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SPILLOVER EFFECTS OF GOVERNMENT SUBSIDIES ON AGRICULTURAL SUPPLY CHAINS UNDER RISK-SHARING MECHANISMS

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Abstract. This paper focuses on the agricultural supply chain and proposes a new financing model of risk sharing: the collateralized loan. In order to examine the operational effectiveness of the risk-sharing mechanism, we consider the effects of government subsidies and risk aversion factors. In the collateralized loan, the purchaser serves as the leader to provide loans to the farmer when the farmer provides collateral. The simulation results demonstrate that government subsidies can bring significant spillover effects to the whole supply chain. The effect's extent increases as the subsidy increases. And the spillover effect is more obvious under the collateralized loan than bank financing. The discount factor also generates a spillover effect for the agricultural supply chain through the inversion mechanism. The lower the discount factor, the more obvious the spillover effect. The increase in government subsidies facilitates the operation of risk-sharing mechanisms, while the increase in risk aversion hinders the operation of the mechanisms. This paper provides new insights into the issue of agricultural finance and managerial implication to precisely assist farmers in greater need.

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1. Introduction

With the achievement of the goal of poverty eradication in 2020, China has effectively moved into the next stage of the long-term objective of achieving a "well-off society" – "rural revitalization". However, the development of agriculture, farmers, and rural area (three rural problems) has encountered many obstacles. The most intuitive manifestation is that farmers are hard to secure loans and banks are cautious to lend [1]. The primary causes are as follows: first, the lengthy cycle of agricultural production and the substantial upfront cost of production materials make farmers bear tremendous financial pressure. Such as the greenhouse planting technology of vegetables and fruits, the initial investment requires about 150 000 yuan [2]. Second, due to the vulnerability to climatic calamities, agricultural production exhibits an amount of uncertainty. There is a discrepancy between farmers' actual output and expected output. In 2014, Typhoon Wimaxon swept through Hongxing and other reclaimed farms with a force of 15 wind, causing 35%–75% damage to local sugarcane [3]. In supply chain research literature, the most common and widely adopted uncertainties are fuzziness and randomness. For instance, Mondal and Roy [4] introduced interval type-2 Pythagorean fuzzy set (IT2PFS) to alleviate the burden of information representation, aiding experts in making secure and reliable decisions. Barman *et al.*

Keywords. Collateralized loan, risk sharing, government subsidies, spillover effects, agricultural supply chain.

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[5] selected single valued trapezoidal neutrosophic number to investigate the reworking and inventory problem. Some scholars argue that both types of uncertainties should be simultaneously considered in the supply chain environment. Mondal *et al.* [6] proposed a a novel fuzzy-random robust flexible programming approach based on Me measure and probabilistic programming. However, given the stochastic nature of agricultural production processes, the quantity of agricultural products is a random issue. In other words, agricultural products yield either a high or low quantity in a given year, and the product quantity is a random variable, without the presence of inherently ambiguous concepts. Referring to Lin *et al.* [7], this study focuses on the stochastic uncertainty of agricultural products. Finally, since three rural loans are an uncollateralized and unsecured form of loan, the lender will be accountable for any losses in the case of default.

Additionally, contract farming, a new advanced agricultural production model, has recently risen to prominence as the principal method of farmer and corporate cooperation. It is a significant element in driving agricultural modernization [8]. It was not until the mid-1980s that contract farming emerged in coastal areas of China [9]. It was only in the 1990s that contract farming became the main direction of agricultural development in China and was generally promoted. Compared to the domestic situation, contract farming in foreign countries started earlier and developed more maturely. In the late 20th century, contract farming was commonly developed in the United States, Europe and Japan, and became one of the main features of modern agricultural development. In 1997, the output value of contract farming in the United States accounted for 31% of the total agricultural output value, and this ratio climbed to 39% in 2001 [10].

Agricultural supply chain finance is carried out based on contract farming, which provides a new idea to solve the capital problem of China's agricultural development [11]. The agricultural supply chain is supported by the agricultural enterprises in the chain, and the credit of the agricultural enterprises is used to increase the credit of farmers. Through the design of scientific credit agreements and products, the uncontrollable risks of individual subjects are converted into controllable risks of the supply chain as a whole, so as to meet the capital needs of the industrial chain [12]. In China, in the past, rural areas as a whole were relatively backward, and most farmers did not have fixed assets that could be pledged as collateral. However, with the achievement of the goal of poverty eradication, Chinese farmers are basically equipped with mechanized production tools.

Therefore, to solve the financing problems of the "three rural problems", this paper considers establishing a "collateralized loan" contract, in which the farmer uses its fixed assets as collateral and the purchaser regards them as a credit guarantee to grant loans. A risk-sharing internal financing approach is proposed, in which the upstream purchaser provides loans to the downstream farmer. The risk-sharing mechanism is reflected as follows. On the one hand, the farmer applies for financing from the purchaser using its fixed assets as collateral. In case of default, the farmer will lose part or all of the collateral, thus transferring a certain default risk from the purchaser to the farmer. On the other hand, this paper takes into account the government subsidies' impact on agricultural loans. In order to support agricultural production, the government issues a specific percentage of subsidy for each agricultural loan as a guarantee for farmers' smooth production and timely repayment.

The contributions of this paper are mainly as follows.

(1) This paper proposes a new internal financing model (collateralized loan) and compares it with traditional bank financing. The farmer uses the fixed assets as collateral when seeking loans from purchasers. This risk-sharing mechanism facilitates the transfer of risk among the farmer, the purchaser, and the government. As a result, the farmer's creditworthiness can be enhanced and the default risk faced by the purchaser can be decreased.
(2) This paper incorporates the uncertainty of output in the agricultural supply chain. By considering several default situations of the farmer, we segment the profit functions of the farmer and purchaser based on the different output conditions. In the agricultural supply chain, it is realistic to focus on the occurrence of farmer default cases.
(3) This paper researches the effect of government subsidies and risk aversion on the risk-sharing mechanism by analyzing the effectiveness of government subsidies. That is, we assess whether government subsidies bring spillover effects by comparing the net profit of the supply chain, accounting for government subsidies

(*i.e.*, the supply chain profit after deducting government subsidies) with the supply chain's profit without government subsidies.

The rest of the paper is organized as follows. Section 2 is the literature review. Section 3 is the model construction. Section 4 is the government subsidy model without risk aversion. Section 5 is the government subsidy model with risk aversion. Section 6 is a numerical simulation. Section 7 is the managerial implications. Section 8 is the conclusions and limitations.

2. Literature review

In response to the risk issues in agricultural supply chain finance, we have categorized the literature into three parts: agricultural supply chain finance, agricultural supply chain risks, and risk-sharing mechanisms.

2.1. Agricultural supply chain finance

Asymmetric information leads to the phenomenon of financing difficulties for agricultural operators, which eventually hinders agricultural development. Asymmetric information theory refers to the fact that in the market economy, different market participants have different information, and those who have more information can profit by passing information to those who have less information. In order to solve the financing problem caused by asymmetric information, many scholars have combined agriculture with supply chain finance and put forward the concept of agricultural supply chain finance (ASCF) [13]. Mesopotamia's agricultural sector saw the emergence of "grain warehouse receipts", marking the beginning of supply chain finance's application to the agricultural production process [14]. Moers [15] introduced the concept of a grain bank, where the warehouse was combined with a rural credit bank to establish a new type of bank. A growing number of developing nations have recently turned to warehouse receipt finance, primarily for agricultural production [16]. According to Bogetoft and Olesen [17], a contract farming contract requires taking into account coordination, incentives, and transaction costs. Furthermore, Kazaz and Webster [18] studied the impact of yield-dependent transaction costs on supply chain pricing and production decisions.

Lin and Ye [7] designed a "company+farmer" type of contract farming business model and obtained that a revenue-sharing contract mechanism based on Nash negotiation increases overall social welfare. Dries *et al.* [19] found that the internal financing model of ASCF is the primary source of credit access for farmers in many regions. Chen *et al.* [20] considered the capital constraint problem and proposed loan service pricing rules and production decisions for agricultural supply chain finance. Different from the above studies on internal financing, Deng *et al.* [21] concentrated on the incentives of green financing policies in agricultural supply chains.

Current research on agricultural supply chain finance primarily pays attention to the comparison between internal financing and external financing [22–24]. Guo and Wang [25] suggested that agricultural enterprises provide internal financing subsidies and additional fee policies to create a win-win situation. Therefore, numerous scholars have started focusing on subsidies' impacts [26, 27]. An additional insurance method based on government subsidies was introduced by Huang and Lin [28]. Sana [29] concluded that governments provide high subsidies to green producers and low subsidies to non-green producers. Given the important role of government guidelines [30], it is essential to incorporate the role of the government in the agricultural supply chain.

2.2. Agricultural supply chain risks

The agricultural supply chain is characterized by widespread information asymmetry, which leads to prominent moral hazard issues. Additionally, due to the particularities of agricultural production, the agricultural supply chain faces more risks internally compared to other supply chains. This is mainly reflected in the uncertainties on the demand side and the supply side. First, because agricultural products are seasonal, market demand also fluctuates [31]. Song *et al.* [32]; Song and Yao [33], and DeCroix *et al.* [34] examined the inventory optimization problem of a product assembly-to-order (ATO) system under stochastic demand conditions. Jiang and Wang [35] analyzed supplier competition's impact on firms' price and quantity decisions in an assembly supply chain. The intelligent replenishment method was presented by Chen *et al.* [36] as the optimal replenishment strategy for an agriculture supply chain with stochastic demand. Yang *et al.* [37] investigated the benefit sharing in contract farming default risk collaboration through a market price fluctuations perspective. In the case where random demand is sensitive to selling prices, Sana [38] deduced the optimal selling price, inventory lot size, and reorder point for enterprise managers to achieve profit maximization.

Second, the output of agricultural products also exhibits uncertainty due to climatic conditions. Gurnani and Gerchak [39] studied the coordination problem of supplier assembly systems with supply uncertainty. Ye and Wang [40] studied the coordination problem of the "company+farmers" contract farming supply chain under output uncertainty. Zhou [41] further incorporated government subsidies into the "company+farmers" guaranteed price contract. In addition to considering the impact of government subsidies, Liu [42] included agricultural insurance factors in his analysis. Cao *et al.* [43] thoroughly examined the necessity of poverty alleviation policies under stochastic output conditions.

However, the above studies all focus on a single aspect of possible risk, ignoring the possibility of the intersection of risks from the demand and supply sides. With the advancement of research technology, the research on the risks has gradually evolved to an integrated examination of uncertainty in demand and output [44]. Qin and Li [45] integrated the uncertainty risks of market demand and agricultural output to analyze the change in supply chain performance after the implementation of the benefiting-farmers policy. Based on stochastic output and demand, Chen [46] studied the impact of different linkage methods on the contract farming supply chain. In addition to considering quantity uncertainty in demand and supply, Gokarn and Kuthambalayan [47] analyzed price uncertainty in sustainable fresh agricultural products supply chains.

2.3. Risk-sharing mechanisms

Due to the many risks involved in agricultural production, how to share the overall supply chain risk among the individual members has become the subject of agricultural supply chain research. Wang and Tsao [48] introduced a single-period two-stage supply contract with a two-way option where the buyer can adjust the initial order up or down. Gomez-Padilla and Mishina [49] proposed another two-echelon supply chain. It used option contracts to coordinate the supply chain in single supplier-single retailer and multiple supplier-single retailer scenarios. Based on it, Xu and Nozick [50] proposed a stochastic programming model for supplier selection. Based on the context of stochastic output and stochastic demand, Xu [51] argued that the application of option contracts enabled suppliers to hedge the risk of low demand and low prices and manufacturers to hedge the risk of high prices and low demand. Four risk-sharing contracts' effects on the parties' profits in a contract farming supply chain under unpredictable output were examined by Ling *et al.* [52].

