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TWO-PERIOD DECISION STRATEGIES IN A DUAL-CHANNEL SUPPLY CHAIN CONSIDERING REFERENCE PRICE AND ONLINE REVIEWS

Srikumar Panda[1] and Tarun Maiti[2,*]

Abstract. The rapid development of e-commerce and Internet technology impacts the consumer purchasing practices and the decision-making of the supply chain. In this regard, dealing with online reviews and reference price together for a competitive interaction in a two-period dual-channel scenario is one of the main challenges. To address this interaction, in the present research, we consider a dual-channel supply chain with a two-period. The selling price and the reference price impact on the retailer's demand in both channels. Whereas, the online reviews influence the demands of the direct channels only. The manufacturer, who announces wholesale prices and direct channel selling prices, is Stackelberg game's leader and the retailer is the follower. Two different decision-making strategies (I, II) are made by both players: (I) The manufacturer and the retailer both make all their decisions at the beginning of the selling season. (II) Here all decisions are made at the beginning of each selling period. In order to compare these strategies, we create a centralized policy as a benchmark scenario. The optimal solutions of the supply chain and each player are determined and analyzed. The numerical and sensitivity analysis suggests that the responsive pricing can bring additional benefits to both the players.

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1. Introduction

The rapid development of the Internet and information technology has led to the tremendous growth of automated trading. This consequence makes different aspects of demand and supply. To fulfill this demand, multiple production and sales possibilities for both (upstream and downstream) firms in a supply chain is a new direction for research. As a consequence, companies may introduce online channels with their existing retail channels or may use multiple periods to meet the on-growing requirement. However, the initiation of online channels creates channel competition between the manufacturers and retailers because of common consumer groups. In case of multi-period, the consumers can observe the previous prices (or reference prices) and previous product reviews. For example, various companies like computer manufacturer (Apple, IBM, Dell, etc.), sports kits manufacturer (Nike, Adidas, Puma, COSCO, etc.), cosmetics manufacturers (Estée Lauder, Lakme, Avon, etc.) are exercised multi-period selling seasons to sell their products. These facts show that the consumers,

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[1] Department of Mathematics, Vidyasagar College, Kolkata 700006, India.

[2] Department of Mathematics, Gurudas College, Kolkata 700054, India.

*Corresponding author: tarun.ju@gmail.com

companies and researchers are aware of the importance of reference prices and product reviews in multi-period (especially two-period) product pricing.

Besides the traditional retail stores, most international manufacturers such as Samsung, Apple, Hewlett Packard, and Lenovo sell their products through online [7]. By using online direct sales, manufacturing companies can improve the customer relationship and can gather some critical demand information [4, 36]. Dual-channel structures are attractive from a customer perspective because they offer more purchasing options, more dedicated customer service, more increased relevance, and lower transaction costs. However, online sales by manufacturers can be fraught with challenges in coordinating the supply chain between them in terms of competitiveness, pricing, independence, advertising, and in-store operations [3,19]. On the demand side, the customers prefer to share the information about the products and services; and leave reviews after purchasing. These online consumer reviews (OCRs) are freely accessible from the manufacturers' and retailers' websites. As OCRs are more user-focused, therefore these are more reliable and persuasive than product descriptions and traditional marketing through newspaper and TV ads. According to the literature, this persuasive material has now become one of the main sources of product information that shoppers rely on when they are shopping online.

In this study, we consider a two-period problem. In the first period, the early customers had no feedback from the previous customers; and had to make purchasing decisions based on their own expectations. Whereas, in the second period, the late arrivals decide their purchases based on the average reviews given by the early customers. In this article, we also consider some issues related to two types of dynamic pricing strategies in a dual-channel supply chain. These are – (a) Pre-announced pricing strategy: In this strategy, a company may advertise full sale prices at the beginning of the sales season. (b) Responsive pricing strategy: In this strategy, only the selling price of the first period is announced and the selling price of the second period is delayed until the start of the second period [27,28]. The current sale is affected by the current selling price and the difference between the previous selling price and the current one.

Unlike the previous studies, the present study looks at a manufacturer that not only sells their products through online channels but also through traditional offline channels. To the best of our knowledge, most of the previous studies are focused only on a dual-channel supply chain pricing strategy. But, in the present study, we consider a two-period dual-channel where the online reviews and reference pricing are considered together. Under these circumstances, we attempted to illustrate the following important and practical questions in the present study:

(1) The pricing decision in inter-temporal and horizontal competitions is the foremost significant factor for a healthy relation between the consumer and seller. Therefore, the primary question which arises naturally is that how do the manufacturer and retailer set their online and offline prices over the two periods?
(2) How does the market environment impact the online and offline prices under two strategies? In addition, how the equilibrium condition for the two-period pricing strategy can be determined?
(3) How do the online reviews and reference pricing affect the selling pricing decisions in a two-period dual-channel supply chain?
(4) How do the online reviews and the reference pricing impact on channel efficiency of two decentralized pricing strategies? In addition, how do the reference pricing and online consumer reviews influence the pricing strategies of the manufacturer and retailer?
(5) Currently pricing strategy plays a very crucial role in the business scenario for the profit of any company. Hence, the choice of strategy is a significant factor. Therefore, the companies are now interested to uncover which pricing strategy is most preferable?

In order to address the above issues, in the present study, we consider a manufacturer-controlled Stackelberg game-theoretic model to discuss the interaction between the manufacturer and the retailer. In the proposed model, the demand of the product is influenced by the online reviews and the reference price. Further, we develop a centralized model as a benchmark case and two decentralized models with two different pricing strategies. Then we optimize the proposed model to derive four selling prices and corresponding profit under three different decision strategies. We also analyze the impact of online reviews and reference pricing effect on the optimal selling prices. Finally, the derived channel efficiency shows which pricing strategy is more preferable by the manufacturer and the retailer. The effects of the key parameters involved in the proposed model are illustrated graphically. From the derived result, the efficiency of the supply chain can be increased when the acceptance of online consumers is limited and the online reviews are sufficiently positive. Therefore, the manufacturers are likely to prefer the dual-channel method including the effect of online reviews.

Below, we summarize the rest of this manuscript. The next section summarizes the related literature. The assumptions and notations which are used throughout the study are listed in Section 3. Section 4 is devoted to the model formulation. Section 5 develops the analysis of the model. Section 6 illustrates the numerical results and sensitivity analysis of the main parameters of the model and the comparison between the dominant domains. The practical significance of the model is discussed in Section 7. Finally, Section 8 presents the conclusions and limitations of the present study with some directions for future research.

2. Literature review

This study mainly focuses on the dual-channel in a two-period supply chain involving decision strategy, reference price and online reviews. Therefore, the literature review focuses on these issues that are discussed in the following subsections.

2.1. Dual-channel and online reviews in supply chain

Huang and Swaminathan [19] considered a stylized deterministic demand model in which the demand depends on the price, the substitution between channels, and the overall market potential. They also considered an oligopolistic case in which a historically mixed retailer faces competition from a pure retailer and characterizes the price parity. Chen *et al.* [4] showed that the dual-channel supply chain can increase profits for the retailer and the manufacturer compared to a single-channel supply chain. They derived certain conditions under which the dual-channel supply chain increases the manufacturer's profit, and also improves the efficiency of the supply chain. Dan *et al.* [7] used a two-step optimization technique and Stackelberg game theory to analyze the optimal retail service and price decisions in centralized and decentralized dual-channel supply chains. Li *et al.* [22] investigated the use of collaborative online review communities in a dual-channel supply chain of manufacturer and retailer. Raza and Govindaluri [32] explored a dual-channel in which a single manufacturer offers a standard product through the direct online channel and green products through a traditional channel where the manufacturer and retailer are said to be risk averse. Zhou *et al.* [52] considered a dual supply chain in which the retailer may add an online channel to the existing offline store.

Xu *et al.* [40] studied some optimal operational decisions and supply chain coordination issues by including the idea that the manufacturer and retailer are selling their products through offline channels and online platforms. Lei *et al.* [21] investigated the impact of strategic consumer behavior between the online and offline retailers in a dual-channel supply chain for four different channel structures. They explained that not all promotional prices have fallen as expected, although the patience of the consumer has increased. Zhao *et al.* [45] modeled the impact of equity concerns on a dual-channel green food procurement, where the manufacturer handles the production and sales through online and retail channels. Huang *et al.* [20] showed the impact of online reviews on the profitability of a supply chain including capacity constraints. They examined the decision to accept the online reviews from a capacity supply chain perspective under capacity constraints, penalties for lost sales, and product quality estimates. Zhang *et al.* [47] reviewed the dynamic pricing strategy and greening issues in a two-tier sourcing dual-channel in decentralized and centralized models. Li *et al.* [23] found that the online reviews have a negative impact on internal (multi-product) brand competition which affects the wholesale pricing strategy and product design strategy in the rapidly changing goods with online and offline distributed supply chains. They also explored the impact of online reviews in a competitive marketplace. Sun *et al.* [35] provided new insights into optimal pricing strategies and return policies for online sellers. They showed that the online merchants' strategic response to the returned product can mislead merchants about their online shopping if the impacts of online reviews are ignored.

Li *et al.* [24] examined the discount pricing strategies for a two-tier dual-channel supply chain consisting of a manufacturer and a retailer. The manufacturer sell their products through their own direct channel and independent retailer. Wang *et al.* [39] proposed a supply chain model in which post-consumers update their beliefs about the quality of the product based on the previous consumer ratings in the retailer's strategic inventory levels. Yang *et al.* [42] investigated the impact of online reviews in a supply chain by introducing several advertising strategies. Yapeng and Wei [43] created a price game model for a dual-channel supply chain, comprising a manufacturer and a retailer, by analyzing the stability of the model's equilibrium. Zhao and Li [50] considered the Stackelberg game approach to study both centralized and decentralized models by considering the market segmentation and online discount. Shi and Liu [33] analyzed the influence of online reviews on the supply chain, consisting of manufacturer and online platform, in the manufacturer's monopoly and competitive markets of the manufacturers. Shi *et al.* [34] established a product improvement strategy based on extracted product features by prioritizing the product features. He *et al.* [18] analyzed the optimal pricing strategy over two phases, and showed how consumers' channel preferences, price competition, and market shifts affect the strategic equilibrium in the market. Further, they showed the prices increase with the rate of market changes most times. Xu *et al.* [41] explored a model with online reviews and channel preferences, which are important factors to influence the consumer purchasing behavior. Deng *et al.* [8] illustrated that the strategic interactions between consumers and supply chains are strongly influenced by the online reviews for centralized and decentralized dual-channel models. They also enlighten some information on how the manufacturer can use online reviews to advance their marketing strategies.

2.2. Pricing strategy in two-period model

Ramani and Giovanni [31] investigated a two-period model of a reliable closed-loop supply chain using the Dell reconnect extension. They showed that reducing the cost of unused goods within the investigated supply chain is not enough to counteract the negative effects of cannibalization. Maiti and Giri [28] considered a two-period supply chain model in which a manufacturer and a retailer are involved in trading a single product. They assumed the demand rate in each period depends not only on the selling price of the current period, but also on the selling price of the previous period. Wang *et al.* [37] found that the quality improvements increase the expected profits for the retailers in both periods, but potentially decrease expected profits for manufacturers in a two-level supply chain. Giri *et al.* [10] examined a two-period supply chain with a manufacturer and a retailer. They assumed that the demand in the first period depends on selling price, product quality, and also on the refund price. However, demand in the second period depends on the selling price and product quality. Zhang *et al.* [51] studied a dynamic supply chain contract problem where a manufacturer sells seasonal products through a retailer in two phases. They argued that introducing rapid response is not necessarily profitable as the manufacturers have to avoid the information rentals. Zhang *et al.* [46] established a two-period Stackelberg game model where the manufacturer cooperates with an online retailer to pre-sell to the consumer. Qiao and Su [30] studied a two-period game model with the presence of online assessment in a closed supply chain. They uniquely solved the problem of original equipment manufacturer's licensing strategy and distribution channel selection of the manufacturer. Maiti [27] studied a closed-loop supply chain model where a manufacturer and a retailer trade for a product. In their model, the demand rate of the retailer is influenced by the selling price, the product quality, and the refund price.

2.3. Pricing strategy with reference pricing impact

Duan and Liu [9] reviewed a dynamic pricing model for perishable goods whose quality and quantity decreased under the influence of reference price. Liu *et al.* [26] analyzed strategic choice and two-stage pricing for supply chains when the underlying market demand is uncertain. Zha *et al.* [44] observed a two-stage pricing problem with two competing sellers, combining horizontal and inter-temporal reference price effects, formed by historical and competitor prices. Wang *et al.* [38] examined the reference price effect, with pricing strategies of online and offline retailers, in a competitive context. Zhao *et al.* [51] looked at its two-stage supply chain

TABLE 1. Comparison of the present study with the existing related literature.

Related articles	Two periods	Dual channels	Different pricing strategies	Online reviews	Reference price
Deng *et al.* [8]		✓	✓	✓	
He *et al.* [18]	✓	✓	✓		
Li *et al.* [22]		✓	✓	✓	
Li *et al.* [24]	✓	✓	✓		
Zhang *et al.* [47]	✓	✓	✓		
Yang *et al.* [42]		✓	✓	✓	
Chen *et al.* [4]		✓			
Dan *et al.* [7]		✓	✓		
Lei *et al.* [21]		✓	✓		
Zhou *et al.* [52]		✓			
Qiao and Su [30]	✓			✓	
Wang *et al.* [37]	✓				
Maiti and Giri [28]	✓		✓		✓
Giri *et al.* [10]	✓		✓		
Present study	✓	✓	✓	✓	✓

where the manufacturer sells the products through its two competing retailers. The manufacturer may offer the same or different wholesale prices to two retailers, but both retailers may use the pre-announced pricing strategy or a responsive pricing strategy. Chenavaz *et al.* [6] presented an optimal dynamic price control framework in which the behavior of the consumer depends on the reference price and online channel subject to the last mile delivery cost. Chen *et al.* [5] found that the optimal online sales strategies may vary depending on the consumer's acceptance of the online channel and also on the spillover effects. Li *et al.* [25] developed a game theory model to examine the impact of the review volume on the pricing strategies of different players under a centralized and decentralized two-stage structure. They showed that both the retailer and manufacturer benefited from high appraised values but are not harmed by low appraised values. Zhang *et al.* [49] addressed the impact of the reference price effect on the advance selling strategy and the corresponding pricing decisions.

