ \nu_1 = 0.5,\ \nu_2 = 0.5,\ C = 1,\ R = 5.$

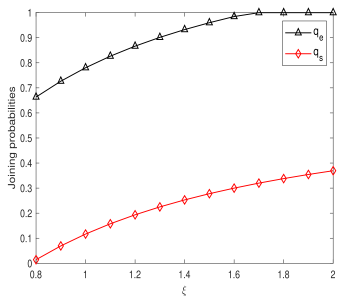

FIGURE 11. Joining probabilities *vs.* ξ when $\lambda = 0.5,\ \theta = 1.3,\ \eta = 0.5,\ b_1 = 0.7,\ b_2 = 0.6,\ \beta_1 = 0.8,\ \beta_2 = 0.5,\ g_1 = 0.8,\ g_2 = 0.9,\ \nu_1 = 0.5,\ \nu_2 = 0.5,\ C = 1,\ R = 5.$

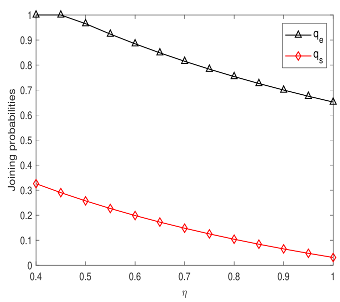

FIGURE 12. Joining probabilities *vs.* η when $\lambda = 0.6,\ \theta = 1.4,\ \xi = 1,\ b_1 = 0.4,\ b_2 = 0.5,\ \beta_1 = 0.9,\ \beta_2 = 0.3,\ g_1 = 0.7,\ g_2 = 0.7,\ \nu_1 = 0.4,\ \nu_2 = 0.5,\ C = 1,\ R = 5.$

6. Numerical experiments

In this section, the influences of system parameters on the q_e and q_s are first investigated in Figures 4–12. We can find that $q_e \geq q_s$ holds. Then, some numerical results are provided to explore the influence of parameters on the maximum of social welfare (see Figs. 13–16). Finally, we compare the influences of p on the $q_e(p)$ and q_s in Figure 17.

From Figures 4 and 5, we can observe that joining probabilities decrease with respect to both λ and b_1. The reason is that as λ and b_1 increase, the orbit becomes more crowded, which leads to an increase in the customer's

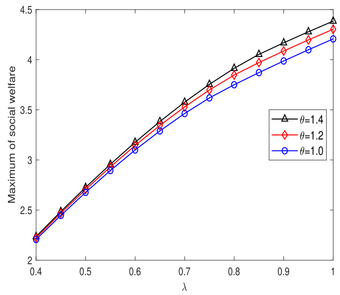

FIGURE 13. Maximum of social welfare *versus vs.* λ for different values of θ when $\eta = 0.4,\ \xi = 1.5,\ b_1 = 0.2,\ b_2 = 0.3,\ \beta_1 = 0.5,\ \beta_2 = 0.3,\ g_1 = 0.7,\ g_2 = 0.6,\ \nu_1 = 0.3,\ \nu_2 = 0.2,\ C = 1,\ R = 6.$

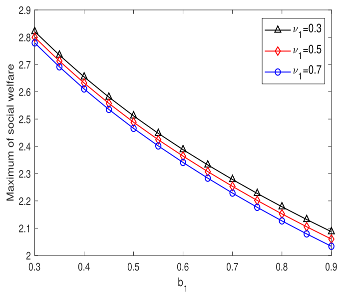

FIGURE 14. Maximum of social welfare *versus vs.* b_1 for different values of ν_1 when $\lambda = 0.8,\ \theta = 1,\ \eta = 0.7,\ \xi = 1.3,\ b_2 = 0.5,\ \beta_1 = 0.6,\ \beta_2 = 0.3,\ g_1 = 0.8,\ g_2 = 0.6,\ \nu_2 = 0.5,\ C = 1,\ R = 6.$

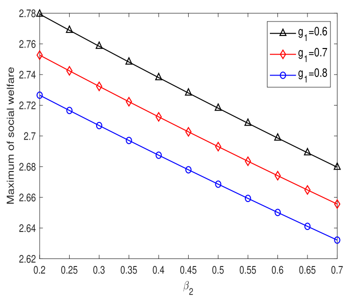

FIGURE 15. Maximum of social welfare *versus vs.* β_2 for different values of g_1 when $\lambda = 1,\ \theta = 1.5, \eta = 0.6,\ \xi = 1.4,\ b_1 = 0.6,\ b_2 = 0.4,\ \beta_1 = 0.8,\ g_2 = 0.6,\ \nu_1 = 0.4,\ \nu_2 = 0.2,\ C = 1,\ R = 6.$

waiting time in the orbit. At this point, the customer's willingness that enters the orbit decreases in both the non-cooperative and cooperative cases to avoid paying excessive waiting costs.

Figures 6 and 7 show the influence of β_1 and β_2 on joining probabilities, respectively. We can find that the joining probabilities are decreasing as β_1 and β_2 increase. The main reason for this phenomenon is that as β_1 and β_2 increase, the frequency of server breakdowns increases, which prolongs the time that customers occupy the server. Then, the waiting cost of customers in the orbit increases, and thus the joining probabilities q_e and q_s decrease.