Few studies have taken into account the possibility that supply chain members might collaborate together. Sana [53] contributed to this by showing through numerical analysis that manufacturers and retailers make collaborative decisions that generate more profit than separate optimisations. On the one hand, such coordination is reflected in the introduction of contracts. Jiang and Li [54] verified that producers' and retailers' profits exceed the profit level under decentralized decision-making through the revenue-sharing contract. In addition to considering the revenue sharing, Chen *et al.* [55] also included cost factors within the scope of sharing and concluded that the benefit sharing-cost sharing contract increased the amount of scrap recycled. On the other hand, many academics have also focused on mitigating uncertainty risk through buyback strategies. Yang *et al.* [56], who suggested a supplier buyback mechanism to lessen the demand uncertainty risk, provided a more intuitive consideration of the positive impact of risk sharing on the supply chain's efficiency. Sun and Zhao [57] took into account the supply chain credit level and constructed a bank pledge rate decision model. Therefore, this research believes that the presence of loan collateral acts as a mechanism for sharing risk between farmers and purchasers. Through a systematic review of the literature, Jin *et al.* [58] combined the external risk-sharing mechanism with the internal risk-sharing mechanism and proposed a revenue-sharing-bidirectional option contract.

2.4. Motivation

Through reviewing the literature, we find that several research gaps remain. (1) There are few quantitative studies analyzing the spillover effects of government subsidies. Previous supply chain literature considering government subsidies mostly focused on studying their impact on supply chain production and operational strategies, without specifically comparing the supply chain economic performance with and without subsidies. (2) There is still a research gap in the innovation of risk-sharing mechanisms. In order to enhance the effectiveness of risk-sharing mechanisms in agricultural loans, this study proposes a novel risk-sharing mechanism formed through collaboration between agricultural product purchasers and the government. (3) There is a lack of literature that simultaneously considers government subsidies, collateral discount factor, and risk aversion factors in agricultural supply chain finance. By taking these real-world factors into account, this study fills the research gap between existing literature and the proposed risk-sharing mechanism.

Based on the above research gaps, the main motivation of this study is to design a novel risk-sharing mechanism under the uncertainty of agricultural production, aiming to address the difficulty faced by farmers in obtaining loans and promote enterprises' willingness to provide loans. (1) The basic motivation places in the fact that the literature considering the spillover effects of government subsidies is nil. This study quantitatively examines the difference in overall supply chain profits under subsidies scenario and without subsidies scenario. (2) The main motivation for introducing government subsidies is to overcome the shortcomings of the existing risk-sharing mechanism. One example is the collateralized loan which lays a foundation for risk-sharing. However, at this point, the risk is only transferred between farmers and purchasers. Therefore, we introduce the government subsidies to enhance the effectiveness of risk diversification among farmers, purchasers and the government. (3) Considering that government subsidies boost farmers' production enthusiasm, while the depreciation of collateral assets hinders their production by reducing the accessibility of loans (*i.e.*, these two factors act in opposing ways). Therefore, we combine these two factors with farmer's risk aversion. We specifically explore how these two factors, under conditions of risk neutrality and risk aversion, affect the implementation effectiveness of the risk-sharing mechanism.

3. Model construction

This section introduces the main assumptions and parameter meanings of the agricultural supply chain model. Then, this section introduces the characteristics and financing processes of bank financing and collateralized loan, respectively, as well as the game order between the farmer and purchaser in the financing process. This section paves the way for Sections 4 and 5.

3.1. Assumptions and parameters

The government's primary objective is to maximize social welfare, prioritizing the overall development of industries over the interests of individuals. Typically, governments utilize subsidies to foster sectoral growth. To account for the government's role, we incorporate a loan subsidy into the model. Within the agricultural supply chain framework, the farmer employs fixed assets for production and subsequently sells the products to the purchaser. The leader in this dynamic is the purchaser with ample capital, while the farmer, constrained by capital, assumes the role of the follower. We set the ratio of government subsidies to the total loan amount as b. The farmer organizes production after receiving a loan. The purchaser buys the agricultural products after taking the farmer's debt into account. We further make the following assumptions.

Assumption 1. *When the production is finished, the purchaser buys all the crops for the price V and sells them in the market. We assume that there is an equilibrium between the supply and demand for agricultural products. The equation for market clearing is as follows.*

$$S = Q \tag{1}$$

TABLE 1. Variable definitions.

Variables	Description
a	The value of farmer's fixed assets
r_b	The interest rate of bank loan
u	The uncertainty variables of farmer output
$g(u)$	The probability density function of output $\int_{-\infty}^{\infty} g(u)\,\mathrm{d}u = 1$
$G(u)$	The cumulative distribution function of output $0 \leq G(u) \leq 1$
V	The purchase price per unit of agricultural product
P	The market price per unit of agricultural product
k	The reciprocal of the demand-price elasticity $k \in (0,1)$
S	The total amount of agricultural products' uncertain output
Q	The market demand of agricultural products
Π	The profit of the purchaser
Ω	The profit of the farmer
b	The government's loan subsidy ratio $b \in [0,1]$
d	Discount ratio of farmer's fixed assets $d \in [0,1]$
λ	The risk aversion coefficient of the farmer $\lambda \in [0,1]$
σ	The degree of risk volatility in the market
Decision variables	Description
L	The loan amount
m	The rate of the purchase price to the market price $m \in (0,1)$
r	The interest rate of collateralized loan

where S is the total amount of uncertain output. Q is the market demand of agricultural products. The purchaser's market price is P. The purchaser's purchase price V is positively related to the market price P. m is purchase price discount, indicating the ratio of the purchase price to the market price (i.e., $V = mP, m \in (0,1)$).

Assumption 2. *Crop production is subject to significant external uncertainties. Therefore, we note the amount of uncertain output S as follows.*

$$S = uL \tag{2}$$

where u is the output uncertainty variable, and it obeys a uniform distribution. Its cumulative distribution function is $G(u)$. Its probability density function is $g(u)$, and $u \in (0, \bar{u})$. L is the loan's amount.

Assumption 3. *We assume that the agricultural products market is elastic [59]. The following is the inverse demand function.*

$$P = \bar{P}Q^{-k} \tag{3}$$

where $k \in (0,1)$ is the reciprocal of demand-price elasticity and $\bar{P}$ is choking price that makes demand zero. That is, demand exists only when the market price is lower than the choking price.

Assumption 4. *Consider the depreciation and impairment of the fixed asset collateral. That is, when the fixed asset collateral is liquidated in the market, there is a discrepancy between the net realizable value and the original value of the asset. The discount ratio d is the ratio between the net realizable value and the original value. If the value of the farmer's output (i.e., income received by farmers from the sale of agricultural products) combined with the government subsidies is still insufficient to cover the repayment of the loan's principal and interest, the collateral assets will be sold by the purchaser at a discounted price.*

The further settings are made in Table 1.

Superscripts B, D, BF, DF respectively represent bank financing under risk-neutral conditions, collateralized loan under risk-neutral conditions, bank financing under risk appetite conditions, and collateralized loan under risk appetite conditions.

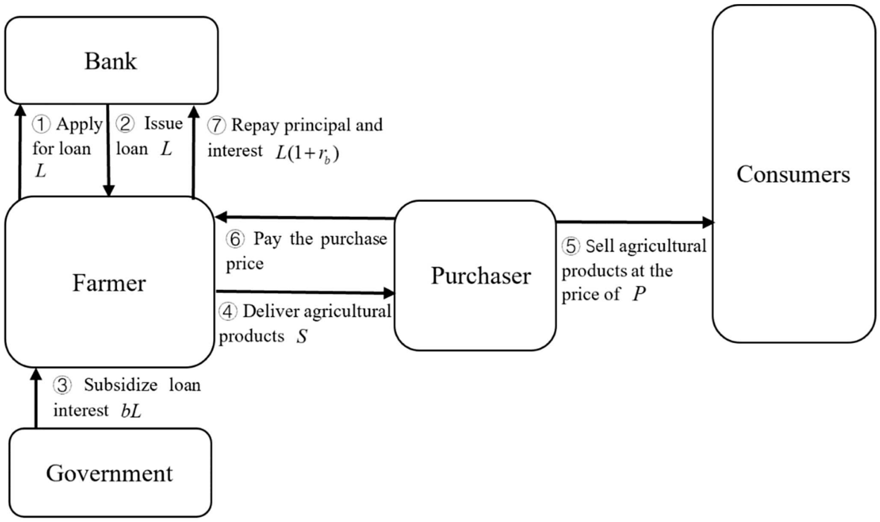


FIGURE 1. Timeline of bank financing.

3.2. Bank financing

The banks provide loans to farmers, and the government offers loan subsidies. Once the farmer completes production and receives payment, they repays the principal and interest to the bank. The main players in the supply chain game are the farmer and the purchaser. The financing occurs between the external financial institutions and the farmer. Even if the farmer encounters difficulties in repaying the principal and interest on time, the risk is borne by the bank and does not affect the purchaser. Therefore, asset collateral is not considered in this process.

The game sequence under bank financing is as follows. The purchaser, as the leader, decides the purchase price V^* before the farmers' decision. According to $V = mP$, the market price P and the purchase price discount m together determine the purchase price V. Because the market determines the price of agricultural products, we regard it as a model exogenous variable. The purchase price discount m^* can therefore be used to explain the purchase price V^*. The farmer then decides the loan amount L^* based on the purchaser's choice. The following is the solution order based on the characteristics of the Stackelberg game. First, the farmer determines the optimal loan amount for the given purchase price discount. At this time, the optimal loan amount $L(m)^*$ is the reaction function of the purchase price discount. Second, substitutes the purchaser's profit function with the reaction function. Then we can solve the optimal purchase price by maximizing the purchaser's profit. Finally, the optimal purchase price discount is then substituted into the loan amount reaction function to obtain the optimal loan amount.

Thus, the bank financing's timeline is shown in Figure 1. Step 3 of the timeline occurs almost simultaneously with Step 2. That is, the government grants a subsidy as soon as the farmer receives the loan from the bank.

3.3. Collateralized loan

The remaining payment is made to the farmer by deducting the loan's principal and interest once the purchaser has sold the agricultural products. Given the risk of loss to the purchaser, we consider the case of collateralization of the farmer's assets. The results can be divided into two main categories.

(1) The total value of the farmer's payment and government subsidy is higher than the principal and interest of the loan. Then, the purchaser pays the remaining product's payment and returns the farmer all of the collateral back.

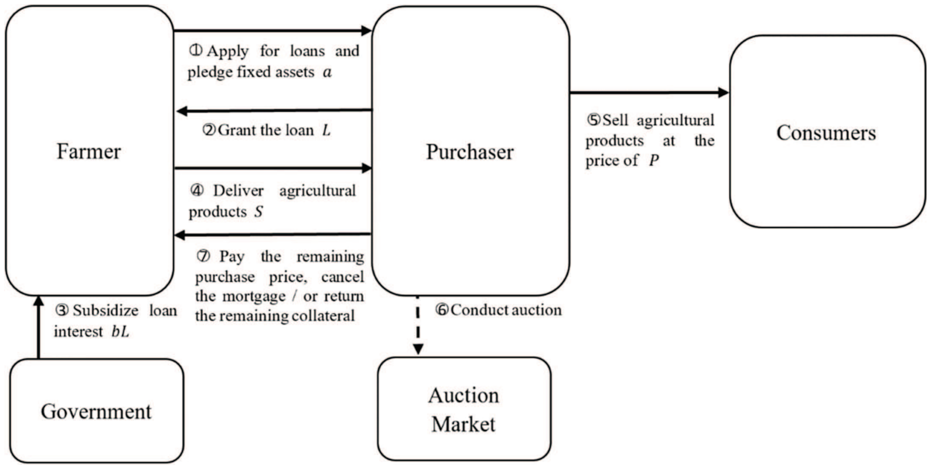


Figure 2. Timeline of collateralized loan.

(2) The total value of the farmer's payment and government subsidies is lower than the principal and interest of the loan. There are two small cases:
- In the first case, the total value of the farmer's payment and subsidies is lower than the loan's principal and interest, but the value of the farmer's discounted collateral can cover the gap. The purchaser does not need to pay the farmer but needs to return part of the collateral.
- In the second case, the total value of payment, subsidy, and collateral cannot pay for the loan's principal and interest. All sales proceeds and collateral are kept by the purchaser, who does not pursue the farmer's remaining loans.