Overall, the above literature review shows that few of the previous studies explored the two-stage model in a dual-channel supply chain. However, some studies have been performed separately based on dual-channel in two periods. Therefore, one aspect of the present model is the dynamic price situation, where the manufacturer can update both prices and product features periodically. Another aim of the present study is to visualize the vertical and horizontal reference prices effect in the supply chain. The timing of announcement plays a vital role in introducing the product to the commercial market. To fill this gap, here we consider a model with decision strategy, online evaluation, and reference price in a two-period dual-channel supply chain scenario. It is worthy to note that there are several types of methods such as robust optimization [13,16], meta-heuristic algorithm [2,11,12,14,15,17] are available in the literature. These methods are used to study the various types of multi-objective optimization problems. In the present study, we adopt the game theoretical approach to solve the two-period dual-channel model comprising online reviews and reference price. In order to understand the contribution of the present article as compared to the previous literature, a comparative study is presented in Table 1.

In this study, a model is developed for a sales season divided into two periods. The manufacturer produces a product and sells it through a retailer and through a direct online channel. The demands of the retailer and the manufacturer in both periods depend on the current selling price, the reference price (competitive channel's price and previous period's price). In addition, the manufacturer's demands are influenced by the reviews from the online customers. In the present model, the manufacturer is the leader of Stackelberg game and the retailer is the follower who follows the manufacturer's decisions. The centralized model is considered as a benchmark case. The optimal decisions derived for the two proposed decentralized strategies are analyzed and compared. Numerical results derived here reveal the important differences between these two strategies.

3. Notations and assumptions

The following notations are used throughout this article:

D_{r1} Demand rate at the retail channel in the first period.
D_{r2} Demand rate at the retail channel in the second period.
D_{e1} Demand rate at the online channel in the first period.
D_{e2} Demand rate at the online channel in the second period.
d_1 Basic demand of the first period.
d_2 Basic demand of the second period.
p_{e1} Selling price at the direct channel in the first period.
p_{e2} Selling price at the direct channel in the second period.
p_{r1} Selling price at the retail channel in the first period.
p_{r2} Selling price at the retail channel in the second period.
w_1 Wholesale price in the first period.
w_2 Wholesale price in the second period.
c Production cost per unit product.
p_{e0} Basic selling price (reference price) of the manufacturer at the beginning of the first period.
p_{r0} Basic selling price (reference price) of the retailer at the beginning of the first period.
Π_{m1} Profit of the manufacturer in the first period.
Π_{m2} Profit of the manufacturer in the second period.
Π_m Profit of the manufacturer in the whole season.
Π_{r1} Profit of the retailer in the first period.
Π_{r2} Profit of the retailer in the second period.
Π_r Profit of the retailer in the whole season.
Π Profit of the whole system in the whole season.

The following assumptions are made to develop the proposed model:

(1) The two-echelon supply chain includes a manufacturer and a retailer. The manufacturer produces a single product and sells it through dual channels. One is a regular retail channel and the other one is a direct online channel for a season divided into two periods.
(2) The demand rate at the retailer point of sale of the two periods depends on the selling price of the current period, and the selling price of the direct channel. In the direct channel, the demand rate also depends on the selling price of this period, the selling price of the previous period and the retail price of the corresponding period. In addition, the online reviews affect the demand of the direct channel. The demand for the second period of the direct channel is influenced by the online reviews of the first period. Following the previous works [1,7,18,28,29] on reference pricing, we assumed the demand rates at the retail channel for both periods D_{r1} and D_{r2} are as follows:

$$D_{r1} = (1-\sigma_1)d_1 - \alpha_1 p_{r1} + \rho p_{r0} + \gamma p_{e1}, \tag{1}$$
$$D_{r2} = (1-\sigma_2)d_2 - \alpha_2 p_{r2} + \rho p_{r1} + \gamma p_{e2}. \tag{2}$$

In a similar manner, the demand rates at the online channel for both periods D_{e1} and D_{e2} can be written as follows:

$$D_{e1} = \sigma_1 d_1 - \beta_1 p_{e1} + \rho p_{e0} + \gamma p_{r1} + \theta(1-\delta_1)O_1, \tag{3}$$
$$D_{e2} = \sigma_2 d_2 - \beta_2 p_{e2} + \rho p_{e1} + \gamma p_{r2} + \theta(1-\delta_2)O_2 + \tau(1-\delta_1)O_1. \tag{4}$$

In the demand functions, $d_1, d_2 > 0$ are the basic demands of the total market in two consecutive periods. With the growth of the Internet, consumers now have a wide range of buying options. As a result, they have a variety of channel alternatives and fidelity depending on their consumption patterns, product qualities, shopping convenience, and other factors. Here σ_i represents the consumer loyalty to the direct channels for the period i where $i \in \{1, 2\}$. The customers' loyalty to the retail channel is represented by $1 - \sigma_i$ where $0 < \sigma_i < 1$. The self-price sensitivity of the demand in retail channel and direct channel are represented by α_i and β_i, respectively. Previous period selling price or inter-temporal reference price sensitivity [28,44,47] is denoted by ρ which should be lesser sensitivity than the self-price sensitivity, *i.e.* $\alpha_i, \beta_i > \rho$. Alternative channel's selling price or horizontal reference price sensitivity is denoted by γ; θ represents the sensitivity of online reviews in online demands; δ_i is the fraction which signifies the negative reviews or no reviews by the customers among all customers. τ is the sensitivity of the first period's online reviews to the second period.

(3) The manufacturer is the Stackelberg leader, and the retailer is the follower. First, the manufacturer declares the wholesale prices and the online selling prices. Then the retailer sets his selling prices for two periods in different decision strategies.
(4) For feasibility of the present model, we assume that $p_{ri}, p_{ei} > w_i > c$, $i \in \{1, 2\}$.
(5) Lead time is assumed to be zero.

4. Model formulation

We develop a dual-channel supply chain model including a manufacturer (he) and a retailer (she). In addition, the sales season is divided into two periods, and each player will set prices over two periods. On the one hand, the manufacturer wholesales the product to a retailer for w_1 and w_2 in two periods respectively, she then resells them to market (*i.e.* "traditional/retail/offline channel"); he sells products directly to customers (*i.e.* "direct channel/online channel"). We assume the manufacturer incurs a production cost c for each product he produces.

We consider the manufacturer as a Stackelberg leader who adopts two decision strategies (namely, I and II) to declare all prices (wholesale and online prices); and the retailer follows him to decide her selling prices. In strategy I, the decisions for both the periods are made sequentially by each player at the beginning of the first period. Whereas, in strategy II, the players announce only the first-period prices at the beginning and delay the second period's prices announcement until the beginning of the second period. In addition, we also ignore some cost components such as set-up cost, ordering cost and transportation cost, model, which have no direct impact on the optimal decisions (Figs. 1 and 2).

Let, Π_{m1}, Π_{m2}, and Π_m are the manufacturer's profits for the first, second, and entire sales season in both channels, respectively. Then we have

$$\Pi_{m1}(w_1, p_{e1}) = \underbrace{(w_1 - c)D_{r1}}_{\text{offline revenue}} + \underbrace{(p_{e1} - c)D_{e1}}_{\text{online revenue}}, \tag{5}$$

$$\Pi_{m2}(w_2, p_{e2}) = \underbrace{(w_2 - c)D_{r2}}_{\text{offline revenue}} + \underbrace{(p_{e2} - c)D_{e2}}_{\text{online revenue}}, \tag{6}$$

$$\begin{aligned}\Pi_m(w_1, w_2, p_{e1}, p_{e2}) &= \Pi_{m1}(w_1, p_{e1}) + \Pi_{m2}(w_2, p_{e2}) \\ &= (w_1 - c)D_{r1} + (p_{e1} - c)D_{e1} + (w_2 - c)D_{r2} + (p_{e2} - c)D_{e2}. \end{aligned} \tag{7}$$

The retailer's profit functions for the first period, second period and the entire season are shown in equations (8)–(10), respectively:

$$\Pi_{r1}(p_{r1}) = (p_{r1} - w_1)D_{r1}, \tag{8}$$

$$\Pi_{r2}(p_{r2}) = (p_{r2} - w_2)D_{r2}, \tag{9}$$

$$\Pi_r(p_{r1}, p_{r2}) = \Pi_{r1}(p_{r1}) + \Pi_{r2}(p_{r2}) = (p_{r1} - w_1)D_{r1} + (p_{r2} - w_2)D_{r2}. \tag{10}$$

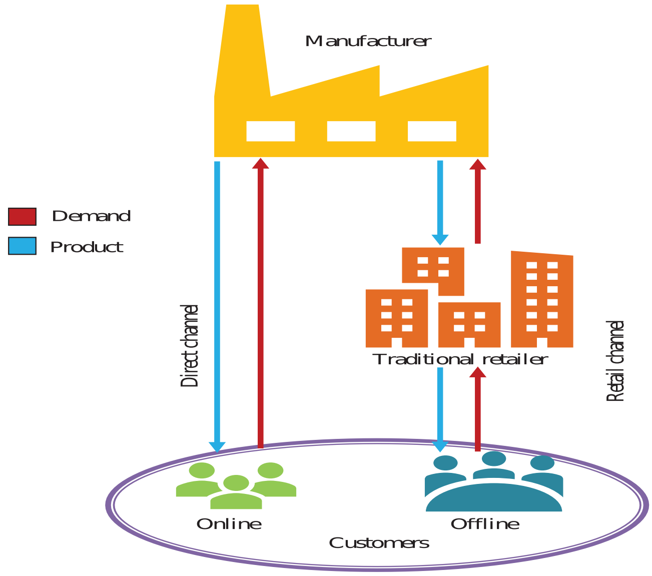


FIGURE 1. The structure of the dual-channel model.

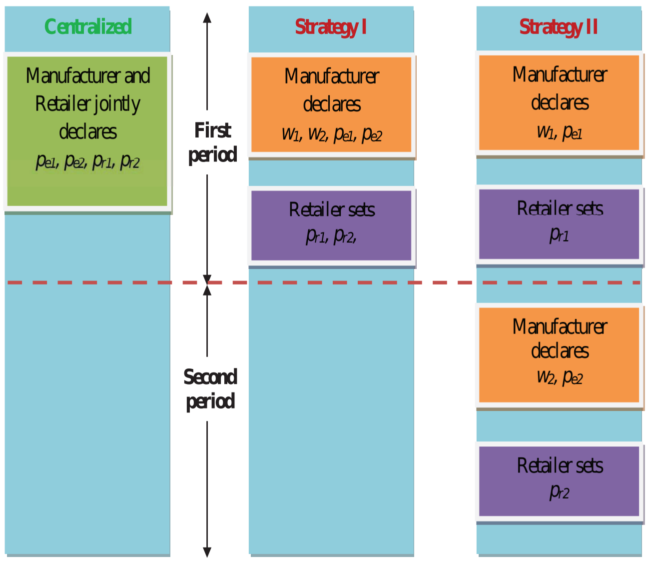


FIGURE 2. Decision sequences of different strategies.

Therefore, the profit of the whole system can be written as

$$\begin{aligned}\Pi(p_{e1}, p_{e2}, p_{r1}, p_{r2}) &= \Pi_m(p_{e1}, p_{e2}) + \Pi_r(p_{r1}, p_{r2}) \\ &= (p_{e1} - c)D_{e1} + (p_{e2} - c)D_{e2} + (p_{r1} - c)D_{r1} + (p_{r2} - c)D_{r2}. \end{aligned} \tag{11}$$

5. Model analysis

In this section, we mainly seek to describe the two-period equilibrium methods in the proposed dual-channel supply chain. We first develop a centralized model which is to be considered a benchmark case. Then, we establish two decentralized models for the manufacturer led Stackelberg game. The impacts of the online reviews and reference price on the optimal decisions are derived and compared.

5.1. Centralized model (C)

To manage the production and distribution processes, the manufacturer and the retailer create a vertically integrated organization in the centralized model. In order to maximize the profit of the supply chain, they jointly establish both channels and periods pricing p_{e1}, p_{e2}, p_{r1} and p_{r2} at the beginning of their selling season.