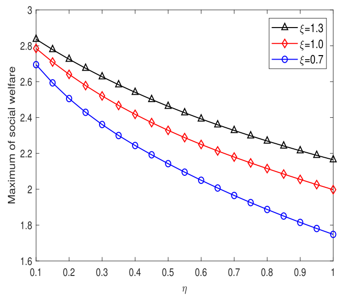

FIGURE 16. Maximum of social welfare *versus vs.* η for different values of ξ when $\lambda = 0.7,\ \theta = 1,\ b_1 = 0.5,\ b_2 = 0.5,\ \beta_1 = 0.6,\ \beta_2 = 0.3,\ g_1 = 0.8,\ g_2 = 0.6,\ \nu_1 = 0.5,\ \nu_2 = 0.5,\ C = 1,\ R = 6$.

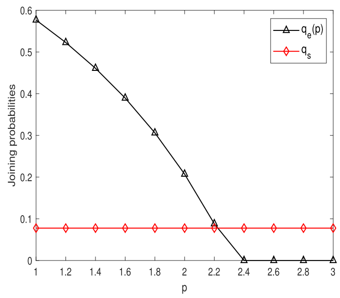

FIGURE 17. Joining probabilities *vs.* p when $\lambda = 0.7,\ \theta = 1.2,\ \eta = 0.7,\ \xi = 1.5,\ b_1 = 0.4,\ b_2 = 0.6,\ \beta_1 = 0.8,\ \beta_2 = 0.6,\ g_1 = 0.9,\ g_2 = 1,\ \nu_1 = 0.5,\ \nu_2 = 0.7,\ C = 1,\ R = 5$.

Similarly, Figures 8 and 9 reveal that an increase in g_1 and ν_1 also leads to a decrease in joining probabilities. This is because when the repair time increases, the server will be unavailable for a longer period, which leads to an increase in the time that customers occupy the server. Faced with these cases, customers who find the server unavailable upon arrival show a negative attitude towards entering the orbit.

When it comes to the increasing of θ in Figure 10, joining probabilities increase as θ increases. The reason is that when θ increases, the probability of retrial customers getting the service increases. Therefore, customers who find the server unavailable upon arrival are more likely to enter the orbit.

It can be easily observed from Figure 11 that joining probabilities increase as ξ increases. This is because the time required for preventive maintenance is reduced, which allows customers to complete service faster. At this point, retrial customers have more opportunities to get service, which leads to a decrease in waiting costs. Thus, joining probabilities are increased.

Figure 12 provides the effect of η on the joining probability. It can be found that joining probabilities decrease as η increases. This is because when the system prefers to perform preventive maintenance, more customers need to spend extra time on preventive maintenance, which leads to an increase in the waiting cost of retrial customers. Faced with the increased cost, the customer's willingness that enters the system is inhibited.

In Figure 13, the maximum of social welfare increases as λ or θ increases. This is the same as our expectation. According to Figure 4, although it is known that as λ increases, arriving customers show a negative attitude towards entering the orbit, it does not have a significant effect on the optimal social welfare. The main reason is that customers who find the server idle upon arrival must enter the system, which is beneficial for the increase of social welfare. On the other hand, the waiting time for retrial customers decreases as θ increases, which facilitates the increase of social welfare.

Figure 14 shows that the optimal social welfare decreases as b_1 and ν_1 increase. This is because as b_1 or ν_1 increases, retrial customers spend more waiting time to receive service, which leads to a discouragement that customers enter the orbit. It is easy to find from Figure 15 that the social welfare decreases with respect to β_2 and g_1. For similar reasons as ν_1, the server availability decreases as β_2 and g_1 increase, which leads to a decrease in social welfare. This phenomenon is consistent with realistic logic.

Figure 16 shows that optimal social welfare increases as ξ increases and decreases as η increases. It is easy to understand that an increase in ξ implies that the time of preventive maintenance decreases. Therefore, this is good for an increase in social welfare. Conversely, an increase in η implies that the weight of preventive maintenance increases, which leads to a decrease in social welfare.

Figure 17 illustrates the effect of p on joining probabilities. This provides strong support for Theorem 5.1, that there exists a price p_e such that $q_e(p_e) = q_s$. It is easy to see that the selfish behavior of non-cooperative customers can be effectively discouraged by the pricing strategy. Therefore, the pricing strategy is necessary to maximize social welfare.

7. Conclusion

In this paper, we extensively analyze the non-cooperative and cooperative joining behavior of customers in an unreliable M/G/1 retrial queue with Bernoulli preventive maintenance. Specifically, we first derive the equilibrium joining strategy of customers in the non-cooperative case. Then, the socially optimal joining strategy in the cooperative case is obtained. Finally, the optimal pricing strategy is discussed from the perspective of social managers. Different from other related queue models, this paper not only introduces unreliable server and preventive maintenance to the retrial queue, but also considers a novel Bernoulli preventive maintenance strategy. Moreover, the discussion and analysis in this paper show that the individual equilibrium entry probability for the non-cooperative case is always no less than the socially optimal joining probability for the cooperative case.

For future research, the pricing problem of the retrial queue with Bernoulli preventive maintenance and setup time is worth investigating, considering resource utilization.

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