Thus, the collateralized loan's timeline is shown in Figure 2. Step 3 of the timeline occurs almost simultaneously with Step 2. That is, the government grants a subsidy as soon as the farmer receives the loan from the purchaser. Step 6 is triggered only when the value of the farmer's output plus the government's subsidy still does not cover the principal and interest of the loan.

The game sequence under the collateralized loan is as follows. The purchaser, as the leader, decides both the optimal purchase price discount m^* and the optimal interest rate r^* before the farmers' decision. The farmer decides the loan amount L^* based on the purchaser's choice. The following is the solution order based on the Stackelberg game's characteristics. First, the farmer determines the optimal loan amount for the given purchase price discount and loan interest rate. At this time, the optimal loan amount $L(m, r)$ is the reaction function of the purchase price discount and the interest rate. Second, substitute the purchaser's profit function with the reaction function. Thus, we can solve the optimal purchase price discount and the interest rate by maximizing the purchaser's profit. Finally, the optimal loan amount can be obtained by substituting the optimal purchase price discount m^* and the optimal interest rate r^* into the loan amount reaction function.

4. Government subsidy model without risk aversion

In this section, the bank financing model and the collateralized loan model are constructed under the government subsidy and no risk aversion scenarios, respectively. Based on the game sequence introduced in the previous section, the optimal decisions of the farmer and purchaser under bank financing and collateralized loan are obtained, respectively.

4.1. Bank financing's optimal strategy

To simplify the analysis process, we only consider the single-period lending scenario. The bank's profit comes from the loan's interest (Lr_b). The sales revenue of the farmer is VS known from the market clearing VS = VQ. Bringing $V = mP$ and $P = \bar{P}Q^{-k}$ into market clearing function, *i.e.*, $\mathrm{VQ} = m\bar{P}Q^{-k+1}$. Therefore, the farmer's sales revenue is $\mathrm{VS} = m\bar{P}(uL)^{-k+1}$. Therefore, the farmer's profit function is as follows:

$$\Omega^B = m\bar{P}(uL)^{1-k} - L(1+r_b) + bL. \tag{4}$$

The farmer's expected return is $E(\Omega^B) = m\bar{P}(uL)^{1-k} - L(1+r_b) + bL$.

The purchaser's expected revenue comes from the difference between market price and purchase price (*i.e.*, sales profit). Therefore, the purchaser's profit function is as follows:

$$\Pi^B = (1-m)\bar{P}(uL)^{1-k}. \tag{5}$$

The purchaser's expected return is $E(\Pi^B) = (1-m)\bar{P}(uL)^{1-k}$.

The purchaser first determines the optimal purchase price discount, and then the farmer decides the optimal loan amount. According to the reverse solution method, the following property is obtained:

Property 1. The optimal loan amount L^{B*} of the farmer and the optimal purchase price discount m^{B*} of the purchaser satisfy the following restrictions:

$$\bar{P}\left(uL^{B^*}\right)^{1-k} = \frac{\left(1-m^{B^*}\right)\bar{P}(1-k)u^{1-k}L^{-k}}{\eta m^{B^*}}\left[\frac{\overline{P}(1-k)u^{1-k}m^{B^*}}{1+r_b-b}\right]^{\frac{1}{k}} \tag{6}$$

$$L^{B^*} = \left[\frac{(1-k)m^{B^*}\bar{P}u^{1-k}}{1+r_b-b}\right]^{\frac{1}{i}} \tag{7}$$

and

$$\left[(1-m)\bar{P}u^{1-k}\left(k^2-k\right)\left(L^{B^*}\right)^{1-k}\frac{\partial L^{B^*}}{\partial m} - 2\bar{P}u^{1-k}(1-k)\left(L^{B^*}\right)^{-k}\right]\frac{\partial L^{B^*}}{\partial m} + (1-m)\bar{P}u^{1-k}(1-k)(L^{B^*})^{-k}\frac{\partial^2 L^{B^*}}{\partial m^2} < 0. \tag{8}$$

The detailed proof is shown in Appendix A.

4.2. Collateralized loan's optimal strategy

Similar to bank financing, we also only consider the single-period lending scenario. The farmer's sales revenue is $\mathrm{VS} = m\overline{P}(uL)^{-k+l}$. Combining analysis of different scenarios in the game process, the farmer's utility function is a segmented function. The expression is as follows.

$$\Omega^D = \begin{cases} m\bar{P}(uL)^{1-k} - L(1+r) + bL, & m\bar{P}(uL)^{1-k} - L(1+r) + bL > 0 \\ \max_{L>0}\left\{\frac{m\bar{P}(uL)^{1-k} - L(1+r) + bL}{d}, -a\right\}, & m\bar{P}(uL)^{1-k} - L(1+r) + bL \le 0. \end{cases} \tag{9}$$

Sales of the crops make up the purchaser's portion of the profit. The loan's interest income consists of another part of the purchaser's profit. The purchaser's profit is Lr when the farmer is able to return the principal and interest in time. However, if the total value of the agricultural products, government subsidies, and the auction value of the collateral is still less than the principal and interest of the loan, the cost that the purchaser can recover will be $m\bar{P}(uL)^{l-k} + da - L$. Therefore, the purchaser's utility function is as follows:

$$\Pi^D = (1-m)\bar{P}(uL)^{1-k} + \min\left\{m\bar{P}(uL)^{1-k} + bL + da, L(1+r)\right\} - L. \tag{10}$$

There are two key points of output uncertainty. The first critical value is the output when the agricultural products' value plus the government subsidy just covers the loan's principal and interest. And the output uncertainty variable is u_0, which satisfies $m\overline{P}(uL)^{l-k} - L(1+r) + bL = 0$, *i.e.*, $u_0 = [\frac{L(1+r-b)}{m\bar{P}}]^{\frac{1}{1-k}} \cdot \frac{1}{L}$. The second critical value is the output when the total value of the products, the government subsidy, and the realizable value of the collateral is equal to the principal and interest of the loan. The output uncertainty variable is u_1, which satisfies $m\bar{P}(uL)^{1-k} - L(1+r) + bL + da = 0$, *i.e.*, $u_1 = [\frac{L(1+r-b)-da}{m\bar{P}}]^{\frac{1}{1-k}} \cdot \frac{1}{L}$.

We note $\varphi(L) = [\frac{L(1+r-b)}{m\bar{P}}]^{\frac{1}{1-k}} \cdot \frac{1}{L}$, $\phi(L) = [\frac{L(1+r-b)-da}{m\bar{P}}]^{\frac{1}{1-k}} \cdot \frac{1}{L}$.

When $u_0 < u < \overline{u}$, the farmer can return the principal and interest of the loan, and its profit is $m\overline{P}(uL)^{l-k} - L(1+r) + bL$. When $u_0 < u < u_1$, the farmer cannot return the principal and interest, but can use the collateral to cover part of them, its profit is $[m\bar{P}(uL)^{l-k} - L(1+r) + bL]/d$. When $0 < u < u_1$, the farmer cannot use collateral to cover debt and all the collateral assets are lost. Therefore, the farmer's expected profit is as follows:

$$\begin{aligned} E\left(\Omega^D\right) = &\int_{u_0}^{\bar{u}} \left[m\overline{P}(uL)^{1-k} - L(1+r) + bL\right] \cdot g(u)\,\mathrm{d}u \\ &+ \int_{u_1}^{u_0} \frac{\left[m\bar{P}(uL)^{1-k} - L(1+r) + bL\right]}{d} \cdot g(u)\,\mathrm{d}u + \int_0^{u_1} (-a) \cdot g(u)\,\mathrm{d}u. \end{aligned} \tag{11}$$

The sale of the product contributes to the purchaser's profit in part $\int_0^{\Pi} (1-m)\bar{P}(uL^{D^*})^{1-k} \cdot g(u)\,\mathrm{d}u$. The loan's interest income makes up for the rest. The purchaser's expected profit function is as follows:

$$\begin{aligned} E\left(\Pi^D\right) = &\int_0^{\bar{u}} (1-m)\bar{P}\left(uL^{D^*}\right)^{1-k} \cdot g(u)\,\mathrm{d}u + \int_{u_1}^{\bar{u}} L^{D^*}(1+r) \cdot g(u)\,\mathrm{d}u \\ &+ \int_0^{u_1} \left[m\bar{P}\left(uL^{D^*}\right)^{1-k} + bL + da\right] \cdot g(u)\,\mathrm{d}u - L^{D^*}. \end{aligned} \tag{12}$$

We can find the following property according to the reverse solution method:

Property 2. The optimal loan amount L^{D*} of the farmer, the optimal interest rate r^{D*} of the purchaser and the optimal purchase price discount m^{D*} of the purchaser satisfy the following restrictions:

$$L^{D^*} = \left[\frac{m^{D^*}\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k)}{1 + r^{D*} - b}\right]^{\frac{1}{k}} \tag{13}$$

$$\begin{aligned} \frac{\partial E\left(\Pi^D\right)}{\partial r} = &\left\{\int_0^u \left[\overline{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k}\right] g(u)\,\mathrm{d}u \right. \\ &\left. - \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \left[m^{D^*}\overline{P}(k-1)u^{1-k}\left(L^{D^*}\right)^{-k} + 1 + r\right] g(u)\,\mathrm{d}u - 1\right\} \frac{\partial L^{D^*}}{\partial r} \\ &+ \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} L^{D^*} g(u)\,\mathrm{d}u = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial E\left(\Pi^D\right)}{\partial m} = &\left\{\int_0^u \left[\overline{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k}\right] \cdot g(u)\,\mathrm{d}u \right. \\ &\left. + \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \left[m\overline{P}(k-1)u^{1-k}\left(L^{D^*}\right)^{-k} + (1+r)\right] \cdot g(u)\,\mathrm{d}u\right\} \cdot \frac{\partial L^{D^*}}{\partial m} \\ &- \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \overline{P}\left(uL^{D^*}\right)^{1-k} \cdot g(u)\,\mathrm{d}u = 0 \end{aligned} \tag{15}$$

$$
\begin{aligned}
\frac{\partial^2 E(\Pi^{D^*})}{\partial r^2} &= \left\{ \int_0^u \left[\bar{P}(1-k)u^{1-\eta}\left(L^{D^*}\right)^{-k} \right] g(u)\,\mathrm{d}u \right.\\
&\quad \left. - \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \left[m^{D^*}\overline{P}(k-1)u^{1-k}\left(L^{D^*}\right)^{-k} + (1+r) \right] \cdot g(u)\,\mathrm{d}u - 1 \right\} \frac{\partial^2 L^{D^*}}{\partial r^2}\\
&\quad + \frac{\partial L^{D^*}}{\partial r} \left\{ \int_0^{\phi\left(L^{D^*}\right)} \left[\overline{P}(k^2-k)u^{1-k}\left(L^{D^*}\right)^{-k-1} \right] \frac{\partial L^{D^*}}{\partial r} g(u)\,\mathrm{d}u \right.\\
&\quad + \left[\frac{(k^2-k)(L(1+r)-b)-da}{m^{D^*}L^2} \right] g\left(\phi\left(L^{D^*}\right)\right)\phi'(r) - G(\overline{u})\\
&\quad \left. + G\left(\phi\left(L^{D^*}\right)\right) + (1+r)\frac{\partial G(\phi(L^{D^*}))}{\partial r} \right\} + \frac{\partial L^{D^*}}{\partial r}\left[G(\overline{u}) - G\left(\phi\left(L^{D^*}\right)\right) \right] < 0
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\frac{\partial E^2(\Pi^{D})}{\partial m^2} &= \left\{ \int_0^{\overline{u}} \left[\overline{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k} \right] g(u)\,\mathrm{d}u \right.\\
&\quad \left. + \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \left[m\overline{P}(k-1)u^{1-k}\left(L^{D^*}\right)^{-k} + \left(1+r^{D^*}\right) \right] \cdot g(u)\,\mathrm{d}u \right\} \frac{\partial^2 L^{D^*}}{\partial m^2}\\
&\quad + \frac{\partial L^{D^*}}{\partial m} \left\{ \int_0^{\phi\left(L^{D^*}\right)} \left[(k^2-k)\overline{P}u^{1-k}\left(L^{D^*}\right)^{-1-k} \right] \frac{\partial L^{D^*}}{\partial m} g(u)\,\mathrm{d}u \right.\\
&\quad + \left[\frac{(k^2-k)\left(L\left(1+r^{D^*}\right)-b\right)-da}{mL^{D^*}} \right] g\left(\phi\left(L^{D^*}\right)\right)\phi'(m)\\
&\quad \left. + \left(1+r^{D^*}\right)\frac{\partial G(\phi(L^{D^*}))}{\partial m} \right\} - \int_{\phi\left(L^{D^*}\right)}^{\overline{u}} \left[\overline{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k}\frac{\partial L^{D^*}}{\partial m} \right] g(u)\,\mathrm{d}u\\
&\quad + \frac{\left(L\left(1+r^{D^*}\right)-b\right)-da}{m} g\left(\phi\left(L^{D^*}\right)\right)\phi'(m) < 0
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\frac{\partial E^2(\Omega^{D})}{\partial L^2} &= \int_{\varphi(L)}^{\bar{u}} \left[(k^2-k)m^{D*}\bar{P}u^{1-k}L^{-k-1} \right] g(u)\,\mathrm{d}u - (1-k)g(u)\varphi'(L)\\
&\quad + \frac{1}{d}\left\{ \int_{\phi(L)}^{\varphi(L)} \left[(k^2-k)m^{D*}\bar{P}u^{1-k}L^{-k-1} \right] g(u)\,\mathrm{d}u + (1-k-da)g(u)\varphi'(L) \right.\\
&\quad \left. - (1-k)g(u)\phi'(L) < 0 \right\}.
\end{aligned}
\tag{18}
$$

The detailed proof is shown in Appendix A.