In order to determine the optimal decision, we optimize the total profit function given in equation (11) using the demand functions given in equations (1)–(4). The criteria of the first order optimal condition are given by $\frac{\partial \Pi}{\partial p_{e1}} = 0$, $\frac{\partial \Pi}{\partial p_{e2}} = 0$, $\frac{\partial \Pi}{\partial p_{r1}} = 0$, and $\frac{\partial \Pi}{\partial p_{r2}} = 0$. Solutions of these equations yield the following optimal prices which are given below in equations (12)–(15):

$$\begin{aligned} p_{e1}^C = &-\frac{1}{2\alpha_2(16XY-4Z\rho^2+\rho^4)}\Big[(4Y+\rho^2)\Big(S(-O_1(1-\delta_1)\theta - p_{e0}\rho + c(-\beta_1+\gamma+\rho) - d_1\sigma_1) \\ &+ 2\gamma\big(c(-2\alpha_1\alpha_2 + 2\alpha_2\gamma + (\alpha_2+\gamma)\rho) - 2d_1\alpha_2(1-\sigma_1) + \rho\big(-2p_{r0}\alpha_2 - d_2(1-\sigma_2)\big)\big)\Big) \\ &+ \rho(4R-\rho^2)(-2O_2\alpha_2(1-\delta_2)\theta - c(2Y-\beta_1\rho+\gamma\rho+\rho^2) + \rho(O_1\theta(1-\delta_1) \\ &+ p_{e0}\rho + d_1\sigma_1) + 2d_2(-\gamma(1-\sigma_2) - \alpha_2\sigma_2) - 2O_1\alpha_2(1-\delta_1)\tau)\Big], \end{aligned} \tag{12}$$

$$\begin{aligned} p_{e2}^C = &\frac{1}{16XY-4Z\rho^2+\rho^4}\Big[8X\big(d_2\gamma + O_2\alpha_2(1-\delta_2)\theta\big) + 4O_1R\delta_1\theta\rho(1-\delta_1) \\ &+ p_{e0}\rho^2(4R-\rho^2) + 2d_2\gamma\rho^2 + 4p_{r0}\gamma\rho^2(\alpha_2+\beta_1) - 2O_2\beta_1\theta\rho^2(1-\delta_2) - O_1\theta\rho^3(1-\delta_1) \\ &+ c\Big(8XY + \rho(-4(\alpha_2+\gamma)(-\alpha_1\beta_1+\gamma^2) - 2(2\alpha_1\alpha_2 + \beta_1\beta_2 + \gamma(\alpha_2+\beta_1+3\gamma))\rho \\ &+ (-\beta_1+\gamma)\rho^2 + \rho^3)\Big) + d_1\rho\big(4(\alpha_2+\beta_1)\gamma + (S - 4(\alpha_2+\beta_1-\gamma)\gamma)\,\sigma_1\big) \\ &+ 8Xd_2\sigma_2(\alpha_2-\gamma) - 2d_2\rho^2\sigma_2(\beta_1+\gamma) - 2O_1(1-\delta_1)(-4X\alpha_2+\beta_1\rho^2)\tau\Big], \end{aligned} \tag{13}$$

$$\begin{aligned} p_{r1}^C = &\frac{1}{16XY-4Z\rho^2+\rho^4}\Big[8Y\big(d_1\beta_1 + O_1\gamma(1-\delta_1)\theta\big) + c\Big(8XY + \rho\big(-4Y(\beta_1+\gamma) \\ &+ \rho(-2R + 2(\alpha_2+\beta_1)\gamma + (\alpha_2-\gamma)\rho)\big)\Big) + d_2\rho\big(4\gamma^2(1-\sigma_2) - \rho^2(1-\sigma_2) \\ &+ 4\alpha_2\gamma\sigma_2 + 4\beta_1(\beta_2 - \beta_2\sigma_2 + \gamma\sigma_2)\big) + 2\Big(\big(2O_2\gamma\theta\rho(\alpha_2+\beta_1)\big)(1-\delta_2) - d_1\alpha_2\rho^2 \\ &+ O_1\gamma\theta\rho^2(1-\delta_1) + p_{e0}\gamma\rho(4Y+\rho^2) - p_{r0}\rho(4\beta_1\gamma^2 - T\alpha_2) - 4d_1Y\sigma_1(\beta_1-\gamma) \\ &+ d_1\rho^2\sigma_1(\alpha_2+\gamma) + 2O_1(\alpha_2+\beta_1)\gamma(1-\delta_1)\rho\tau\Big)\Big], \end{aligned} \tag{14}$$

and

$$\begin{aligned} p_{r2}^C = &\frac{1}{16XY-4Z\rho^2+\rho^4}\Big[8\big(\alpha_1\beta_1-\gamma^2\big)(d_2\beta_2 + O_2\gamma(1-\delta_2)\theta) + 4d_1\beta_1\beta_2\rho - 2d_2\alpha_1\rho^2 \\ &+ 4p_{r0}W\rho^2 + 4O_1\alpha_1\gamma\theta\rho(1-\delta_1)(\alpha_1+\beta_1) + 4p_{e0}\gamma\rho^2(\alpha_1+\beta_2) + 2O_2\gamma\theta\rho^2(1-\delta_2) \\ &- p_{r0}\rho^4 + c\Big(8XY + \rho(4X(\beta_2+\gamma) - 2\big(2\beta_1\beta_2 + \alpha_1(\alpha_2+\gamma) + \gamma(\beta_2+3\gamma)\big)\rho \\ &- (\alpha_1-\gamma)\rho^2 + \rho^3)\Big) + d_1\rho\big(4\gamma^2(1-\sigma_1) - \rho^2(1-\sigma_1) - 4\beta_1\beta_2\sigma_1 + 4(\alpha_1+\beta_2)\gamma\sigma_1\big) \\ &- 8d_2X\sigma_2(\beta_2-\gamma) + 2d_2\rho^2\sigma_2(\alpha_1+\gamma) + 2O_1\gamma(1-\delta_1)(4X+\rho^2)\tau\Big], \end{aligned} \tag{15}$$

where,

$$X = \alpha_1\beta_1 - \gamma^2, \quad Y = \alpha_2\beta_2 - \gamma^2, \quad W = \beta_1\beta_2 + \gamma^2, \quad R = \alpha_1\alpha_2 + \gamma^2,$$
$$S = 4\alpha_1\alpha_2 - \rho^2, \quad T = 4\beta_1\beta_2 - \rho^2, \quad Z = \alpha_1\alpha_2 + \beta_1\beta_2 + 2\gamma^2,$$
$$P = 2\beta_1\beta_2 - \gamma^2, \quad U = 2\alpha_1\alpha_2 + \gamma^2, \quad Q = 2\alpha_1\alpha_2 - \gamma^2, \quad V = 2\beta_1\beta_2 + \gamma^2.$$

In order to verify the uniqueness of the derived optimal solution, we compute the corresponding Hessian matrix (H^C):

$$H^C = \begin{pmatrix} \frac{\partial^2\Pi}{\partial p_{e1}} & \frac{\partial^2\Pi}{\partial p_{e1}\partial p_{e2}} & \frac{\partial^2\Pi}{\partial p_{e1}\partial p_{r1}} & \frac{\partial^2\Pi}{\partial p_{e1}\partial p_{r2}} \\ \frac{\partial^2\Pi}{\partial p_{e2}\partial p_{e1}} & \frac{\partial^2\Pi}{\partial\partial p_{e2}^2} & \frac{\partial^2\Pi}{\partial p_{e2}\partial p_{r1}} & \frac{\partial^2\Pi}{\partial p_{e2}\partial p_{r2}} \\ \frac{\partial^2\Pi}{\partial p_{r1}\partial p_{e1}} & \frac{\partial^2\Pi}{\partial p_{r1}\partial p_{e2}} & \frac{\partial^2\Pi}{\partial p_{r1}^2} & \frac{\partial^2\Pi}{\partial p_{r1}\partial p_{r2}} \\ \frac{\partial^2\Pi}{\partial p_{r2}\partial p_{e1}} & \frac{\partial^2\Pi}{\partial p_{r2}\partial p_{e2}} & \frac{\partial^2\Pi}{\partial p_{r2}\partial p_{r1}} & \frac{\partial^2\Pi}{\partial p_{r2}^2} \end{pmatrix} = \begin{pmatrix} -2\beta_1 & \rho & 2\gamma & 0 \\ \rho & -2\beta_2 & 0 & 2\gamma \\ 2\gamma & 0 & -2\alpha_1 & \rho \\ 0 & 2\gamma & \rho & -2\alpha_2 \end{pmatrix}.$$

It can be noticed that, all the diagonal elements of the Hessian matrix are negative. Also, the second order minor of the Hessian matrix is given by $|H_2^C| = 4\beta_1\beta_2 - \rho^2 > 0$ *i.e.* $4\beta_1\beta_2 > \rho^2$ which is always true as the self-price sensitivity on the direct channel is always greater than the sensitivity of the previous period's price. Similarly, third order minor is determined as $|H_3^C| = -8\alpha_1\beta_1\beta_2 + 8\beta_2\gamma^2 + 2\alpha_1\rho^2 < 0$ if $8\alpha_1\beta_1\beta_2 > 8\beta_2\gamma^2 + 2\alpha_1\rho^2$ *i.e.* $4\beta_1\beta_2 > \frac{4\beta_2\gamma^2}{\alpha_1} + \rho^2$ holds. Further, the determinant value of the Hessian matrix $|H^C| = 16(\alpha_2\beta_2 - \gamma^2)(\alpha_1\beta_1 - \gamma^2) - 4(\alpha_1\alpha_2 + \beta_1\beta_2 + 2\gamma^2)\rho^2 + \rho^4 > 0$ if $16XY + \rho^4 > 4Z\rho^2$ holds. Hence, it can be conclude that the above Hessian matrix is negative definite. Therefore, the optimal solution derived above is always unique.

The above analysis is summarized by the following proposition.

Proposition 1. *Under the centralized policy, the manufacturer's and the retailer's optimal selling prices* (p_{e1}^C, p_{e2}^C, p_{r1}^C *and* p_{r2}^C) *for the entire sales season are given in equations* (12)–(15)*; and are unique if* $4\beta_1\beta_2 > (\frac{4\beta_2\gamma^2}{\alpha_1} + \rho^2)$ *and* $(16XY + \rho^4) > 4Z\rho^2$ *holds.*

5.2. Manufacturer's led Stackelberg games

This section examines a decentralized structure where the manufacturer acts as the leader and the retailer acts as the follower. The manufacturer first announces the wholesale prices and selling prices, and the retailer then follows his lead and sets her selling prices for the two subsequent periods. The manufacturer and the retailer make two different decision strategies, which are covered in the following two subsections.

5.2.1. Decision strategy I

This section discusses the strategic game that arises when a pre-announced pricing policy is used by both the manufacturer and the retailer. By maximizing their respective total profits for the entire selling season, the manufacturer and the retailer declare the full price path (w_1, w_2, p_{e1}, p_{e2} and p_{r1}, p_{r2}, respectively) in the first period. Customers make the first-period purchases based on this announcement as provided. Customers who are still in the market during the second period read the first-period buyers' reviews, revise their opinions of the quality of the products, and then decide which one to buy. The decision process in this strategy is as follows:

– At the start of the first period, the manufacturer announces his two separate wholesale prices, w_1 and w_2; and online selling prices p_{e1} and p_{e2} for the two periods.
– At the start of the first period, the retailer determines her selling prices, p_{r1} and p_{r2} by maximizing her combined profit for the two periods.

In this strategy, the retailer first returns a response by maximizing $\Pi_r(p_{r1}, p_{r2})$ with respect to p_{r1} and p_{r2}. The first order optimal conditions are: $\frac{\partial\Pi_r}{\partial p_{r1}} = 0 = \frac{\partial\Pi_r}{\partial p_{r2}}$. Solving these two equations simultaneously, we have the following optimal selling price response, given in equations (16) and (17), respectively:

$$p_{r1}^{\mathrm{I}} = \frac{\big(2\alpha_2\big(w_1^{\mathrm{I}}\alpha_1 + p_{e1}^{\mathrm{I}}\gamma + p_{r0}\rho + d_1(1-\sigma_1)\big) + \big(p_{e2}^{\mathrm{I}}\gamma - w_2^{\mathrm{I}}\alpha_2 + d_2(1-\sigma_2)\big)\rho\big)}{S}, \tag{16}$$

$$p_{r2}^{\mathrm{I}} = \frac{\big(2\alpha_1\big(w_2^{\mathrm{I}}\alpha_2 + p_{e2}^{\mathrm{I}}\gamma + d_2(1-\sigma_2)\big) + \big(p_{r0} - w_2^{\mathrm{I}}\big)\rho^2 + \big(w_1^{\mathrm{I}}\alpha_1 + p_{e1}^{\mathrm{I}}\gamma + d_1(1-\sigma_1)\big)\rho\big)}{S}. \tag{17}$$

This reaction is unique as the Hessian matrix which is given below

$$H^{\mathrm{I}} = \begin{pmatrix} \frac{\partial^2\Pi_r}{\partial p_{r1}^2} & \frac{\partial^2\Pi_r}{\partial p_{r1}\partial p_{r2}} \\ \frac{\partial^2\Pi_r}{\partial p_{r2}\partial p_{r1}} & \frac{\partial^2\Pi_r}{\partial p_{r2}^2} \end{pmatrix} = \begin{pmatrix} -2\alpha_1 & \rho \\ \rho & -2\alpha_2 \end{pmatrix}$$

is negative definite due to the facts (i) all the diagonal elements of the Hessian matrix are negative; and (ii) $|H^{\mathrm{I}}| = 4\alpha_1\alpha_2 - \rho^2$ which is always greater than 0.

After obtaining this response, the manufacturer maximizes the total profit $\Pi_m(w_1, w_2, p_{e1}, p_{e2})$ with respect to w_1, w_2, p_{e1} and p_{e2}. To determine the optimal reaction, the following four equations $\frac{\partial\Pi_m}{\partial w_1} = 0 = \frac{\partial\Pi_m}{\partial w_2}$, $\frac{\partial\Pi_m}{\partial p_{e1}} = 0 = \frac{\partial\Pi_m}{\partial p_{e2}}$ are solved simultaneously; and the optimal solutions are given in equations (18)–(21):

$$\begin{aligned} w_1^{\mathrm{I}} = \Big[& 32O_1\alpha_1^2\alpha_2^2\gamma\theta X(1-\delta_1) + 32p_{r0}\alpha_1^2\alpha_2^2\beta_1\rho Y + 32p_{e0}\alpha_1^2\alpha_2^2\gamma\rho Y + 16d_2\alpha_1^2\alpha_2\gamma^2\rho(\alpha_2+\beta_1) \\ &+ 8d_2\alpha_1\alpha_2\gamma^2\rho Y + 16O_2\alpha_1^2\alpha_2^2\gamma\theta\rho(1-\delta_2)(\alpha_2+\beta_1) + 8O_1\alpha_1\alpha_2^2\gamma\theta\rho^2(1-\delta_1)(\alpha_1-\beta_2) \\ &+ 12O_1\alpha_1\alpha_2\gamma^3\theta\rho^2(1-\delta_1) + 8p_{r0}\alpha_1\alpha_2\beta_1\rho^3(2\gamma^2-\alpha_2\beta_2) - 8p_{e0}\alpha_1\alpha_2^2\gamma\rho^3(\beta_2-\alpha_1) \\ &+ 4p_{r0}\alpha_1\alpha_2^2\rho^3U - 2d_2\alpha_1\beta_1\gamma^2\rho^3 + 12p_{e0}\alpha_1\alpha_2\gamma^3\rho^3 - 4O_2\alpha_1\alpha_2\gamma\theta\rho^3(1-\delta_2)(\alpha_2+\beta_1) \\ &+ d_2\gamma^4\rho^3 - O_1\gamma\theta\rho^4U(1-\delta_1) + p_{r0}\rho^5(\alpha_2Q - 2\beta_1\gamma^2) - p_{e0}\gamma\rho^5U \\ &+ c\Big(32XY\alpha_1^2\alpha_2^2 - 8Y\alpha_1\alpha_2\gamma(Q+2\alpha_1\beta_1+\alpha_2\gamma)\rho + 4\alpha_1\alpha_2\big(2\alpha_1\alpha_2((\alpha_2+\beta_1)\gamma \\ &- (\alpha_1\alpha_2+\beta_1\beta_2)) + (-3\alpha_1\alpha_2 + 2\alpha_1\beta_1 + 2\alpha_2\beta_2)\gamma^2 + (\alpha_2+\beta_1)\gamma^3 - 3\gamma^4\big)\rho^2 \\ &+ \gamma\big(4\alpha_1\alpha_2(\alpha_2+\beta_1)\beta_2 - 2\alpha_1\alpha_2(2\alpha_2+\beta_1)\gamma - 2\alpha_1\beta_1\gamma^2 + \gamma^3(-\alpha_2+\gamma)\big)\rho^3 \\ &+ U\big(\alpha_1\alpha_2 - (\alpha_2+\beta_1)\gamma + \gamma^2\big)\rho^4\Big) + d_1S\Big(\gamma^2\rho^2(\alpha_2 + 2\beta_1(1-\sigma_1) + \sigma_1(\gamma-\alpha_2)) \\ &+ 2\alpha_1\alpha_2\big(\alpha_2((1-\sigma_1)T + 4\beta_2\sigma_1\gamma) + \gamma(-4\beta_1\gamma(1-\sigma_1) + (-4\gamma^2+\rho^2)\sigma_1)\big)\Big) \\ &+ 16d_2\alpha_1^2\alpha_2\gamma\rho\sigma_2\big(\alpha_2^2 + \beta_1(\alpha_2-\gamma)\big) - 8d_2\alpha_1\alpha_2\gamma^2\rho\sigma_2(\alpha_2\beta_2+Q) - 4d_2\alpha_1\alpha_2\gamma\rho^3\sigma_2(\alpha_2+\beta_1) \\ &+ d_2\gamma^2\rho^3\sigma_2(2\alpha_1\beta_1-\gamma^2) + 4O_1S\alpha_1\alpha_2(\alpha_2+\beta_1)\gamma(1-\delta_1)\rho\tau\Big] \\ &\times \Big[64XY\alpha_1^2\alpha_2^2 - 8\alpha_1\alpha_2\big(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2) + (2U-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2\big)\rho^2 + U^2\rho^4\Big]^{-1}, \end{aligned} \tag{18}$$

$$\begin{aligned} w_2^{\mathrm{I}} = \Big[& 32O_2\alpha_1^2\alpha_2^2\beta_1\gamma\theta X(1-\delta_2) + 16d_1\alpha_1\alpha_2^2\rho(\alpha_1V - \beta_2\gamma^2) - 8d_1\alpha_1\alpha_2\gamma^2\rho X \\ &+ 16O_1\alpha_1^2\alpha_2^2\gamma\theta\rho(1-\delta_1)(\alpha_1+\beta_2) + 16p_{e0}\alpha_1^2\alpha_2^2\gamma\rho^2(\alpha_1+\beta_2) + 16p_{r0}\alpha_1\alpha_2^2\rho^2(\alpha_1V-\beta_2\gamma^2) \\ &- 8p_{r0}\alpha_1\alpha_2\gamma^2\rho^2X + 8O_2\alpha_1^2\alpha_2\gamma\theta\rho^2(\alpha_2-\beta_1)(1-\delta_2) + 12O_2\alpha_1\alpha_2\gamma^3\theta\rho^2(1-\delta_2) \\ &- 8d_1\alpha_1\alpha_2\rho^3(\beta_1\beta_2+\alpha_1\alpha_2) - 12d_1\alpha_1\alpha_2\gamma^2\rho^3 + d_1\gamma^2\rho^3P - 4O_1\alpha_1\alpha_2\gamma\theta\rho^3(1-\delta_1)(\alpha_1+\beta_2) \\ &- 8p_{r0}\alpha_1\alpha_2\rho^4(\alpha_1\alpha_2+\beta_1\beta_2) - 4p_{e0}\alpha_1\alpha_2\gamma\rho^4(\alpha_1+\beta_2) - 12p_{r0}\alpha_1\alpha_2\gamma^2\rho^4 + p_{r0}\gamma^2\rho^4P \\ &- O_2\gamma\theta\rho^4U(1-\delta_2) + d_1\rho^5U + p_{r0}\rho^6U + c\Big(8X\alpha_1\alpha_2\big(4Y\alpha_1\alpha_2 + \gamma(2\alpha_1\alpha_2+P+\alpha_1\gamma)\rho\big) \\ &+ \rho^2\big(-4\alpha_1\alpha_2(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2) + (\alpha_1+\beta_2)\gamma U + 5\alpha_1\alpha_2\gamma^2 + \gamma^4\big) \\ &+ \gamma(-4\alpha_1\alpha_2\beta_1(\alpha_1+\beta_2) + 2\alpha_1\alpha_2(2\alpha_1+\beta_2)\gamma + 2\alpha_2\beta_2\gamma^2 + \alpha_1\gamma^3 - \gamma^4)\rho \end{aligned}$$

$$
\begin{aligned}
&+U(\alpha_1\alpha_2+(\alpha_1+\beta_2)\gamma)\rho^2\big)\Big)+16d_1\alpha_1^2\alpha_2^2\gamma\rho\sigma_1(\alpha_1+\beta_2)-16d_1\alpha_1^2\alpha_2^2\rho\sigma_1 V\\
&+8d_1\alpha_1\alpha_2\gamma^2\rho\sigma_1 X+16d_1\alpha_1\alpha_2^2\beta_2\gamma^2\rho\sigma_1+8d_1\alpha_1\alpha_2\rho^3\sigma_1(\alpha_1\alpha_2+\beta_1\beta_2)+12d_1\alpha_1\alpha_2\gamma^2\rho^3\sigma_1\\
&-d_1\gamma^2\rho^3\sigma_1 P-d_1\rho^5\sigma_1 U-4d_1\alpha_1\alpha_2\gamma\rho^3\sigma_1(\alpha_1+\beta_2)+d_2 S\big(\gamma^3\rho^2\sigma_2\\
&+\alpha_1\gamma\big(-8\alpha_2\beta_2\gamma-\gamma\rho^2(1-\sigma_2)+2\alpha_2(4(\beta_2-\gamma)\gamma+\rho^2)\sigma_2\big)+2\alpha_1^2\alpha_2\big(-\rho^2(1-\sigma_2)\\
&+4\beta_1(\beta_2-\beta_2\sigma_2+\gamma\sigma_2)\big)\big)+O_1S\gamma(1-\delta_1)(8X\alpha_1\alpha_2+U\rho^2)\tau\Big]\\
&\times\Big[64XY\alpha_1^2\alpha_2^2-8\alpha_1\alpha_2\big(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(2U-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2\big)\rho^2+U^2\rho^4\Big]^{-1},
\end{aligned}\tag{19}
$$

$$
\begin{aligned}
p_{e1}^{\mathrm{I}}=&\Big[32O_1\alpha_1^3\alpha_2^2\theta Y(1-\delta_1)+32p_{e0}\alpha_1^3\alpha_2^2\rho Y+32p_{r0}\alpha_1^2\alpha_2^2\gamma\rho Y+8d_2\alpha_1^2\alpha_2\gamma\rho(\alpha_2\beta_2+\gamma^2)\\
&+16O_2\alpha_1^2\alpha_2^2\theta\rho R(1-\delta_1)-8O_1\alpha_1^2\alpha_2^2\beta_2\theta\rho^2(1-\delta_1)+4O_1\alpha_1^2\alpha_2\gamma^2\theta\rho^2(1-\delta_1)\\
&+12p_{r0}\alpha_1\alpha_2\gamma\rho^3 R-4p_{e0}\alpha_1^2\alpha_2\rho^3 P-2O_2\alpha_1\alpha_2\theta\rho^3 U(1-\delta_2)-p_{r0}\gamma\rho^5 U-d_2\alpha_1\gamma\rho^3 U\\
&+16d_2\alpha_1^3\alpha_2^2\gamma\rho+c\alpha_1\big(32\alpha_1\alpha_2^2XY-8Y\alpha_1\alpha_2(U+\alpha_2\gamma)\big)\rho\\
&+4\alpha_2\big(-2\alpha_1\alpha_2\beta_1\beta_2+\alpha_1\alpha_2(\alpha_1+\beta_2)\gamma+(\alpha_1(-\alpha_2+\beta_1)+\alpha_2\beta_2)\gamma^2-\gamma^4\big)\rho^2\\
&-U\big(\gamma^2+\alpha_2(-2\beta_2+\gamma)\big)\rho^3+d_1\gamma\big(-\gamma^2\rho^4-2\alpha_1\alpha_2\rho^2(2\alpha_2\beta_2-6\gamma^2+\rho^2)\\
&+4\alpha_1^2\alpha_2^2(8Y+3\rho^2)\big)+d_1\big(32Y\alpha_1^2\alpha_2^2(\alpha_1-\gamma)+4\alpha_1\alpha_2\big(\alpha_2\beta_2\gamma-3\gamma^3-\alpha_1(P+3\alpha_2\gamma)\big)\rho^2\\
&+U\gamma\rho^4\big)\sigma_1+16d_2\alpha_1^3\alpha_2^3\rho\sigma_2(1-\gamma)-8d_2\alpha_1^2\alpha_2\gamma\rho\sigma_2 Y+16d_2\alpha_1^2\alpha_2^2\gamma^2\rho\sigma_2+2d_2\alpha_1^2\alpha_2\gamma\rho^3\sigma_2\\
&-2d_2\alpha_1\alpha_2\rho^3\sigma_2 U+d_2\alpha_1\gamma^3\rho^3\sigma_2+2O_1\alpha_1\alpha_2(1-\delta_1)\rho(8\alpha_1\alpha_2 R-U\rho^2)\tau\Big]\\
&\times\Big[64XY\alpha_1^2\alpha_2^2-8\alpha_1\alpha_2\big(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(2U-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2\big)\rho^2+U^2\rho^4\Big]^{-1},
\end{aligned}\tag{20}
$$

and

$$
\begin{aligned}
p_{e2}^{\mathrm{I}}=&\Big[c\Big(8X\alpha_1\alpha_2^2\big(4Y\alpha_1+(U+\alpha_1\gamma)\rho\big)+\rho^2\big(-4\alpha_1\alpha_2\big(2\alpha_1\alpha_2(2\alpha_1\alpha_2+\beta_1\beta_2)+\alpha_1\alpha_2(\alpha_2+\beta_1)\gamma\\
&+(7\alpha_1\alpha_2-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2+3\gamma^4\big)+U\alpha_2(\gamma^2+\alpha_1(-2\beta_1+\gamma))\rho+U^2\rho^2\big)\Big)\\
&+\alpha_2\Big(16d_1\alpha_1^2\alpha_2^2\gamma\rho(1-\sigma_1)+24d_1\alpha_1^2\alpha_2\beta_1\gamma\rho(1-\sigma_1)-8d_1\alpha_1\alpha_2\gamma^3\rho(1-\sigma_1)\\
&+16O_1\alpha_1^2\alpha_2^2\theta\rho R(1-\delta_1)+16p_{e0}\alpha_1^2\alpha_2\rho^2 R+8p_{r0}\alpha_1\alpha_2\gamma\rho^2 Q+24p_{r0}\alpha_1^2\alpha_2\beta_1\gamma\rho^2\\
&-4d_1\alpha_1\beta_1\gamma\rho^3-d_1\gamma\rho^3 Q+2O_1\alpha_1\theta\rho^3(Q+\delta_1 U)+2p_{e0}\alpha_1\rho^4 Q-p_{r0}\gamma\rho^4 Q\\
&-4p_{r0}\alpha_1\beta_1\gamma\rho^4+4O_2\alpha_1\alpha_2(1-\delta_2)\theta(8\alpha_1\alpha_2X-2\alpha_1\beta_1\rho^2+\gamma^2\rho^2)+16d_1\alpha_1^2\alpha_2\rho\sigma_1(R+\gamma^2)\\
&-24d_1\alpha_1^2\alpha_2\beta_1\gamma\rho\sigma_1+8d_1\alpha_1\alpha_2\gamma^3\rho\sigma_1-4d_1\alpha_1^2\alpha_2\rho^3\sigma_1+2d_1\alpha_1\gamma\rho^3\sigma_1(\alpha_2-\gamma)\\
&+4d_1\alpha_1\beta_1\gamma\rho^3\sigma_1-d_1\gamma^3\rho^3\sigma_1+4d_2\alpha_1(8X\alpha_1\alpha_2(\gamma+\alpha_2\sigma_2-\gamma\sigma_2)+\rho^2(\gamma^2(\gamma+\alpha_2\sigma_2-\gamma\sigma_2)\\
&+\alpha_1(-\beta_1\gamma(1-\sigma_2)+\alpha_2(\gamma-(2\beta_1+\gamma)\sigma_2))))+\big(4O_1\alpha_1\alpha_2(1-\delta_1)(8\alpha_1\alpha_2X-2\alpha_1\beta_1\rho^2+\gamma^2\rho^2)\tau\big)\Big)\Big]\\
&\times\Big[64XY\alpha_1^2\alpha_2^2-8\alpha_1\alpha_2\big(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(2U-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2\big)\rho^2+U^2\rho^4\Big]^{-1}.
\end{aligned}\tag{21}
$$