5. Government subsidy model with risk aversion

To further examine the risk-sharing effect of the collateralized loan in the agricultural supply chain, this section introduces the risk aversion factor and considers the purchaser as risk-neutral and the farmer as risk-averse in the agricultural supply chain model. Based on this, the impact of government subsidies on the optimal decisions of the farmer and the purchaser is studied. To avoid triviality, we evaluate the profit function based on the mean-variance theory. λ ($\lambda \in [0,1]$) represents the risk aversion coefficient of the farmer, where a larger value indicates a higher degree of risk aversion, *i.e.*, the farmer being more conservative. Therefore, in the presence of risk preferences, farmers also consider the risk cost $\lambda[m\bar{P}(uL)^{-k}]^2\sigma^2$ [60] in their decision-making. Here, σ^2 represents the degree of risk volatility.

5.1. Bank financing's optimal strategy

The bank's profit is Lr_b. The farmer's profit function is as follows:

$$\Omega^{\mathrm{BF}} = m\bar{P}(uL)^{1-k} - \lambda\left[m\bar{P}(uL)^{-k}\right]^2\sigma^2 - L(1+r_b) + bL. \tag{19}$$

The farmer's expected return is $E(\Omega^{\mathrm{BF}}) = m\overline{P}(uL)^{1-k} - \lambda[m\overline{P}(uL)^{-k}]^2\sigma^2 - L(1+r_b) + bL$.

The purchaser's profit function is as follows:

$$\Pi^{\mathrm{BF}} = (1-m)\bar{P}(uL)^{l-k}. \tag{20}$$

The purchaser's expected return is $E(\Pi^{\mathrm{BF}}) = (1-m)\bar{P}(uL)^{l-k}$.

We can find the following property according to the reverse solution method:

Property 3. The optimal loan amount L^{BF} of the farmer and the optimal purchase price discount m^{BF} of the purchaser satisfy the following restrictions:

$$\left[1 - u(1-k)m^{\mathrm{BF}^*}\overline{P}\left(uL^{\mathrm{BF}^*}\right)^{-k}\right]\lambda\sigma^2 m^{\mathrm{BF}^*}\overline{P}\left(uL^{\mathrm{BF}^*}\right)^{1-k} = 1 + r_b - b \tag{21}$$

$$\left(1-m^{\mathrm{BF}^*}\right)\bar{P}(1-k)u^{1-k}\left(L^{\mathrm{BF}^*}\right)^{-k}\frac{\partial L^{\mathrm{BF}^*}}{\partial m} = \bar{P}\left(uL^{\mathrm{BF}^*}\right)^{1-k} \tag{22}$$

$$1 - u(1-k)m^{\mathrm{BF}^*}\bar{P}(uL)^{-k} < 0 \tag{23}$$

$$\begin{aligned}&\left[(1-m)\bar{P}u^{1-k}\left(k^2-k\right)\left(L^{\mathrm{BF}^*}\right)^{1-k}\frac{\partial L^{\mathrm{BF}^*}}{\partial m} - 2\bar{P}u^{1-k}(1-k)\left(L^{\mathrm{BF}^*}\right)^{-k}\right]\frac{\partial L^{\mathrm{BF}^*}}{\partial m}\\ &\qquad + (1-m)\bar{P}u^{1-k}(1-k)\left(L^{\mathrm{BF}^*}\right)^{-k}\frac{\partial^2 L^{\mathrm{BF}^*}}{\partial m^2} < 0.\end{aligned} \tag{24}$$

The proof process of Property 3 is similar to that of Property 1, and therefore, it will not be repeated here.

5.2. Collateralized loan's optimal strategies

The farmer's profit function is as follows:

$$\Omega^{\mathrm{DF}} = \begin{cases} m\bar{P}(uL)^{1-k} - \lambda\left[m\bar{P}(uL)^{-k}\right]^2\sigma^2 - L(1+r) + bL, \\ m\bar{P}(uL)^{1-k} - \lambda\left[m\bar{P}(uL)^{-k}\right]^2\sigma^2 - L(1+r) + bL > 0 \\ \max_{L>0}\left\{\frac{m\bar{P}(uL)^{1-k} - \lambda\left[m\bar{P}(uL)^{-k}\right]^2\sigma^2 - L(1+r) + bL}{d}, -a\right\}, \\ m\bar{P}(uL)^{1-k} - \lambda\left[m\bar{P}(uL)^{-k}\right]^2\sigma^2 - L(1+r) + bL \le 0. \end{cases} \tag{25}$$

The purchaser's profit function is as follows:

$$\Pi^{\mathrm{DF}} = (1-m)\bar{P}(uL)^{1-k} + \min\left\{m\bar{P}(uL)^{1-k} - \lambda\left[m\overline{P}(uL)^{-k}\right]^2\sigma^2 + bL + da, L(1+r)\right\} - L. \tag{26}$$

As mentioned in the previous section, there are two additional key points of output uncertainty in this case. When the farmer's sales profit plus the government subsidy is just enough to repay the loan principal and interest, *i.e.*, $m\bar{P}(uL)^{l-k} - \lambda\sigma^2[m\bar{P}(uL)^{-k}]^2 + bL = L(1+r)$. The output uncertainty variable is u_2, which satisfies $m\bar{P}(Lu_2)^{-k}[Lu_2 - \lambda\sigma^2 m\bar{P}(u_2L)^{-k}] = L(1+r) - bL$. When the farmer's sales profit plus the government subsidy, as well as the value of the discounted collateral, is just enough to offset the loan principal and interest, *i.e.*, $m\bar{P}(uL)^{l-k} - \lambda\sigma^2[m\bar{P}(uL)^{-k}]^2 + bL + da = L(1+r)$. The output uncertainty variable is u_3, which satisfies $m\bar{P}(Lu_3)^{-k}[Lu_3 - \lambda\sigma^2 m\bar{P}(u_3L)^{-k}] = L(1+r) - bL - da$.

Therefore, the farmer's expected return is as follows:

$$E\left(\Omega^{\mathrm{DF}}\right) = \int_{u_2}^{\bar{u}} \left\{ m\overline{P}(uL)^{1-k} - \lambda\sigma^2\left[m\overline{P}(uL)^{-k}\right]^2 - L(1+r) + bL \right\} \cdot g(u)\,\mathrm{d}u + \int_{u_3}^{u_2} \frac{\left\{ m\bar{P}(uL)^{1-k} - \lambda\sigma^2\left[m\bar{P}(uL)^{-k}\right]^2 - L(1+r) + bL \right\}}{d} \cdot g(u)\,\mathrm{d}u + \int_0^{u_3} (-a) \cdot g(u)\,\mathrm{d}u. \tag{27}$$

The purchaser's expected return is as follows:

$$E\left(\Pi^{\mathrm{DF}}\right) = \int_0^{\bar{u}} (1-m)\bar{P}\left(uL^{\mathrm{DF}^*}\right)^{1-k} \cdot g(u)\,\mathrm{d}u + \int_{u_3}^{\bar{u}} L^{\mathrm{DF}^*}(1+r) \cdot g(u)\,\mathrm{d}u + \int_0^{u_3} \left\{ m\bar{P}\left(uL^{\mathrm{DF}^*}\right)^{1-k} - \lambda\sigma^2\left[m\bar{P}\left(uL^{\mathrm{DF}^*}\right)^{-k}\right]^2 + bL + da \right\} \cdot g(u)\,\mathrm{d}u - L^{\mathrm{DF}^*}. \tag{28}$$

We can find the following property according to the reverse solution method:

Property 4. The optimal loan amount $L^{\mathrm{DF}*}$ of the farmer, the optimal interest rate $r^{\mathrm{DF}*}$ of the purchaser and the optimal purchase price discount $m^{\mathrm{DF}*}$ of the purchaser satisfy the following restrictions:

$$(1-k)m^{\mathrm{DF}^*}\overline{P}\left(u^{L^*}\right)^{1-k}\left(L^{\mathrm{DF}^*}\right)^{-k} - 2\lambda\sigma^2\left(m^{\mathrm{DF}^*}\overline{P}\right)^2(-k)\left(u^{L^*}\right)^{-2k}\left(L^{\mathrm{DF}^*}\right)^{-2k-1} = 1 + r - b \tag{29}$$

$$\frac{\partial E\left(\Pi^{\mathrm{DF}}\right)}{\partial r} = \left\{ \int_0^{\Phi\left(L^{\mathrm{DF}^*}\right)} \left\{ \left[(1-k)\overline{P} + 2k\left(m\overline{P}\right)^2\left(uL^{\mathrm{DF}^*}\right)^{-k-1}\right] u^{1-k}\left(L^{\mathrm{DF}^*}\right)^{-k} \frac{\partial L^{\mathrm{DF}^*}}{\partial r} \right\} \mathrm{g}(u)\,\mathrm{d}u + \int_{\Phi\left(L^{\mathrm{DF}^*}\right)}^{\overline{u}} \left\{ (1-m)\overline{P}(1-k)u^{1-k}\left(L^{\mathrm{DF}^*}\right)^{-k} \frac{\partial L^{\mathrm{DF}^*}}{\partial r} \right\} + \frac{\partial L^{\mathrm{DF}^*}}{\partial r}(1+r) + L^{\mathrm{DF}^*} \right\} g(u)\,\mathrm{d}u - \frac{\partial L^{\mathrm{DF}^*}}{\partial r} = 0 \tag{30}$$

$$\frac{\partial E\left(\Pi^{\mathrm{DF}}\right)}{\partial m} = \left\{ \int_0^{\Phi\left(L^{\mathrm{DF}^*}\right)} \left\{ \left[(1-k)\overline{P} + 2k\left(m\overline{P}\right)^2\left(uL^{\mathrm{DF}^*}\right)^{-k-1}\right] u^{1-k}\left(L^{\mathrm{DF}^*}\right)^{-k} \frac{\partial L^{\mathrm{DF}^*}}{\partial m} \right\} g(u)\,\mathrm{d}u + \int_{\Phi\left(L^{\mathrm{DF}^*}\right)}^{\overline{u}} \left\{ (1-m)\overline{P}(1-k)u^{1-k}\left(L^{\mathrm{DF}^*}\right)^{-k} \frac{\partial L^{\mathrm{DF}^*}}{\partial m} \right\} + \frac{\partial L^{\mathrm{DF}^*}}{\partial m}(1+r) - \overline{P}\left(uL^{\mathrm{DF}^*}\right)^{1-k} \right\} g(u)\,\mathrm{d}u - \frac{\partial L^{\mathrm{DF}^*}}{\partial m} = 0 \tag{31}$$

$$\frac{\partial^2 E\left(\Pi^{\mathrm{DF}}\right)}{\partial r^2} < 0 \tag{32}$$

$$\frac{\partial^2 E\left(\Pi^{\mathrm{DF}}\right)}{\partial m^2} < 0 \tag{33}$$

$$\frac{\partial^2 E\left(\Omega^{\mathrm{DF}}\right)}{\partial L^2} < 0 \tag{34}$$