The above solution is unique as the Hessian matrix (H^{Im}) which is given below

$$
H^{Im}=\begin{pmatrix}
\frac{\partial^2\Pi_m}{\partial w_1^2} & \frac{\partial^2\Pi_m}{\partial w_1\partial w_2} & \frac{\partial^2\Pi_m}{\partial w_1\partial p_{e1}} & \frac{\partial^2\Pi_m}{\partial w_1\partial p_{e2}}\\
\frac{\partial^2\Pi_m}{\partial w_2\partial w_1} & \frac{\partial^2\Pi_m}{\partial w_2^2} & \frac{\partial^2\Pi_m}{\partial w_2\partial p_{e1}} & \frac{\partial^2\Pi_m}{\partial w_2\partial p_{e2}}\\
\frac{\partial^2\Pi_m}{\partial p_{e1}\partial w_1} & \frac{\partial^2\Pi_m}{\partial p_{e1}\partial w_2} & \frac{\partial^2\Pi_m}{\partial p_{e1}^2} & \frac{\partial^2\Pi_m}{\partial p_{e1}\partial p_{e2}}\\
\frac{\partial^2\Pi_m}{\partial p_{e2}\partial w_1} & \frac{\partial^2\Pi_m}{\partial p_{e2}\partial w_2} & \frac{\partial^2\Pi_m}{\partial p_{e2}\partial p_{e1}} & \frac{\partial^2\Pi_m}{\partial p_{e2}^2}
\end{pmatrix}
$$

$$= \begin{pmatrix} -\frac{4\alpha_1^2\alpha_2}{4\alpha_1\alpha_2-\rho^2} & \frac{2\alpha_1\alpha_2\rho}{4\alpha_1\alpha_2-\rho^2} & \gamma & 0 \\ \frac{2\alpha_1\alpha_2\rho}{4\alpha_1\alpha_2-\rho^2} & -\frac{4\alpha_1\alpha_2^2}{4\alpha_1\alpha_2-\rho^2} & 0 & \gamma \\ \gamma & 0 & -2\beta_1+\frac{4\alpha_2\gamma^2}{4\alpha_1\alpha_2-\rho^2} & \rho+\frac{2\gamma^2\rho}{4\alpha_1\alpha_2-\rho^2} \\ 0 & \gamma & \rho+\frac{2\gamma^2\rho}{4\alpha_1\alpha_2-\rho^2} & -2\beta_2+\frac{4\alpha_1\gamma^2}{4\alpha_1\alpha_2-\rho^2} \end{pmatrix}$$

is negative definite. This is because of:

(i) the diagonal elements of this Hessian matrix are negative if $S > \max\{\frac{2\alpha_2\gamma}{\beta_1}, \frac{2\alpha_1\gamma}{\beta_2}\}$,

(ii) the second order minor is $|H_2^{Im}| = \frac{4\alpha_1^2\alpha_2^2}{S}$ which is always greater than zero as S is always greater than zero,

(iii) the third order minor is $|H_3^{Im}| = -\frac{4\alpha_1\alpha_2^2(8\alpha_1^2\alpha_2\beta_1+\gamma^2\rho^2-2\alpha_1(4\alpha_2\gamma^2+\beta_1\rho^2))}{s^2}$ which is less than zero if $8\alpha_1^2\alpha_2\beta_1 + \gamma^2\rho^2 - 2\alpha_1(4\alpha_2\gamma^2+\beta_1\rho^2) > 0$ *i.e.* $S > \frac{\gamma^2 Q}{\beta_1}$ holds,

(iv) the determinant of the Hessian matrix is $|H| = \frac{1}{S^2}(64\alpha_1^2\alpha_2^2(\alpha_1\beta_1-\gamma^2)(\alpha_2\beta_2-\gamma^2) - 8\alpha_1\alpha_2(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(4\alpha_1\alpha_2-\alpha_1\beta_1-\alpha_2\beta_2)\gamma^2+2\gamma^4)\rho^2+(2\alpha_1\alpha_2+\gamma^2)^2\rho^4)$ which is greater than zero if $64\alpha_1^2\alpha_2^2XY+U^2\rho^4 > 8\alpha_1\alpha_2(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(2U-X-Y)\gamma^2-2\gamma^4)\rho^2$ both holds.

The above analysis shows that the strategy I always produces an optimal solution which is summarized below as Proposition 2.

Proposition 2. *Under the strategy I, the manufacturer's optimal wholesale prices* w_1^{I} *and* w_2^{I}*; the selling prices* p_{e1}^{I} *and* p_{e2}^{I}*; and the retailer's optimal selling prices* p_{r1}^{I} *and* p_{r2}^{I} *for the entire sales season are given in equations* (18)–(21) *and equations* (16), (17). *These optimal prices are unique if* $S > \max\{\frac{2\alpha_2\gamma}{\beta_1}, \frac{2\alpha_1\gamma}{\beta_2}, \frac{\gamma^2 Q}{\beta_1}\}$ *and* $64\alpha_1^2\alpha_2^2XY + U^2\rho^4 > 8\alpha_1\alpha_2(2\alpha_1\alpha_2(\alpha_1\alpha_2+\beta_1\beta_2)+(2U-X-Y)\gamma^2-2\gamma^4)\rho^2$ *hold.*

5.2.2. Decision strategy II

By setting prices under the real demand circumstances and the capacity, a firm can affect the demand for its output under the responsive pricing strategy. High-tech consumer gadgets (like smartphones, tablets, computers, etc.), media items (like movies, books, etc.), and digital accessories (such as computer software, smartphone apps, etc.) are a few examples of these types of products. The decision process in this choice is as follows:

– The manufacturer publishes the wholesale price w_1 and the online sale price p_{e1} for the first period only taking into account the profit of the first period.
– Retailer sets selling price p_{r1} according to the profit of the first period.
– At the end of first period, the manufacturer announces the wholesale price w_2 and online selling price p_{e2} for the second period.
– The retailer sets the selling price p_{r2} for the second period.

Each phase can be thought as a sales season or a phase in a product generation. Therefore, each time period in our proposed model can span a quarter, 6 months or 2 years depending on the nature of the product under consideration. The retailer's reaction in the first period can be obtained by optimizing the profit function Π_{r1} with respect to p_{r1}. The optimal selling price for the first period is determined as:

$$p_{r1}^{\mathrm{II}} = \frac{\left(d_1 + w_1^{\mathrm{II}}\alpha_1 + p_{e1}^{\mathrm{II}}\gamma + p_{r0}\rho - d_1\sigma_1\right)}{2\alpha_1}. \tag{22}$$

The second order derivative of this profit function is $\frac{\partial^2\Pi_{r1}}{\partial p_{r1}^2} = -2\alpha_1$ which is always negative. This indicates that the selling price is unique.

The manufacturer then maximizes the profit function Π_{m1} of the first period and determines the optimal wholesale price and the online selling price respectively:

$$w_1^{\mathrm{II}} = \frac{cX + O_1\gamma\theta(1-\delta_1) + p_{r0}\beta_1\rho + p_{e0}\gamma\rho + d_1(\beta_1 - \beta_1\sigma_1 + \gamma\sigma_1)}{2X}, \tag{23}$$

$$p_{e1}^{\mathrm{II}} = \frac{cX + d_1\gamma + O_1\alpha_1\theta(1-\delta_1) + p_{e0}\alpha_1\rho + p_{r0}\gamma\rho + d_1(\alpha_1 - \gamma)\sigma_1}{2X}. \tag{24}$$

It is worthy to note that the above reaction is unique as the Hessian matrix (H^{II})

$$H^{\mathrm{II}} = \begin{pmatrix} \frac{\partial^2\Pi_{m1}}{\partial w_1^2} & \frac{\partial^2\Pi_{m1}}{\partial w_1\partial p_{e1}} \\ \frac{\partial^2\Pi_{m1}}{\partial p_{e1}\partial w_1} & \frac{\partial^2\Pi_{m1}}{\partial p_{e1}^2} \end{pmatrix} = \begin{pmatrix} -\alpha_1 & \gamma \\ \gamma & -2\beta_1 + \gamma^2/\alpha_1 \end{pmatrix}$$

is negative definite due to the following facts:

(i) as the diagonal elements are negative if $2\alpha_1\beta_1 > \gamma^2$ holds,
(ii) the determinant of Hessian matrix is $|H^{\mathrm{II}}| = 2\alpha_1\beta_1 - 2\gamma^2$ which is greater than zero if $\alpha_1\beta_1 > \gamma^2$ holds.

In order to determine the retailer's selling price, the manufacturer's wholesale and selling prices for the second period, the same process as described above for the first period, is followed. And the retailer's optimal selling price, which is unique $\left(\text{as } \frac{\partial^2\Pi_{r2}}{\partial p_{r2}^2} = -2\alpha_2 < 0\right)$, is obtained as:

$$\begin{aligned} p_{r2}^{\mathrm{II}} = \frac{1}{8\alpha_1\alpha_2 X}\Big(& 4p_{e2}^{\mathrm{II}}\alpha_1\gamma X + 4w_2^{\mathrm{II}}\alpha_1\alpha_2 X + 3d_1\alpha_1\beta_1\rho(1-\sigma_1) + c\alpha_1\beta_1\rho(\alpha_1+\gamma) \\ & - d_1\gamma^2\rho(1-\sigma_1) - c\gamma^2\rho(\alpha_1+\gamma) + 2O_1\alpha_1\gamma\theta\rho(1-\delta_1) + p_{r0}\rho^2(3\alpha_1\beta_1 - \gamma^2) \\ & + 2p_{e0}\alpha_1\gamma\rho^2 + 2d_1\alpha_1\gamma\rho\sigma_1 + 4d_2\alpha_1 X(1-\sigma_2)\Big). \end{aligned} \tag{25}$$

Similarly, the unique wholesale price and selling price of the manufacturer are determined as:

$$\begin{aligned} w_2^{\mathrm{II}} = -\frac{1}{8XY\alpha_1}\Big[& 4O_2\alpha_1\gamma\theta X(1-\delta_2) + 3d_1\alpha_1\beta_1\beta_2\rho + d_1\gamma^2\rho(2\alpha_1 - \gamma^2) + 2p_{e0}\gamma\rho^2\alpha_1(\alpha_1+\beta_2) \\ & + 2O_1\alpha_1\gamma\theta\rho(\alpha_1+\beta_2)(1-\delta_1) + p_{r0}\rho^2(3\alpha_1\beta_1\beta_2 + 2\alpha_1\gamma^2 - \beta_2\gamma^2) \\ & + cX\big(\beta_2\gamma\rho + \alpha_1(4Y + \beta_2\rho + 2\gamma\rho)\big) + d_1\gamma\rho\sigma_1(2\alpha_1^2 + 2\alpha_1\beta_2 - 2\alpha_1\gamma + \beta_2\gamma) \\ & - 3d_1\alpha_1\beta_1\beta_2\rho\sigma_1 + 4d_2\alpha_1 X(\beta_2 - \beta_2\sigma_2 + \gamma\sigma_2) + 4O_1\alpha_1\gamma X(1-\delta_1)\tau\Big], \end{aligned} \tag{26}$$

and

$$\begin{aligned} p_{e2}^{\mathrm{II}} = \frac{1}{8XY\alpha_1}\Big[& 4O_2\alpha_1\alpha_2\theta X(1-\delta_2) + d_1\gamma\rho Q + 3d_1\alpha_1\beta_1\gamma\rho + 2O_1\alpha_1\theta\rho R(1-\delta_1) + 2p_{e0}\alpha_1\rho^2 R \\ & + p_{r0}\gamma\rho^2 Q + 3p_{r0}\alpha_1\beta_1\gamma\rho^2 + cX\big(\gamma^2\rho + \alpha_1(4Y + 2\alpha_2\rho + \gamma\rho)\big)2d_1\alpha_1\rho\sigma_1 R \\ & - d_1\gamma\rho\sigma_1 Q - 3d_1\alpha_1\beta_1\gamma\rho\sigma_1 + 4d_2\alpha_1 X\big(\gamma(1-\sigma_2) + \alpha_2\sigma_2\big) + 4O_1\alpha_1\alpha_2 X(1-\delta_1)\tau\Big]. \end{aligned} \tag{27}$$

The above prices (wholesale and selling) are unique as the Hessian matrix ($H^{\mathrm{II}m}$) which is given below

$$H^{\mathrm{II}m} = \begin{pmatrix} \frac{\partial^2\Pi_{m2}}{\partial w_2^2} & \frac{\partial^2\Pi_{m2}}{\partial w_2\partial p_{e2}} \\ \frac{\partial^2\Pi_{m2}}{\partial p_{e2}\partial w_2} & \frac{\partial^2\Pi_{m2}}{\partial p_{e2}^2} \end{pmatrix} = \begin{pmatrix} -\alpha_2 & \gamma \\ \gamma & -2\beta_2 + \gamma^2/\alpha_2 \end{pmatrix}$$

is negative definite due to the following facts:

(i) as the diagonal elements are negative if $2\alpha_2\beta_2 > \gamma^2$ holds,
(ii) the determinant of the Hessian matrix is $|H^{\mathrm{II}m}| = 2\alpha_2\beta_2 - 2\gamma^2$ which is greater than zero if $\alpha_2\beta_2 > \gamma^2$ holds.

The above analysis is summarized as the following proposition:

Proposition 3. *Under strategy II, the manufacturer's optimal wholesale prices w_1^{II} and w_2^{II}; and the selling prices p_{e1}^{II} and p_{e2}^{II} for the entire sales season are given in equations (23), (26), (24) and (27); and the retailer's optimal selling prices p_{r1}^{II} and p_{r2}^{II} are given in equations (22), (25) and those are unique if $\alpha_1\beta_1 > \gamma^2$ and $\alpha_2\beta_2 > \gamma^2$ both hold.*

5.3. Comparative analysis

This subsection provides development ideas and guidance for decision makers by analyzing the optimal decisions in decentralized and centralized decision strategies. For this purpose, the determination of traditional retail prices, wholesale prices and online selling prices of both periods are compared in different scenarios. Here also we illustrate the impact of online reviews and reference prices on the decision-making of individual members of the supply chain in different strategies from both centralized and decentralized decision-making perspectives. For this purpose, we assume the same value for all the parameters which are used to differentiate the different channels or different periods of sensitivity on our developed model. So, in this present analysis, we use $d_1 = d_2$, $\sigma_1 = \sigma_2$, $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\delta_1 = \delta_2$.