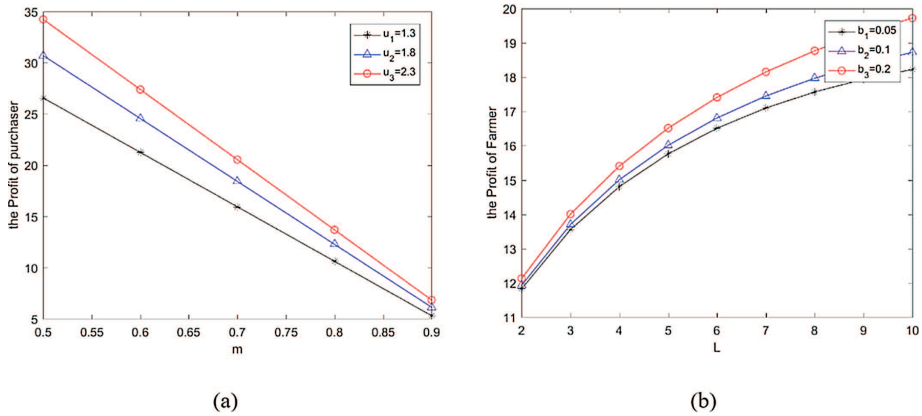


FIGURE 3. Impacts of decision variables on the purchaser's and farmer's profit.

where $\Phi(L^{\mathrm{DF}^*})$ satisfies $m\bar{P}(Lu)^{-k}[Lu - \lambda\sigma^2 m\bar{P}(uL)^{-k})] = L(1+r) - bL - da$, $\frac{\partial L^{\mathrm{DF}^*}}{\partial r} = \frac{(L^{\mathrm{DF}^*})^{1+k}}{-km^{\mathrm{DF}^*}\overline{P}[(1-k)(u^{L^*})^{1-k}+2\lambda\sigma^2 m^{\mathrm{DF}^*}\overline{P}(u^{L^*})^{-2k}(2k+1)(L^{\mathrm{DF}^*})^2}$, $\frac{\partial L^{\mathrm{DF}^*}}{\partial m} = \frac{(L^{\mathrm{DF}^*})^{1+k}[4\lambda\sigma^2 m^{\mathrm{DF}^*}\bar{P}^2 k(u^{L^*})^{-2k}(L^{\mathrm{DF}^*})^{-2k-1}+(1-k)\bar{P}(u^{L^*})^{1-k}(L^{\mathrm{DF}^*})^{-k}]}{km^{\mathrm{DF}^*}\bar{P}[(1-k)(u^{L^*})^{1-k}+2\lambda\sigma^2(u^{L^*})^{-2k}(2k+1)(L^{\mathrm{DF}^*})^2}$. The proof process of Property 4 is similar to that of Property 2, and therefore, it will not be repeated here.

6. NUMERICAL SIMULATION

Next, we aim to assess the influence of optimal decisions on the profits of both the farmer and the purchaser, utilizing specific values that match the actual situation. To examine the sensitivity changes in the profit of both parties, we also consider discount factors and government subsidies as the crucial variables.

6.1. Parameters setting

In this subsection, we give preference to using the values of the parameters from previous representative literature. The parameters are set as follows. We set u obeys a uniform distribution $[1.3, 2.3]$ [7]. In addition, we set $k = 1/1.8$ and $a = 10$ [61]. For the choking price $\bar{P}$ of agricultural products, we set the output and price data of corn and early rice between 13 and 23, *i.e.*, $\bar{P} = 17$ [20].

6.2. Sensitivity analysis under bank financing

6.2.1. Impacts of decision variables on profits

Combining the purchaser's profit function and Figure 3a, it can be seen that the purchase price discount is negatively related to the purchaser's profit. This means that when the purchase price deviates further from the market price, the purchaser earns less. As the output uncertainty variable increases, the purchaser's profit then increases. This is attributed to the simultaneous increase in demand and output, leading to a positive impact on the purchaser's profit. From Figure 3b, it is clear that the farmers' profit increases with the amount of loans. However, it's important to note that the marginal increase in profit diminishes over time. As government subsidies increase, farmers receive more subsidy income based on the same loan amount and profits naturally increase.

6.2.2. Effectiveness of government subsidies

Based on the above analysis, it is clear that government subsidies have a positive effect on farmers' profits. In this section, we need to further assess the effectiveness of the government subsidy. An effective subsidy should

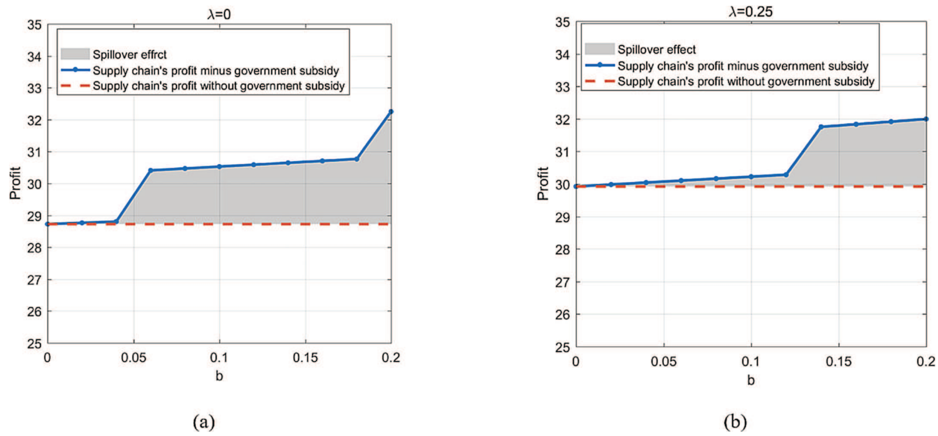


FIGURE 4. The spillover effect of government subsidies for the supply chain.

increase the supply chain's profit which is greater than the amount of government subsidy. That is, we study whether the subsidy brings additional spillover effects to the supply chain.

In Figure 4, the solid line represents the supply chain's profit deducting the government subsidies. The dashed line represents the supply chain's profit without government subsidies. Therefore, the shaded area reflects the additional incremental value of government subsidies (spillover effects). As can be seen from Figure 4, the spillover effect increases with the subsidy coefficient. On the one hand, government subsidies for agricultural loans improve farmers' repayment ability and reduce default risk. On the other hand, government subsidy promotes agricultural production and enables the supply chain to function normally. Therefore, government subsidies in bank financing are effective, and this effectiveness increases with the subsidy coefficient.

Comparing Figures 4a and 4b, we observe the following two differences: first, the overall profit of the supply chain increases with the risk aversion coefficient. The increase in risk aversion implies that the more conservative the farmer is, the greater the role of government subsidies in protecting the farmer. The farmers' profit increases with the subsidy coefficient, which ultimately leads to an increase in the supply chain's profit. Second, the spillover effect of government subsidies is segmented. When the subsidy coefficient is small, the spillover effect is not obvious; when the subsidy coefficient is large, the spillover effect is obvious. That is, there is a key point of the subsidy coefficient, which makes the spillover effect change in stages. When the risk aversion coefficient increases, the key point of subsidy coefficient increases, *i.e.*, the increasing trend of spillover effect becomes delayed in response to the subsidy coefficient. From the perspective of the transmission mechanism of the spillover effect of government subsidies, with the same subsidy coefficient, as the risk aversion increases, the farmer will reduce production quantity in order to prevent agricultural products from being sold late, which then decreases its profit.

6.3. Sensitivity analysis under the collateralized loan

6.3.1. Impacts of government subsidies and collateral discount factor

Another risk-sharing mechanism in this paper is government subsidies. Further analyzing the risk-sharing transmission mechanism, we can find that farmers receive a partial reduction of loan principal and interest, while the government bears this reduction. The beneficiaries are not only farmers but also purchasers. Due to the volatility of agricultural production, the loans initially faced complete uncertainty regarding the recovery of principal and interest. But now it gains certainty in terms of the government subsidy. Thus, the government subsidy is a partial risk sharing by the government for the agricultural supply chain.

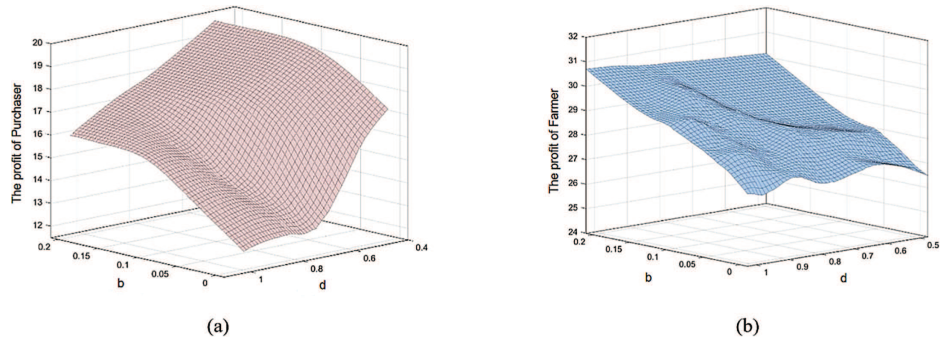


FIGURE 5. The combined effect of the subsidy and discount factors on the profit of each participant.

Analyzing the discount factor's transmission mechanism. When farmers must withhold a portion of the collateral to pay the principal and interest, the discount on the collateral exposes them to greater losses. As for purchasers, if they liquidate all the collateral in the worst case, the value that they can obtain will be reduced due to the discount factor.

Figure 5 shows the sensitivity of the purchaser's (and the farmer's) profit to changes in government subsidies and discount factor. Both the purchaser's and the farmer's profit increase with the subsidy factor. This is consistent with our previous findings on the transmission mechanism. Government subsidies prove effective in reducing risk for both the purchaser and the farmer, subsequently exerting a positive influence on the expected profits of both parties.

We also observe that the discount factor has different effects on the purchaser and the farmer respectively. The farmer's profit decreases as the discount factor decreases. This is because greater depreciation of collateral is less advantageous for the farmer. Conversely, the purchaser's profit increases as the discount factor decrease. This phenomenon arises from the interaction between the previously analyzed transmission mechanism, which can be detrimental to the purchaser, and the discount factor's influence, which makes the farmer more cautious in production to avoid losses. We can call it a "pushback mechanism". Specifically, when the farmer needs the discounted collateral to repay part of the loan, a lower discount factor necessitates a higher original value for the collateral. In response to this, the farmer exerts more effort in production to ensure the loan's success. As a result, the purchaser's overall expected profit increases.

6.3.2. Effectiveness of government subsidies and collateral discount factor

We find that the spillover effect of government subsidies increases with the subsidy factor. From the transmission mechanism perspective, this increase is due to government subsidies not only expanding production incentives but also distributing risks for the purchaser and then stimulating credit easing.

In Figures 6a and 6b, we compare two scenarios: one without the discount phenomenon and the other with a discount factor of 0.75. Comparing Figure 6a with 6b, we find that when the subsidy factors are the same, the lower the discount factor, the higher the supply chain's profit. That is, the depreciation of collateral also brings a spillover effect to the supply chain. In Figure 5, the purchaser's profits increase much more than the farmer's profits decrease as the discount factor decreases. As a result, the supply chain's overall profit increases.

In Figures 6a and 6c, we compare two scenarios: one without risk aversion and the other with risk aversion coefficient of 0.5. Comparing Figure 6a with 6c, we observe that the overall profit of the supply chain increases with the risk aversion coefficient. In addition, we find again that the increasing trend of the spillover effect diminishes with the increase in the risk aversion coefficient for the same subsidy coefficient. Under the collateralized loan, the more conservative the farmer is, the less the production quantity or loan amount is, which

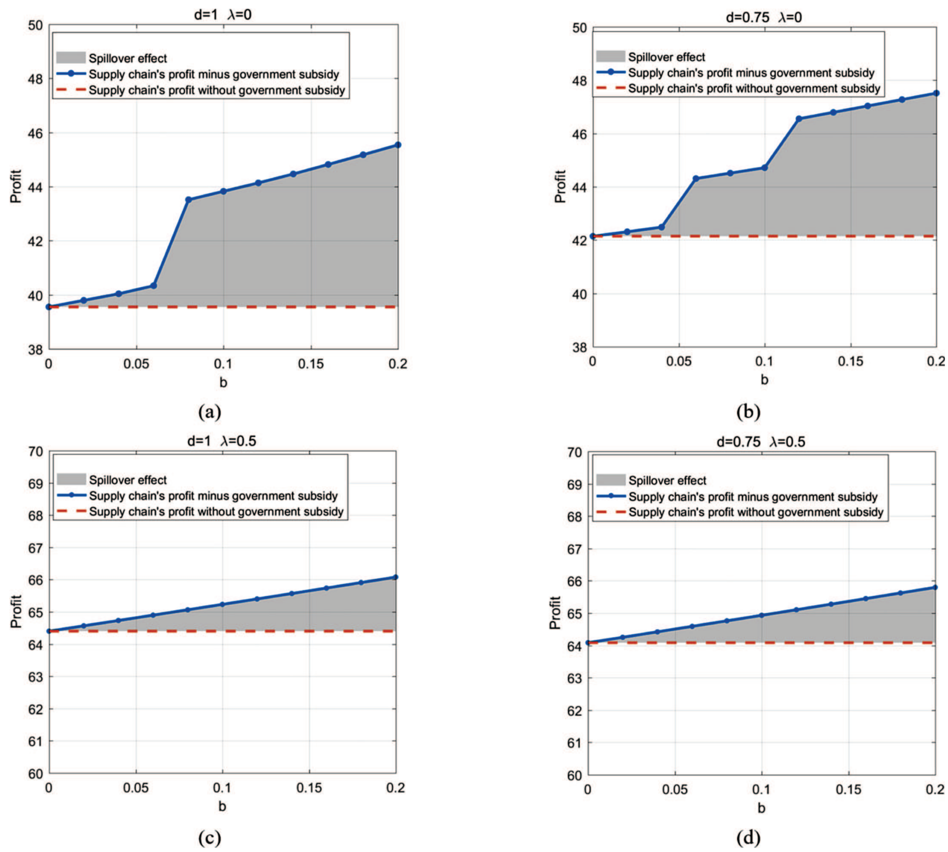


FIGURE 6. The spillover effect of government subsidies for the supply chain.

hinders the operation of the risk-sharing mechanism. That is, an increase in risk aversion inhibits the profit spillover from government subsidies to the supply chain.

Differing from bank financing, the spillover effect of government subsidies is more obvious under collateralized loans. That is, with the increase of the subsidy coefficient, the net increase of the supply chain's profit is more significant under the collateralized loan. The risk of the collateralized loan is shared among the farmer, the purchaser, and the government, whereas the risk-sharing in bank financing is only between the farmer and the government. Therefore, government subsidies are more effective in the collateralized loan.

6.3.3. Interaction of government subsidies

In this section, we further evaluate the interactions (constraints or synergies) of government subsidies with other critical variables. Given that the purchaser holds a position of leadership in the supply chain, they become the primary focus of this study.

First, Figure 7a represents the effect of the subsidy factor and collateral value on the purchaser's profit. When the subsidy keeps constant, we notice that the purchaser's profit decreases with the fixed assets' value. When the subsidy increases, the declining trend of the purchaser's profit is diminished. This suggests that higher government subsidies provide additional security, reducing the purchaser's credit risk. Consequently, uncertainties for the purchaser are more effectively managed. In other words, government subsidies not only lower the repayment risk for farmers but also mitigate the risk of unrealized collateral value, protecting the purchaser's profit.

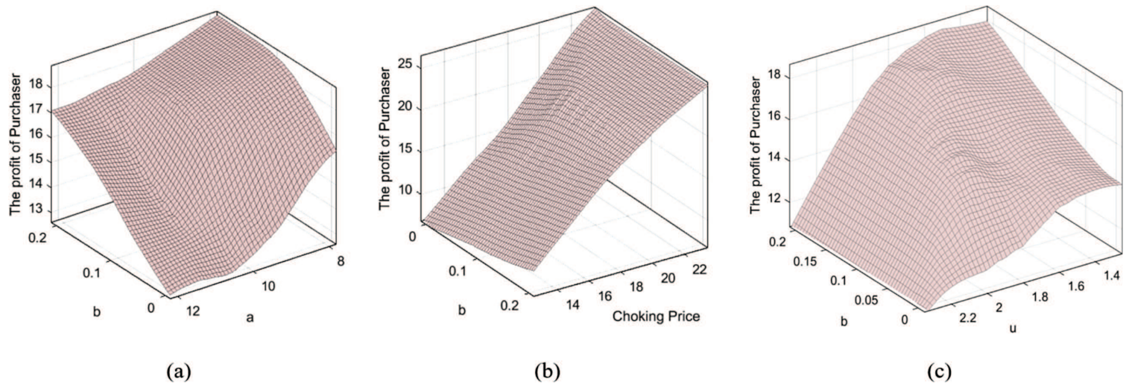


FIGURE 7. Interaction of subsidy factor with other critical variables.

The choking price is a characteristic unique to agricultural products and is directly linked to the type of product. Agricultural products with high choking prices usually correspond to high-end products and *vice versa* for low-end products. In Figure 7b, we find that government subsidies have a greater impact on the rising tendency of low-end products than high-end ones. This is because high-end products yield higher output values, making them less susceptible to subsidies at the same level. High-end product producers typically have stronger financial positions and are less reliant on government aid. Therefore, the choice of crops should not solely hinge on their categorization as high or low-end, but should consider a broader range of factors.

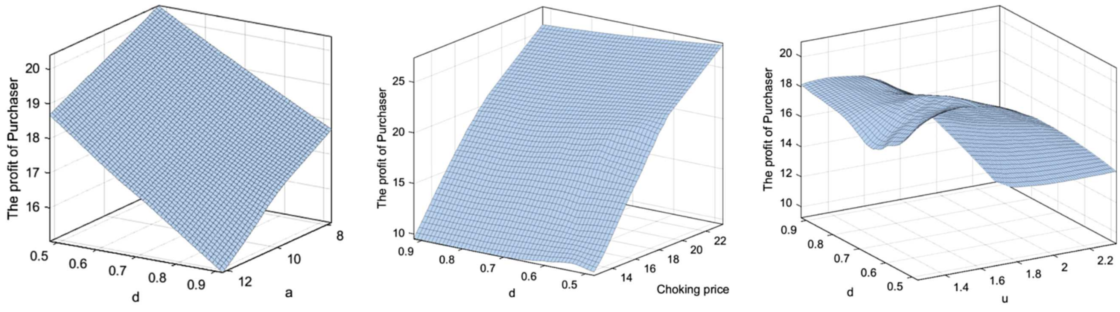
Output uncertainty indicators are key variables for measuring risk, and low output is the main source of credit risk in this paper. However, from the purchaser's perspective, higher output values are not always preferable. Higher output causes the market's supply exceeds the demand, declining the products' price and the purchaser's profit. Figure 7c demonstrates that as the subsidy factor increases, the purchaser's profit is negatively affected by the output's value to a deeper extent. This is because, incentivized by government subsidies in years of high output, farmers may tend to overproduce. Supply further increases and prices further fall, thus deepening the extent to which low grain prices hurt farmers. The purchaser's profit further declines. Therefore, we believe that government subsidies should no longer be provided in years with better output.

6.3.4. Interaction of discount factors with other variables

The discount factor affects the risk-sharing mechanism by influencing the realized value of the collateral. Therefore, this paper also considers the interaction between the discount factor and these critical variables.

Figure 8a represents the interaction of the discount factor and the fixed assets' value. Consistent with the analysis above, the purchaser's profit decreases with the value of fixed assets. Moreover, this declining trend intensifies with the discount factor. The incentive for farmers to produce decreases when fixed asset values rise. The purchaser's profit experiences a further reduction due to the increased uncertainty in selling the fixed assets brought about by a higher discount factor. To mitigate potential losses from substantial discounts on fixed assets, the purchaser should implement stricter supervision over the production process, especially when dealing with high-value collateral.

Figure 8b represents the interaction of the discount factor and the choking price. The choking price worsens the falling trend in purchasers' profits. That is, the sensitivity of the purchaser's profit is greater for high-end products than for low-end products. The discount on farmers' collateral poses a higher risk for high-end products. This is due to the narrow audience and smaller demand for high-end products. Purchasers seek high returns mainly by raising prices. Once consumers notice the severe discounting of farmers' fixed assets, they will quickly leave. As a result, the purchaser sells at lower prices to give up some of the profits. Therefore, the purchaser should sell more low-end products during the period of substantial depreciation of fixed assets.



(a) (b) (c)

FIGURE 8. Interaction of discount factor with other critical variables.

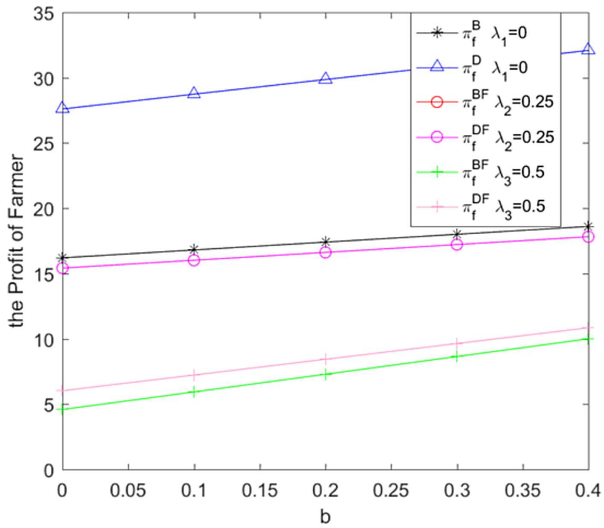


FIGURE 9. Effect of government subsidy coefficient and risk aversion coefficient on the farmer's profit.

Figure 8c represents the interaction of the discount factor and output uncertainty. The discount factor exerts a lesser impact on the purchaser's profit in scenarios with fewer output volumes. However, as output conditions improve, the discount factor wields a greater influence on the purchaser's losses. Consistent with the analysis of the subsidy scenario above, better output years prompt farmers to blindly expand production scale. This ultimately lowers the agricultural products' price and the purchaser's profit. Therefore, in better output years, it is also necessary to monitor the auction market in time to avoid substantial depreciation of collateral.

6.4. Comparison between bank financing and collateralized loan

To verify the effects of the government subsidy coefficient and risk aversion coefficient on risk sharing mechanism, we select the two variables in this section. We compare their effects on the supply chain's profit under bank financing and collateralized loan. Since there is no difference in the purchaser's profit under different risk aversion coefficients, we only analyze the farmer's profit below.

As shown in Figure 9, when both the government subsidy coefficient and risk aversion coefficient keep constant, the farmer's profit under the collateralized loan is not lower than those under the bank financing. This indicates that the risk-sharing mechanism represented by the collateralized loan is effective. When the risk aversion coefficient keeps constant, the farmer's profit under the collateralized loan increases with the government subsidy coefficient. This indicates that the increase in the government subsidy coefficient facilitates the operation of the risk-sharing mechanism. When the government subsidy coefficient keeps constant, the farmer's profit under the collateralized loan decreases with the risk aversion coefficient. This indicates that the increase in the risk aversion coefficient hinders the operation of the risk-sharing mechanism.