Corollaries 1 and 2 illustrate a comparison between two periods pricing decisions for both strategies. Whereas, Corollaries 3 and 4 demonstrate the comparison of the same pricing decision between two different strategies.

Corollary 1. *In equilibrium,*

(i) *selling prices of both strategies for two periods satisfy: $\frac{\partial p_{e2}}{\partial O_1} > \frac{\partial p_{e1}}{\partial O_1} > 0$, $\frac{\partial p_{r2}}{\partial O_1} > \frac{\partial p_{r1}}{\partial O_1} > 0$; and $\frac{\partial p_{e2}}{\partial O_2} > \frac{\partial p_{e1}}{\partial O_2} > 0$, $\frac{\partial p_{r2}}{\partial O_2} > \frac{\partial p_{r1}}{\partial O_2} > 0$. But, in strategy II, we have $\frac{\partial p_{e1}}{\partial O_2} = 0$ and $\frac{\partial p_{r1}}{\partial O_2} = 0$;*
(ii) *wholesale prices of both strategies for two periods satisfy: $\frac{\partial w_2}{\partial O_1} > \frac{\partial w_1}{\partial O_1} > 0$ and $\frac{\partial w_2}{\partial O_2} > \frac{\partial w_1}{\partial O_2} > 0$ but for strategy II, $\frac{\partial w_1}{\partial O_2} = 0$.*

Corollary 1 suggests that if the online reviews are prioritized more in the first period, this will influence the pricing decisions in the second period as compared to the first period. Also, the online reviews in the second period have a larger impact on the prices in the second period. Further, in strategy II, which is the rapid and responsive pricing strategy, the online reviews in the second period have no impact on the prices of the first period. Moreover, in strategy II, all players make their optimal price decisions only in the second period after the end of first period. Therefore, this corollary suggests that the company should maintain the quality of their products by taking into consideration the online reviews. Hence, they produce better quality products at higher prices, creating more revenue and brand in the market.

Corollary 2. *In equilibrium,*

(i) *the selling prices of both strategies for the two periods satisfy:*

$$\frac{\partial p_{e1}}{\partial p_{e0}} > \frac{\partial p_{e2}}{\partial p_{e0}} > 0, \frac{\partial p_{r1}}{\partial p_{e0}} > \frac{\partial p_{r2}}{\partial p_{e0}} > 0; \quad \frac{\partial p_{e1}}{\partial p_{r0}} > \frac{\partial p_{e2}}{\partial p_{r0}} > 0, \frac{\partial p_{r1}}{\partial p_{r0}} > \frac{\partial p_{r2}}{\partial p_{r0}} > 0;$$

(ii) *the wholesale prices of both strategies for the two periods satisfy:*

$$\frac{\partial w_1}{\partial p_{e0}} > \frac{\partial w_2}{\partial p_{e0}} > 0 \quad \text{and} \quad \frac{\partial w_1}{\partial p_{r0}} > \frac{\partial w_2}{\partial p_{r0}} > 0.$$

This corollary suggests that if the reference price or the base price is high, then the pricing decisions in the first period will be higher than the second period. Note that this reference price has a direct impact on the demand of the first period. Hence, the reference price has more impact on the pricing decisions in the first period in both channels.

Corollary 3. *In equilibrium,*

(i) *the online selling prices satisfy:*

$$\frac{\partial p_{e1}^{\mathrm{I}}}{\partial O_1} > \frac{\partial p_{e1}^{\mathrm{II}}}{\partial O_1} > 0, \frac{\partial p_{e1}^{\mathrm{I}}}{\partial O_2} > \frac{\partial p_{e1}^{\mathrm{II}}}{\partial O_2} > 0, \frac{\partial p_{e2}^{\mathrm{I}}}{\partial O_1} > \frac{\partial p_{e2}^{\mathrm{II}}}{\partial O_1} > 0, \frac{\partial p_{e2}^{\mathrm{I}}}{\partial O_2} > \frac{\partial p_{e2}^{\mathrm{II}}}{\partial O_2} > 0;$$

(ii) *the retailer's selling prices satisfy:*

$$\frac{\partial p_{r1}^{\mathrm{I}}}{\partial O_1} > \frac{\partial p_{r1}^{\mathrm{II}}}{\partial O_1} > 0, \frac{\partial p_{r1}^{\mathrm{I}}}{\partial O_2} > \frac{\partial p_{r1}^{\mathrm{II}}}{\partial O_2} > 0, \frac{\partial p_{r2}^{\mathrm{I}}}{\partial O_1} > \frac{\partial p_{r2}^{\mathrm{II}}}{\partial O_1} > 0, \frac{\partial p_{r2}^{\mathrm{I}}}{\partial O_2} > \frac{\partial p_{r2}^{\mathrm{II}}}{\partial O_2} > 0.$$

Corollary 3 reveals that the pre-announced pricing strategy is more sensitive in the presence of online reviews. As the price commitment was done at the beginning of the selling season, therefore both players are unaware of demand uncertainty. Hence, they fixed a higher selling price in the pre-announced pricing strategy.

Corollary 4. *In equilibrium,*

(i) *the online selling prices satisfy:*

$$\frac{\partial p_{e1}^{\mathrm{I}}}{\partial p_{e0}} > \frac{\partial p_{e1}^{\mathrm{II}}}{\partial p_{e0}} > 0, \frac{\partial p_{e1}^{\mathrm{I}}}{\partial p_{r0}} > \frac{\partial p_{e1}^{\mathrm{II}}}{\partial p_{r0}} > 0, \frac{\partial p_{e2}^{\mathrm{I}}}{\partial p_{e0}} > \frac{\partial p_{e2}^{\mathrm{II}}}{\partial p_{e0}} > 0, \frac{\partial p_{e2}^{\mathrm{I}}}{\partial p_{r0}} > \frac{\partial p_{e2}^{\mathrm{II}}}{\partial p_{r0}} > 0;$$

(ii) *the retailer's selling prices satisfy:*

$$\frac{\partial p_{r1}^{\mathrm{I}}}{\partial p_{e0}} > \frac{\partial p_{r1}^{\mathrm{II}}}{\partial p_{e0}} > 0, \frac{\partial p_{r1}^{\mathrm{I}}}{\partial p_{r0}} > \frac{\partial p_{r1}^{\mathrm{II}}}{\partial p_{r0}} > 0, \frac{\partial p_{r2}^{\mathrm{I}}}{\partial p_{e0}} > \frac{\partial p_{r2}^{\mathrm{II}}}{\partial p_{e0}} > 0, \frac{\partial p_{r2}^{\mathrm{I}}}{\partial p_{r0}} > \frac{\partial p_{r2}^{\mathrm{II}}}{\partial p_{r0}} > 0.$$

Corollary 4 indicates that the reference price is more sensitive on the optimal selling prices in the strategy I. Here also we can argue the similar arguments as stated above in the case of Corollary 3. Further, we can conclude that the optimal solutions are highly sensitive in case of a high risk decision strategy.

6. Numerical results

In this section, first some numerical examples are employed to determine the optimal outcomes for three different strategies considered in this study. Then, the optimal outcome ratios of the two periods for two decentralized strategies are calculated and compared between them to study the sensitivity analysis. Finally, a graphical representation of several areas where one decision-making strategy prevails over others is presented.

6.1. Numerical example

Based on the existing literature and industry practice, we consider the values of the parameters. Referring to Maiti and Giri [28]; Zhang *et al.* [47]; Maiti [27], the parameters are adopted here as follows:

$d_1 = 160$;	$d_2 = 170$;	$\sigma_1 = 0.28$;	$\sigma_2 = 0.26$;	$\alpha_1 = 0.42$;	$\alpha_2 = 0.43$;
$\beta_1 = 0.22$;	$\beta_2 = 0.23$;	$\rho = 0.03$;	$\gamma = 0.06$;	$\theta = 0.18$;	$\tau = 0.22$;
$\delta_1 = 0.22$;	$\delta_2 = 0.25$;	$O_1 = 100$;	$O_2 = 105$;	$c = 120$;	$p_{r0} = 300$; $p_{e0} = 305$.

TABLE 2. The optimal results of all strategies.

Optimal decisions	Centralized	Strategies Strategy I	Strategy II
w_1	–	239.522	236.821
w_2	–	250.257	248.383
p_{e1}	276.165	275.672	262.747
p_{e2}	290.999	291.399	290.486
p_{r1}	243.274	289.15	285.035
p_{r2}	246.998	301.824	300.68
Π_{r1}	–	957.695	976.339
Π_{r2}	–	1143.46	1176.05
Π_r	–	2101.16	2152.38
Π_{m1}	–	6150.15	6260.87
Π_{m2}	–	8868.91	8793.78
Π_m	–	15 019.1	15 054.7
Π	19 221.5	17 120.2	17 207.0
Channel efficiency (CE)	–	0.891	0.895

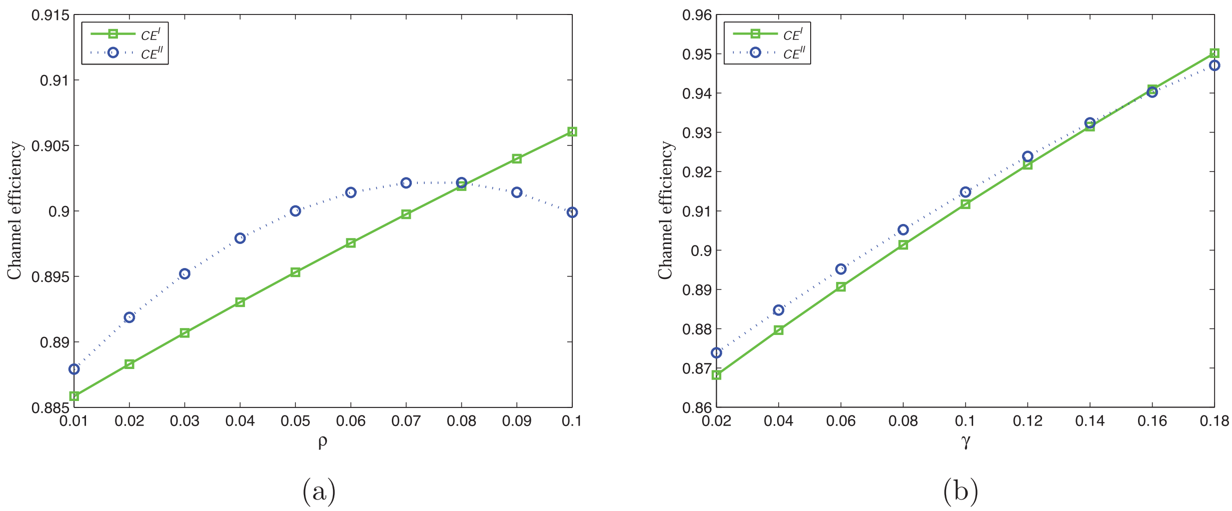


FIGURE 3. Impacts of ρ and γ on channel efficiency of two strategies. (a) ρ *vs.* CE. (b) γ *vs.* CE.

Considering these data, the optimal results for the proposed model are derived using Mathematica 9.0 software; and are shown in Table 2. As expected, Table 2 shows that the centralized decision can achieve higher profits as compared to the decentralized decisions. From this table, it is also clear that the profits of all players in both periods are higher in decision strategy II as compared to strategy I. Further, the manufacturer gains more under strategy I in the second period only. In addition, we calculate the channel efficiency (CE) of strategy I and strategy II, and shown in Table 2 to compare them. From this table, it is clear that strategy II is more efficient.

For further illustration, the channel efficiency of the two strategies are depicted graphically in Figures 3a and 3b with respect to ρ and γ, respectively. It can be observed from Figure 3a that the CE of strategy II is higher than the strategy I. But, when $\rho > 0.08$, strategy I is more profitable than strategy II. Similarly, when γ is 0.16 or more, strategy I will be more efficient, as observed in Figure 3b. This illustrates that

TABLE 3. Comparison of selling prices in different levels with respect to O_1.

Differences of prices in different levels		Variation of online reviews (O_1) {90, 95, 100, 105, 110}
Horizontal	$p_{e2}^{\mathrm{I}} - p_{e1}^{\mathrm{I}}$	{15.2248, 15.4761, 15.7275, 15.9788, 16.2301}
	$p_{r2}^{\mathrm{I}} - p_{r1}^{\mathrm{I}}$	{12.5989, 12.6363, 12.6738, 12.7113, 12.7487}
	$w_2^{\mathrm{I}} - w_1^{\mathrm{I}}$	{10.6796, 10.7074, 10.7352, 10.763, 10.7908}
	$p_{e2}^{\mathrm{II}} - p_{e1}^{\mathrm{II}}$	{26.9586, 27.3488, 27.7389, 28.1291, 28.5192}
	$p_{r2}^{\mathrm{II}} - p_{r1}^{\mathrm{II}}$	{15.5222, 15.5835, 15.6448, 15.7062, 15.7675}
	$w_2^{\mathrm{II}} - w_1^{\mathrm{II}}$	{11.4476, 11.5048, 11.562, 11.6192, 11.6764}
Vertical	$p_{r1}^{\mathrm{I}} - p_{e1}^{\mathrm{I}}$	{16.5697, 15.0242, 13.4787, 11.9332, 10.3877}
	$p_{r2}^{\mathrm{I}} - p_{e2}^{\mathrm{I}}$	{13.9438, 12.1844, 10.4251, 8.66567, 6.90629}
	$p_{r1}^{\mathrm{II}} - p_{e1}^{\mathrm{II}}$	{25.1346, 23.7116, 22.2886, 20.8656, 19.4427}
	$p_{r2}^{\mathrm{II}} - p_{e2}^{\mathrm{II}}$	{13.6981, 11.9463, 10.1945, 8.44273, 6.69095}
Strategical	$p_{e1}^{\mathrm{I}} - p_{e1}^{\mathrm{II}}$	{12.6268, 12.776, 12.9251, 13.0743, 13.2234}
	$p_{e2}^{\mathrm{I}} - p_{e2}^{\mathrm{II}}$	{0.89295, 0.90330, 0.91364, 0.92399, 0.93434}
	$p_{r1}^{\mathrm{I}} - p_{r1}^{\mathrm{II}}$	{4.0620, 4.08861, 4.11522, 4.14183, 4.16844}
	$p_{r2}^{\mathrm{I}} - p_{r2}^{\mathrm{II}}$	{1.13871, 1.14145, 1.14419, 1.14694, 1.14968}
	$w_1^{\mathrm{I}} - w_1^{\mathrm{II}}$	{2.63744, 2.66902, 2.70059, 2.73217, 2.76375}
	$w_2^{\mathrm{I}} - w_2^{\mathrm{II}}$	{1.86942, 1.8716, 1.87379, 1.87598, 1.87816}

the pre-announce pricing strategy is more beneficial for the system when the sensitivity of the horizontal and inter-temporal reference prices increases. It is also clear from these figures (Figs. 3a and 3b) that the responsive pricing dominates the entire ρ and γ domain, as these have a lower impact on the demand functions.