7. Managerial implications

While discussions on financing methods in agricultural supply chain literature abound, few practical studies guide policymakers to allocate resources reasonably and reduce enterprises' agricultural loan risks. As a result, agricultural subsidies fail to fully realize their potential in boosting farmers' productivity or benefiting the overall supply chain. Agricultural development serves as a vital artery for a nation's economic growth. Enterprise managers must be aware of the timely need of farmers for financial assistance. Policymakers also need to adjust the degree of subsidy bias, increase the output value of agricultural products, and promote agricultural production. Therefore, this study proposes the following management implications:

(1) The aim of this study is not only to reduce the agricultural loan risk for businesses, but also to enhance the overall welfare brought to society by government subsidies. Policymakers should increase subsidies for agriculture loans to maximize social welfare. This is because increasing subsidies can bring a profit spillover effect to the whole agricultural supply chain. Government subsidies are no longer recommended when the output is high. Policymakers should optimize the marginal utility of subsidies. In other words, subsidies should be provided when farmers' output is relatively low, in order to maximize the positive externalities of subsidizing agricultural production.
(2) This study shows that the farmer with low-fixed assets needs more capital. It is more difficult for farmers with less fixed assets to obtain loans than those with high fixed assets. Due to the low quality of collateral assets for these farmers, they often struggle to gain the trust of enterprises. However, with the protection of a risk-sharing mechanism, the default risk faced by enterprises may be significantly reduced. Therefore, the entrepreneur should accurately identify the types of farmers and allocate more funds to those in worse conditions.
(3) The findings of this study provide agricultural policymakers with a clear path to optimize the allocation of government subsidy funds. The policymakers should provide more agricultural loan subsidies to conservative farmers. The greater the risk aversion of farmers, the less obvious the risk-sharing effect of the collateralized loan. In order to ensure the effective operation of the risk-sharing mechanism, policymakers should take advantage of the positive impact of agricultural loan subsidies and focus on providing more subsidies to farmers with greater risk aversion. Because subsidies can offset the negative impact of risk aversion on this mechanism.
(4) The introduction of the concept of depreciation helped enterprise managers to overcome the problem of impairment losses on collateralized assets. The increase in the discount coefficient can bring a profit spillover effect. Entrepreneurs should push the establishment of an effective and standardized auction market, which can promote the collateral to obtain a fair realizable value. In other words, the discount factor can reach an appropriate range by market means.

8. Conclusions and limitations

Agricultural supply chain finance is considered an effective approach to address the financing difficulties faced by farmers. However, the risks associated with the financing process in the agricultural supply chain need to be adequately addressed [62]. Therefore, this paper proposes a collateralized loan approach based on the farmers' own collateral to share the financing risks of the purchaser. This section mainly summarizes the results of this study and proposes future research directions. This can improve the agricultural supply chain's operation efficiency and help the development of rural revitalization.

8.1. Conclusions

(1) In order to address the issues of "difficulty in obtaining loans" and "high loan costs" for farmers, a new risk-sharing mechanism consisting of mortgage loans and government subsidies has been established. This study also introduces the government's loan subsidies and the farmers' risk aversion to examine the effectiveness of this risk-sharing mechanism. By applying Stackelberg game theory, the optimal decisions for both farmers and purchasers are determined in each scenario.
(2) By comparing the two financing methods, this study found that government subsidies are more effective under the collateralized loan than the bank financing. In other words, under the collateralized loan, government subsidies can generate more profit for the whole supply chain.
(3) Compared to the risk-sharing mechanism provided solely by collateralized loans, the addition of government subsidies further disperses the risk distribution among farmers, purchasers, and the government. Government subsidies for farmer loans facilitate the operation of risk-sharing mechanisms. Government subsidies bring significant profit spillover effects to the agricultural supply chain, which increases with the subsidy amount. Government subsidies effectively enhance the supply chain's overall risk resistance while relieving farmers' production pressure.
(4) This study suggests that the discount factor also generates a profit spillover effect for the entire agricultural supply chain through an inversion mechanism. The overall profit of the supply chain increases with the degree of collateral depreciation in the market. When the discount factor increases, loan failure scenarios due to poor production cause the farmer to suffer more losses. Therefore, this mechanism pushes the farmer to produce aggressively to ensure the loan's success.
(5) Different from the government subsidies, the risk aversion coefficient of farmers hinders the operation of the risk-sharing mechanism. The higher the risk aversion, the lower the degree of profit spillover from the collateralized loan to the supply chain. In the presence of risk aversion, farmers' conservative production and financing preferences cause them to shrink their production quantity or reduce loans. With the same level of subsidy, farmers' profits decline. Therefore, the effectiveness of government subsidies under the collateralized loan is diminished.

This study has been conducted for the first time in order to establish a novel risk-sharing mechanism to explore the spillover effects of government subsidies on the agricultural supply chain when the output is uncertain. The results of this study demonstrate the applicability and superiority of the collateralized loan + government subsidy model in alleviating the difficulty of farmers in obtaining loans. It not only meets the production capital needs of farmers but also maximizes the profitability of the entire supply chain.

8.2. Limitations

Future studies can be expanded in the following areas based on this paper. First, we only considered the uncertainty of the farmer's output while ignoring demand uncertainty. There is a discrepancy between the demand for agricultural products and the quantity of farmers' production. This difference might intensify the risks associated with the agricultural supply chain. It would be more realistic to consider this uncertainty. Second, blockchain technology can be used in agricultural supply chain risk management. The tamper-evident and traceable characteristics of blockchain technology can measure risk exposure more accurately and prevent risks in time [63]. Therefore, it can also well address the risks brought by the seasonal and perishable characteristics of agricultural products to the supply chain. Finally, the issue of agricultural supply chain disruptions caused by the impact of the COVID-19 pandemic outbreak was not considered. The epidemic shock not only affects global economic development [64], but also an inventory management in socially and economically [65]. It remains to be seen whether uncertain risks arising from such emergencies will affect the operation of risk-sharing mechanisms. Therefore, to solve this problem, the future study will analysis the effect of the COVID-19 pandemic on the agricultural supply chains and consider the mixed uncertainty (as considered in the model which developed by Mondal and Roy [66]).

Appendix A.

Proof of Property 1. According to the solution order, the farmer decides the optimal loan amount based on the profit maximization principle under the assumption that the purchaser's optimal decision is given. The steps are as follows.

$$\frac{\partial \Omega^B}{\partial L} = (1-k)L^{-k} m\bar{P}u^{1-k} - (1+r_b) + b.$$

Let $\frac{\partial \Omega^B}{\partial L} = 0$, then we can get the loan amount reaction function $L(m)$ about m:

$$L(m) = \left[\frac{(1-k)m\bar{P}u^{1-k}}{1+r_b-b}\right]^{\frac{1}{k}}.$$

The purchaser makes profit maximization decisions based on the farmer's decisions. Substitute $L(m)$ into the purchaser's profit function. By considering the partial derivative of the purchaser's profit function with regard to the purchase price discount m, the best result m^{B^*} is obtained.

$$\begin{aligned}\frac{\partial \Pi^B}{\partial m} &= -\bar{P}(uL)^{1-k} + (1-m)\bar{P}u^{1-k}(1-k)L^{-k}\frac{\partial L(m)}{\partial m}\\ \frac{\partial^2 \Pi^B}{\partial m^2} &= \left[(1-m)\bar{P}u^{1-k}(k^2-k)L^{1-k}\frac{\partial L}{\partial m} - 2\bar{P}u^{1-k}(1-k)L^{-k}\right]\frac{\partial L}{\partial m}\\ &\quad + (1-m)\bar{P}u^{1-k}(1-k)L^{1-k}\frac{\partial^2 L}{\partial m^2}\end{aligned}$$

where $\frac{\partial L(m)}{\partial m} = \frac{1}{k}[\frac{(1-k)\bar{P}u^{1-k}}{1+r_b-b}]^{\frac{1}{k}} m^{\frac{1}{k}-1}$, $\frac{\partial^2 L(m)}{\partial m^2} = \frac{1}{k}(\frac{1}{k}-1)[\frac{(1-k)\bar{P}u^{1-k}}{1+r_b-b}]^{\frac{1}{k}} m^{\frac{1}{k}-2}$.

Let $\frac{\partial \Pi^B}{\partial m} = 0$, $\frac{\partial^2 \Pi^B}{\partial m^2} < 0$. Then the optimal purchase price discount m^{B^*} can be solved as follows.

$$\begin{aligned}\bar{P}\left(uL^{B^*}\right)^{1-k} &= \frac{(1-m)\bar{P}(1-k)u^{1-k}L^{-k}}{km^{B^*}}\left[\frac{\bar{P}(1-k)u^{1-k}m^{B^*}}{1+r_b-b}\right]^{\frac{1}{k}}\\ &\left[(1-m)\bar{P}u^{1-k}\left(k^2-k\right)\left(L^{B^*}\right)^{1-k}\frac{\partial L^{B^*}}{\partial m} - 2\bar{P}u^{1-k}(1-k)\left(L^{B^*}\right)^{-k}\right]\frac{\partial L^{B^*}}{\partial m}\\ &+ (1-m)\bar{P}u^{1-k}(1-k)\left(L^{B^*}\right)^{-k}\frac{\partial^2 L^{B^*}}{\partial m^2} < 0.\end{aligned}$$

Substitute m^{B^*} into $L(m)$, we can get the optimal loan amount:

$$L^{B^*} = \left[\frac{(1-k)m^{B^*}\bar{P}u^{1-k}}{1+r_b-b}\right]^{\frac{1}{k}}.$$

Proof completed. □

Proof of Property 2. In order to resolve the farmer's optimization strategy, the optimal solution L^* is obtained by taking the partial derivative of the farmer's expected profit function with respect to the loan amount L.

$$\frac{\partial E\left(\Omega^D\right)}{\partial L} = \int_{\left[\frac{L(1+r-b)}{m\overline{P}}\right]^{1-k}\frac{1}{L}}^{u} \left[(1-k)m\overline{P}u^{1-k}L^{-k} - (1+r) + b\right]g(u)\,\mathrm{d}u$$

$$
\begin{aligned}
&+\left[-m\overline{P}\left(\left[\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}L\right)^{1-k}-Lb+L(1+r)\right]\cdot g\left(\left(\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right)\cdot\varphi(L)'\\
&+\frac{1}{d}\left[m\overline{P}\left(\left(\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}L\right)^{1-k}+Lb-L(1+r)\right]\cdot g\left(\left(\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right)\cdot\varphi(L)\\
&-\frac{1}{d}\left[m\bar{P}\left(\left(\frac{L(1+r-b)-da}{m\bar{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}L\right)^{1-k}+Lb-L(1+r)\right]\\
&\cdot g\left(\left(\frac{L(1+r-b)-da}{m\bar{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right)\cdot\phi(L)'\\
&+\frac{1}{d}\int_{\left[\frac{L(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L}}^{\left[\frac{L(1+r-b)}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L}}\left[(1-k)m\overline{P}u^{1-k}L^{-k}-(1+r)+b\right]g(u)\,\mathrm{d}u\\
&-a\cdot f\left(\left[\frac{L(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L}\right)\cdot\phi(L)'\\
=&\int_{\left[\frac{L(1+r-b)}{m\bar{P}}\right]^{1-k}\frac{1}{L}}^{\overline{u}}\left[(1-k)m\overline{P}u^{1-k}L^{-k}-(1+r)+b\right]g(u)\,\mathrm{d}u\\
&+\frac{1}{d}\int_{\left[\frac{L(1+r-b)-da}{\cdots D}\right]^{\frac{1}{1-k}}\frac{1}{r}}^{\left[\frac{L(1+r-b)}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L}}\left[(1-k)m\overline{P}u^{1-k}L^{-k}-(1+r)+b\right]g(u)\,\mathrm{d}u.
\end{aligned}
$$

Let $\frac{\partial E(\Omega^D)}{\partial L}=0$. The condition is satisfied only if both parts of the formula are equal to zero and use the integral median theorem:

$$
\begin{aligned}
&\exists u^{L1}\in\left(\left(\frac{L(1+r-b)}{m\bar{P}}\right)^{\frac{1}{1-k}}\frac{1}{L},\overline{u}\right)\\
&\exists u^{L2}\in\left(\left(\frac{L(1+r-b)-d\alpha}{m\bar{P}}\right)^{\frac{1}{1-k}}\frac{1}{L},\left(\frac{L(1+r-b)}{m\bar{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right).
\end{aligned}
$$

Satisfy:

$$
\left[(1-k)m\overline{P}u^{1-k}L^{-k}-1-r-b\right]g\left(u^{L1}\right)\cdot\left[\overline{u}-\left(\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right]=0
$$

or

$$
\left[(1-k)m\overline{P}u^{1-k}L^{-k}-1-r-b\right]g\left(u^{L2}\right)\cdot\left[\left(\frac{L(1+r-b)}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}-\left(\frac{L(1+r-b)-da}{m\overline{P}}\right)^{\frac{1}{1-k}}\frac{1}{L}\right]=0.
$$

Because $g(u^{L1})\cdot[\overline{u}-(\frac{L(1+r-b)}{m\overline{P}})^{\frac{1}{1-k}}\frac{1}{L}]>0$ and $g(u^{L2})\cdot[(\frac{L(1+r-b)}{m\bar{P}})^{\frac{1}{1-k}}\frac{1}{L}-(\frac{L(1+r-b)-da}{m\overline{P}})^{\frac{1}{1-k}}\frac{1}{L}]>0$, Only when $(1-k)m\overline{P}u^{1-k}L^{-k}-1-r-b=0$.