Now, some variations are made to establish the derived optimal outcomes, shown in Table 2, more firmly. Two main features: online reviews and reference price are considered for this observation. Table 3 shows the differences of prices in vertical, horizontal and strategy levels with respect to online reviews O_1. It is clear from Table 3 that the prices are higher in the second period irrespective of both strategies and both channels. This affirms our observation made above in Table 2. It is also noted that the differences between the second period price and the first period price are gradually increasing with the increase in online reviews. This incidence suggests that the increment of the second period prices is higher than the increment of the first period prices. This establishes the result shown in Corollary 1. The differences in prices of both channels (*i.e.* vertical level) show that the retail channel always offers higher prices which generate the lower profit. This phenomenon is also observed in Table 2. The differences of prices with cross strategies show that the strategy I always offer higher price, which leads to lower profit (shown in Tab. 2). Further, the differences between prices increase with the increment of online reviews. This phenomenon reflects the analysis derived in Corollary 3.

Noted that similar kinds of arguments are observed in the case of online reviews O_2. Hence, the corresponding table are omitted here.

In Table 4, we variate the reference price of the retail channel (p_{r0}) and establish some observations on price differences. Table 4 shows that prices are higher in the second period irrespective of both strategies and both channels, but the differences between the second period price and first period price is gradually decreasing. This suggests that the increment in the price of second period is lower than the price of the first period. This establishes the result described above in Corollary 2. Further, the differences of prices in both channels on

TABLE 4. Comparison of selling prices in different levels with respect to p_{r0}.

Differences of prices in different levels		Variation of reference price (p_{r0}) {280, 290, 300, 310, 320}
Horizontal	$p^{\mathrm{I}}_{e2} - p^{\mathrm{I}}_{e1}$	{15.9078, 15.8176, 15.7275, 15.6373, 15.5471}
	$p^{\mathrm{I}}_{r2} - p^{\mathrm{I}}_{r1}$	{13.7078, 13.1908, 12.6738, 12.1568, 11.6398}
	$w^{\mathrm{I}}_{2} - w^{\mathrm{I}}_{1}$	{11.4246, 11.0799, 10.7352, 10.3905, 10.0458}
	$p^{\mathrm{II}}_{e2} - p^{\mathrm{II}}_{e1}$	{27.9175, 27.8282, 27.7389, 27.6496, 27.5603}
	$p^{\mathrm{II}}_{r2} - p^{\mathrm{II}}_{r1}$	{16.6843, 16.1645, 15.6448, 15.1251, 14.6054}
	$w^{\mathrm{II}}_{2} - w^{\mathrm{II}}_{1}$	{12.2635, 11.9127, 11.562, 11.2112, 10.8605}
Vertical	$p^{\mathrm{I}}_{r1} - p^{\mathrm{I}}_{e1}$	{12.5821, 13.0304, 13.4787, 13.927, 14.3753}
	$p^{\mathrm{I}}_{r2} - p^{\mathrm{I}}_{e2}$	{10.3821, 10.4036, 10.4251, 10.4465, 10.468}
	$p^{\mathrm{II}}_{r1} - p^{\mathrm{II}}_{e1}$	{21.3909, 21.8398, 22.2886, 22.7375, 23.1863}
	$p^{\mathrm{II}}_{r2} - p^{\mathrm{II}}_{e2}$	{10.1577, 10.1761, 10.1945, 10.2129, 10.2313}
Strategical	$p^{\mathrm{I}}_{e1} - p^{\mathrm{II}}_{e1}$	{12.9232, 12.9242, 12.9251, 12.9261, 12.927}
	$p^{\mathrm{I}}_{e2} - p^{\mathrm{II}}_{e2}$	{0.91351, 0.91358, 0.91364, 0.91371, 0.91378}
	$p^{\mathrm{I}}_{r1} - p^{\mathrm{II}}_{r1}$	{4.11442, 4.11482, 4.11522, 4.11561, 4.11601}
	$p^{\mathrm{I}}_{r2} - p^{\mathrm{II}}_{r2}$	{1.13791, 1.14105, 1.14419, 1.14733, 1.15047}
	$w^{\mathrm{I}}_{1} - w^{\mathrm{II}}_{1}$	{2.7002, 2.7004, 2.70059, 2.70079, 2.70098}
	$w^{\mathrm{I}}_{2} - w^{\mathrm{II}}_{2}$	{1.8613, 1.86755, 1.87379, 1.88003, 1.88628}

vertical level indicates that the retail channel always offers a higher price which generates a lower profit. In cross strategical level, the differences of prices made a similar impact as online reviews are made in Table 3. Moreover, the differences increase negligibly with the reference price, which reflects the analysis derived in Corollary 4. Similar types of phenomena are observed when the reference price p_{e0} is varied, and hence the table is omitted here.

Tables 3 and 4 illustrate that the quick responsive decisions can bring a higher profit to the company. After establishing the brand value of a high-quality product, the company can increase the selling price of their product.

In the next subsection, we illustrate the effects of several key parameters on the optimal decisions. Also some domains where the demand or profit dominate over the others are established.

6.2. Sensitivity analysis

The effects of the key parameters γ, ρ, τ and θ on optimal decisions are depicted in Figures 4–7. The values of these parameters are varied based on optimal results of both channels and both periods of all strategies and by keeping other parameters' values unchanged.

From these figures, the following observations are made:

(i) It is worthy to note here that the increment in the price sensitivity (γ) of the alternative channel's price illustrate the other channel has more impact on the demand function of its rivals. From Figure 4a, it can be observed that the profit margins of both the periods increase in centralized policy and strategy I in direct channel. However, a very insignificant impact is observed on other strategies (*refer* Fig. 4a). Further, it is also observed that the value of p_1/p_2 is always less than 1, which indicates that the selling price of the second period is higher than the selling price of the first period. From Figure 4b, it is clear that the profits

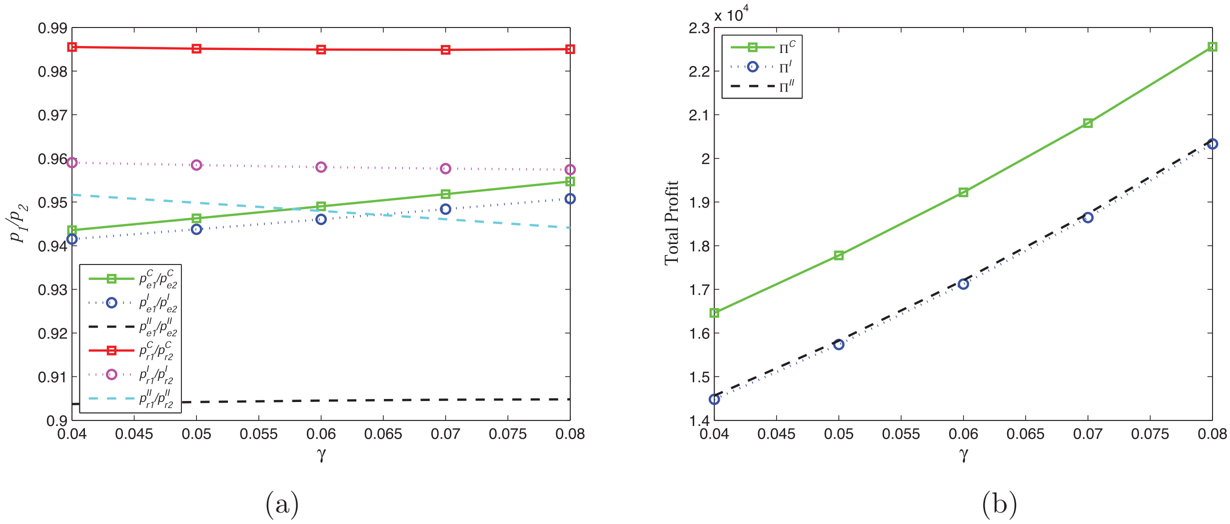


FIGURE 4. Effect of γ on optimal decisions. (a) Selling price ratios. (b) Profits.

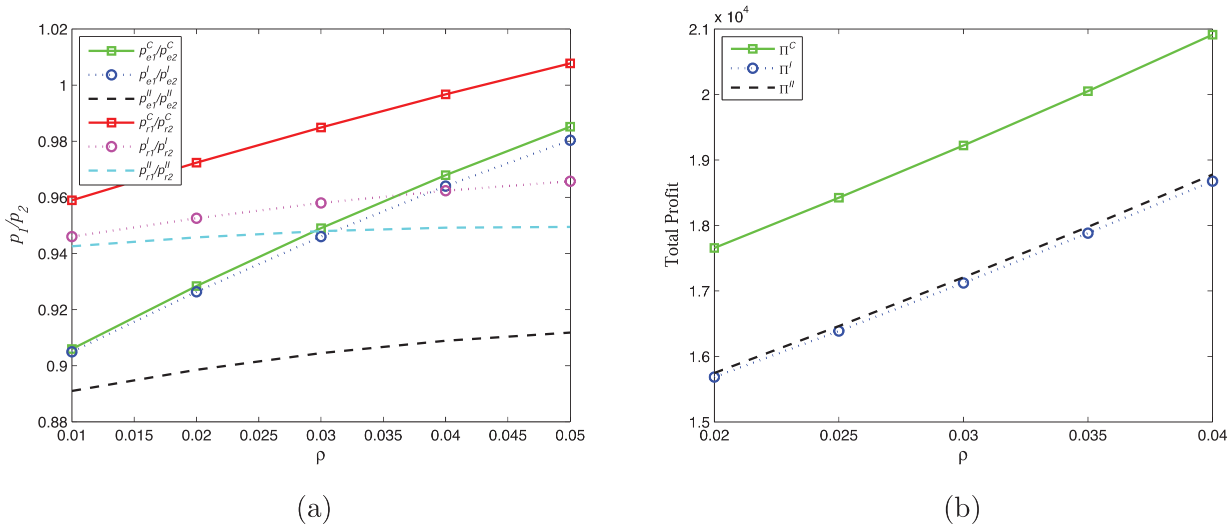
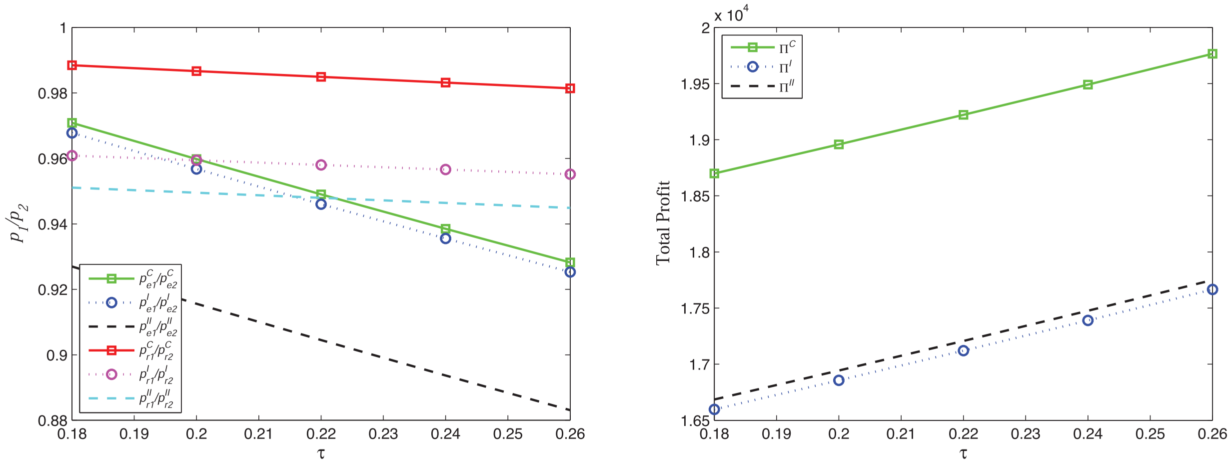


FIGURE 5. Effect of ρ on optimal decisions. (a) Selling price ratios. (b) Profits.

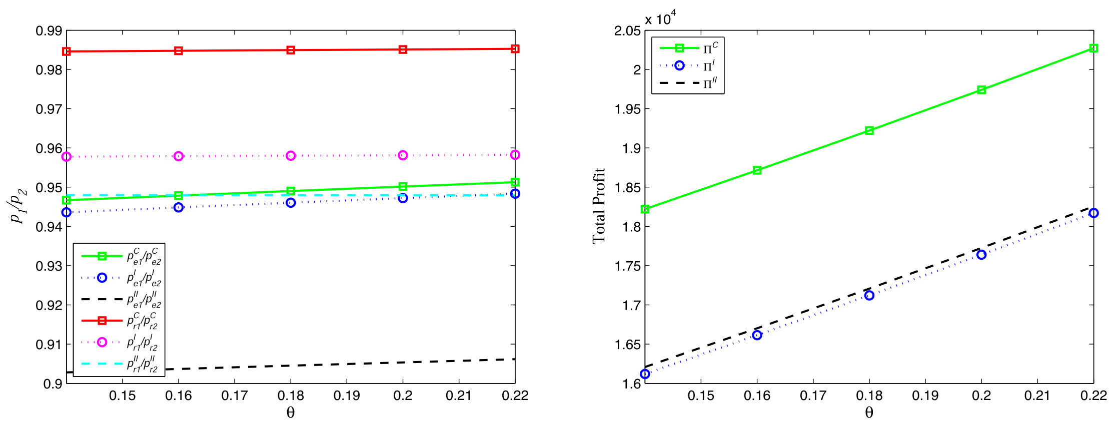
in all strategies increase when the value of the parameter γ is high. This is because of high demand in both channels and in both periods.