Let $\frac{\partial E(\Omega^D)}{\partial L}=0$. we can get the optimal loan amount:

$$
L^{D^*}=\left[\frac{m\overline{P}\left(u^{L^*}\right)^{1-k}\cdot(1-k)}{1+r-b}\right]^{\frac{1}{k}}.
$$

Solving the purchaser's optimization problem, the purchaser's expected profit function is solved for the partial derivatives of r and m, respectively.

$$\begin{aligned}
\frac{\partial E(\Pi^D)}{\partial r} &= \int_0^{\left[\frac{L^{D^*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}} \left[(1-k)\bar{P}u^{1-k}\left(L^{D^*}\right)^{-k}\right]\frac{\partial L^{D^*}}{\partial r} g(u)\,\mathrm{d}u \\
&\quad + \left[\frac{L^{D^*}(1+r-b)-d\alpha}{m} + bL + d\alpha\right]\cdot g\left(\left[\frac{L^{D^*}(1+r-b)-d\alpha}{m\overline{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}\right)\cdot \phi(r)' \\
&\quad + \int_{\left[\frac{L^{D^*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}}^{\overline{u}} \left[(1-m)\bar{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k}\frac{\partial L^{D^*}}{\partial r} + L^{D^*} + \frac{\partial L^{D^*}}{\partial r}(1+r)\right] g(u)\,\mathrm{d}u \\
&\quad - \left[(1-m)\frac{L^{D*}(1+r-b)-da}{m} + L^{D^*}(1+r)\right]\cdot g\left(\left[\frac{L^{D*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}\right)\cdot\phi(r)' \\
&\quad - \frac{\partial L^{D^*}}{\partial r} \\
&= \int_0^{\overline{u}}\left[(1-k)\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k}\right]g(u)\,\mathrm{d}u\cdot\frac{\partial L^{D^*}}{\partial r} \\
&\quad + \int_{\phi(L^*)}^{\overline{u}}\left[(k-1)m\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k}\frac{\partial L^{D^*}}{\partial r}(1+r)\right]g(u)\,\mathrm{d}u - \frac{\partial L^{D^*}}{\partial r} \\
&= \left\{\int_0^{\overline{u}}\left[(1-k)\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k}\right]g(u)\,\mathrm{d}u - \int_{\phi(L^*)}^{\overline{u}}\left[(k-1)m\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k} + (1+r)\right]g(u)\,\mathrm{d}u - 1\right\} \\
&\quad \cdot\frac{\partial L^{D^*}}{\partial r} + \int_{\phi(L^*)}^{\overline{u}} Lg(u)\,\mathrm{d}u \\
\frac{\partial E^2(\Pi^D)}{\partial r^2} &= \left\{\int_0^{\overline{u}}\left[(1-k)\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k}\right]g(u)\,\mathrm{d}u - \int_{\phi(L^*)}^{\overline{u}}\left[(k-1)m\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k} + (1+r)\right]g(u)\,\mathrm{d}u - 1\right\} \\
&\quad \times\frac{\partial^2 L^{D^*}}{\partial r^2} + \frac{\partial L^{D*}}{\partial r}\left\{\int_0^{\phi(L^*)}\left[\left(k^2-k\right)\overline{P}u^{1-k}\left(L^{D^*}\right)^{-k-1}\right]\frac{\partial L^{D^*}}{\partial r}g(u)\,\mathrm{d}u + \left[\frac{L(1+r-b)\left(k^2-k\right)}{m\left(L^{D^*}\right)^2}\right]\right. \\
&\quad \left.\times\, g(\phi(L^*))\phi'(r) - G(\overline{u}) + G(\phi(L^*)) + (1+r)\frac{\partial G(\phi(L^*))}{\partial r}\right\} + \frac{\partial L^{D^*}}{\partial r}\left[G(\overline{u}) - G(\phi(L^*))\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E(\Pi^D)}{\partial m} &= \int_0^{\left[\frac{L^{D^*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}\frac{1}{\left[(1-k)\bar{P}u^{1-k}\left(L^{D^*}\right)^{-k}\right]}\frac{\partial L^{D^*}}{\partial m}g(u)\,\mathrm{d}u} + \left[\frac{L^{D*}(1+r-b)-da}{m} + a\right] \\
&\quad \cdot g\left(\left[\frac{L^{D*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}\right)\cdot\phi(m)' + \int_{\left[\frac{L^{D^*}(1+r-b)-da}{m\bar{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}}^{\overline{u}} \\
&\quad \times\left[(1-m)\overline{P}(1-k)u^{1-k}\left(L^{D^*}\right)^{-k}\frac{\partial L^{D^*}}{\partial m} - \overline{P}\left(uL^{D^*}\right)^{1-k} + \frac{\partial L^{D^*}}{\partial m}(1+r)\right]g(u)\,\mathrm{d}u \\
&\quad - \left[(1-m)\frac{L^{D^*}(1+r-b)-d\alpha}{m} + L^{D^*}(1+r)\right]\cdot g\left(\left[\frac{L^{D^*}(1+r-b)-d\alpha}{m\overline{P}}\right]^{\frac{1}{1-k}}\frac{1}{L^{D^*}}\right)\cdot\phi(m)' \\
&\quad - \frac{\partial L^{D^*}}{\partial m}
\end{aligned}$$

$$
\begin{aligned}
&= \left\{ \int_0^{\overline{u}} \left[(1-k)\overline{P} u^{1-k} \left(L^{D^*} \right)^{-k} \right] g(u)\,\mathrm{d}u + \int_{\phi(L^*)}^{\overline{u}} \left[(k-1) m \overline{P} u^{1-k} \left(L^{D^*} \right)^{-k} \operatorname{gcd}(u) + (1+r) \right] \mathrm{d}u \right\} \\
&\quad \cdot \frac{\partial L^{D^*}}{\partial m} - \int_{\phi\left(L^*\right)}^{u} \overline{P} \left(u L^{D^*} \right)^{1-k} g(u)\,\mathrm{d}u \\
\frac{\partial E^2\left(\Pi^D\right)}{\partial m^2} &= \left\{ \int_0^{\overline{u}} \left[(1-k)\overline{P} u^{1-k} \left(L^{D^*} \right)^{-k} \right] g(u)\,\mathrm{d}u + \int_{\phi(L^*)}^{\overline{u}} \left[(k-1) m \overline{P} u^{1-k} \left(L^{D^*} \right)^{-k} g(u) + (1+r) \right] \mathrm{d}u \right\} \\
&\quad \times \frac{\partial^2 L^D}{\partial m^2} + \frac{\partial L^{D^*}}{\partial m} \left\{ \int_0^{\phi(L^*)} \left[\left(k^2-k\right) \overline{P} u^{1-k} \left(L^{D^*} \right)^{-k-1} \right] \frac{\partial L^{D^*}}{\partial m} g(u)\,\mathrm{d}u \right. \\
&\quad \left. + \left[\frac{L(1+r-b)\left(k^2-k\right)}{m L^2} \right] g(\phi(L^*))\phi'(m) + (1+r) \frac{\partial G(\phi(L^*))}{\partial m} \right\} \\
&\quad - \int_{\phi(L^*)}^{\overline{u}} \left[(1-k)\overline{P} u^{1-k} \left(L^{D^*} \right)^{-k} \frac{\partial L^{D^*}}{\partial m} \right] g(u)\,\mathrm{d}u + \left[\frac{L(1+r-b) - d\alpha}{m} \right] g(\phi(L^*))\phi'(m)
\end{aligned}
$$

where:

$$
\begin{aligned}
L^{D^*} &= \left[\frac{m\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k)}{1+r-b} \right]^{\frac{1}{k}} \\
\frac{\partial L^{D^*}}{\partial r} &= \frac{1}{k} m^{\frac{1-k}{k}} \left[\frac{\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k)}{1+r-b} \right]^{\frac{1}{k}} \\
\frac{\partial^2 L^{D^*}}{\partial r^2} &= \frac{1-k}{k^2} m^{\frac{1}{k}-2} \left[\frac{\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k)}{1+r-b} \right]^{\frac{1}{k}} \\
\frac{\partial L^{D^*}}{\partial m} &= \frac{1}{k} m^{-\frac{1+k}{k}} \left[\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k) \right]^{\frac{1}{k}} \\
\frac{\partial^2 L^{D^*}}{\partial m^2} &= \frac{-1-k}{k^2} m^{-\frac{1}{k}-2} \left[\overline{P}\left(u^{L^*}\right)^{1-k} \cdot (1-k) \right]^{\frac{1}{k}} \\
\phi'(r) &= \frac{\partial \left[\left(\frac{L^{D^*}(1+r-b) - da}{m\overline{P}} \right)^{\frac{1}{1-k}} \frac{1}{L^{D^*}} \right]}{\partial r} \\
&= -\left(\frac{L^{D^*}(1+r-b) - da}{m\overline{P}} \right)^{\frac{1}{1-k}} \left(\frac{1}{L^{D^*}} \right)^2 \frac{\partial L^{D^*}}{\partial r} \\
&\quad + \frac{1}{\left(m\overline{P}\right)^2 L^{D^*}(1-k)} \cdot \left[\frac{\left(L^{D^*}(1+r-b) - d\alpha \right)^k}{m\overline{P}} \right]^{\frac{1}{1-k}} \cdot \left[m\overline{P}(1+r-b) \frac{\partial L^{D^*}}{\partial r} - m\overline{P} L^{D^*} \right] \\
\phi'(m) &= \frac{\partial \left[\left(\frac{L^{D^*}(1+r-b) - da}{m\overline{P}} \right)^{\frac{1}{1-k}} \frac{1}{L^{D^*}} \right]}{\partial m} \\
&= -\left(\frac{L^{D^*}(1+r-b) - da}{m\overline{P}} \right)^{\frac{1}{1-k}} \left(\frac{1}{L^{D^*}} \right)^2 \frac{\partial L^{D^*}}{\partial m}
\end{aligned}
$$

$$+\frac{1}{m^2 L^{D^*}(1-k)}\cdot\left[\frac{(L^{D^*}(1+r-b)-d\alpha)^k}{m\bar{P}}\right]^{\frac{1}{1-k}}\cdot\left[(1+r-b)\frac{\partial L^{D^*}}{\partial m}-\Big(L^{D^*}(1+r-b)\Big)-d\alpha\right].$$

Let the partial derivatives of both be zero and second order partial derivatives less than zero at the same time:

$$\begin{cases}\dfrac{\partial E\big(\Pi^D\big)}{\partial r}=0\\ \dfrac{\partial E\big(\Pi^D\big)}{\partial m}=0\\ \dfrac{\partial E^2\big(\Pi^D\big)}{\partial r^2}<0\\ \dfrac{\partial E^2\big(\Pi^D\big)}{\partial m^2}<0.\end{cases}$$

Proof completed. □

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