(ii) Note that the higher influence of the reference price on the demand functions (ρ) signifies that the reference price of the previous period has more impact on the current period (here the first period). As a result, the demand and selling prices increase with the increased value of ρ. This happens for the centralized model and strategy I as seen in Figure 5a. Figure 5a also illustrates that the prices in the first period rise faster than the second period in centralized model and strategy I. From Figure 5b, it can be observed that the profits in all strategies are highly affected by the reference price sensitivity ρ.



(a) (b)

FIGURE 6. Effect of τ on optimal decisions. (a) Selling price ratios. (b) Profits.

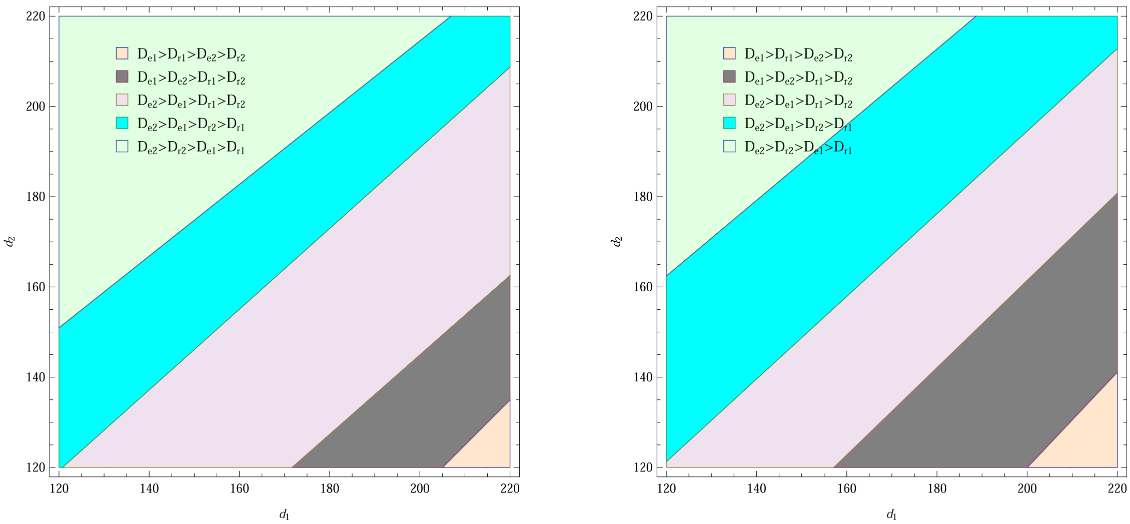


(a) (b)

FIGURE 7. Effect of θ on optimal decisions. (a) Selling price ratios. (b) Profits.

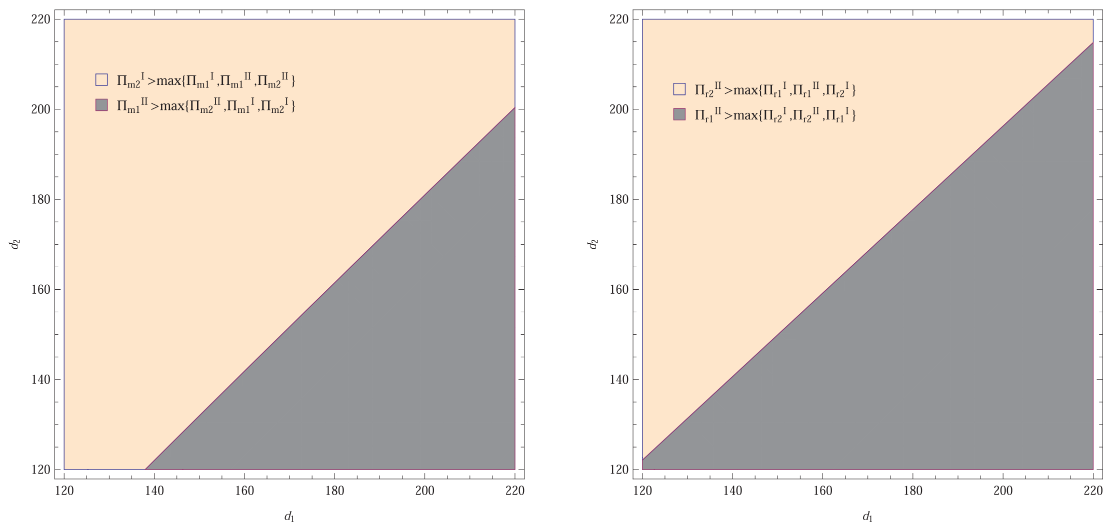
(iii) The sensitivity of online reviews, posted in the first period, has a direct impact on the second period's online demand. If it rises, the price of the second period becomes higher. As a result, the price ratio decreases with the sensitivity of the first period's online reviews. This illustration also followed in Figure 6a. In Figure 6b, it is noticed that the profits of all strategies increase as the online demand of the second stage increases.

(iv) In Figure 7, we have analyzed the effect of sensitivity of online reviews (θ) in the current period's demand on the optimal decisions. As the sensitivity of online reviews in the current period's demand increases, there will be more positive reviews in this period. This turns to more demand in the current period. From Figure 7a, it is observed that the ratio of selling prices is almost fixed with θ. However, the profits increase with the sensitivity of θ as seen in Figure 7b.



(a) (b)

FIGURE 8. Dominant demand rates over (d_1, d_2) region. (a) Strategy I. (b) Strategy II.

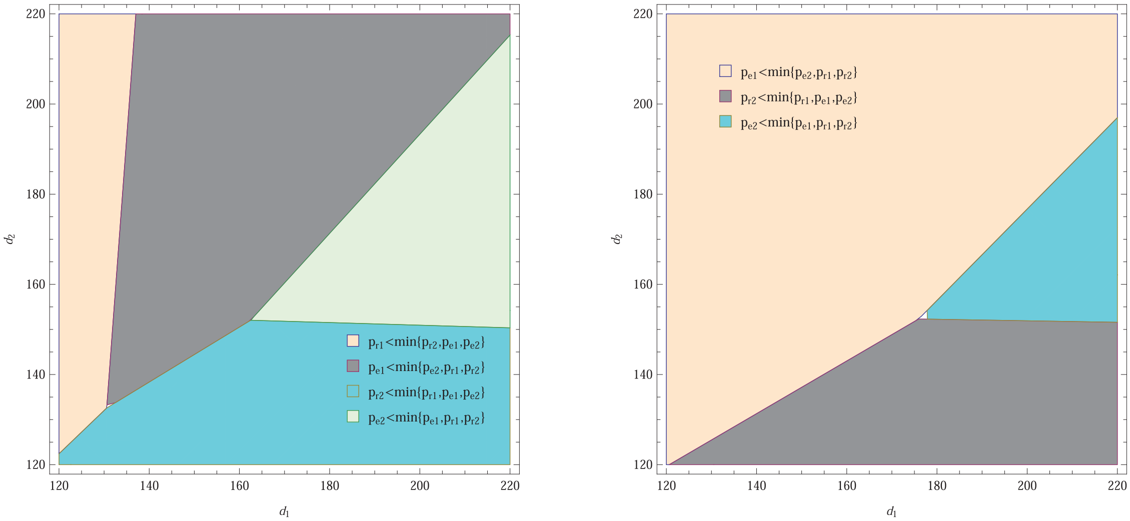


(a) (b)

FIGURE 9. Dominant different strategies' manufacturer's and retailer's profit over (d_1, d_2) region. (a) Manufacturer's profits. (b) Retailer's profits.

6.3. Comparison of dominating areas

This subsection is intended for a graphical representation of the dominant strategic sequences in (d_1, d_2) and (ρ, γ) regions. In Figures 8–10 we have demonstrated all the demand rates, the manufacturer profits and the retailer profits of the three developed strategies for their comparison. The illustration of all these depicted figures are given as follows:



(a) (b)

FIGURE 10. Dominance of different strategies' selling prices over (d_1, d_2) region. (a) Strategy I. (b) Strategy II.

(i) In Figures 8a and 8b, the dominant demand rates are depicted over (d_1, d_2) region. The domain is created as a variation of d_1 and d_2 which lies in $(120, 220)$. Figures 8a and 8b illustrates that the behavior of the demand for both the strategies (I and II) is almost the same. It is also observed that the demand in the first period is much higher in the direct channel when d_1 is high. Similarly, as d_2 increases, the demand in the second period, especially in the direct channel, increases. It is also observed, from Figures 8a and 8b that when d_1 and d_2 have the same value in the diagonal direction (bottom left to top right, mostly cyan color), the demand sequence is $D_{e2} > D_{e1} > D_{r2} > D_{r1}$. This indicates the complete dominance of the online demand in both strategies.

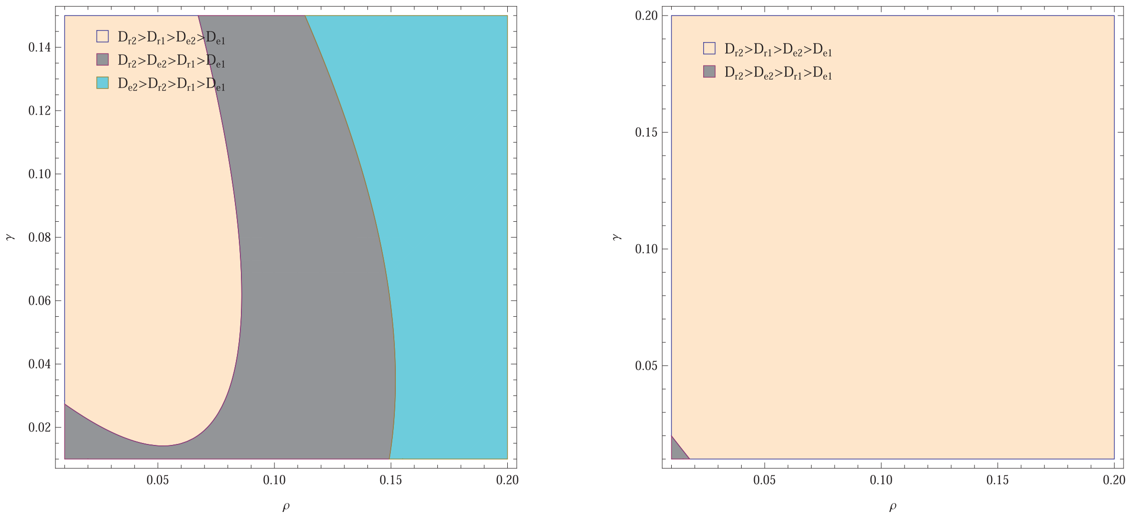
(ii) The profit of the manufacturer is depicted in Figure 9a over the range (d_1, d_2). In this figure, we observe that the manufacturer's profit in the first period in strategy II dominates over the other components of the manufacturer's profit for higher value of d_1. But, in the second period, his profit in strategy I prevail when d_2 is high (*refer* Fig. 9a).

(iii) The profit of the retailer is illustrated in Figure 9b over the region (d_1, d_2). In this figure, we notice that strategy II prevails for all cases with retailer margin. It is also observed that the first period's profit dominates when d_1 is high. But, the profit of the second period is higher when d_2 is higher (*refer* Fig. 9b).

(iv) Figure 10a illustrates that, when d_2 is low, both the manufacturer and retailer offer a lower price in the second period of strategy I. Similarly, when d_2 is higher than d_1, both parties offer a lower selling price in the first period. However, with strategy II, the manufacturer offers a lower selling price in the maximum zone especially when d_2 is high. But, when d_2 is low, the retailer offers the lowest selling price in the second period (*refer* Fig. 10b).

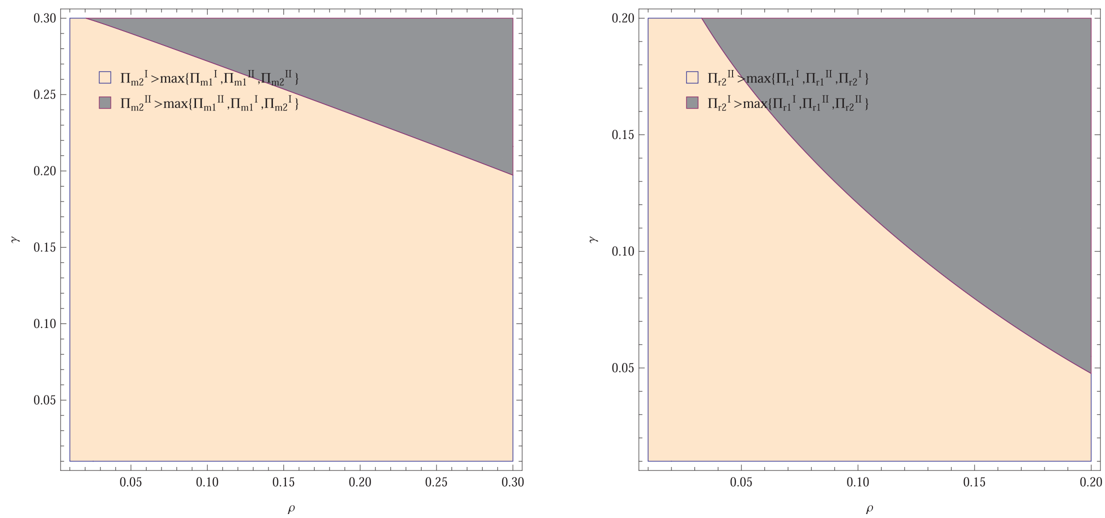
We now discuss some of the region-dominating sequences over (ρ, γ) which are the most important sensitivity parameters of the demand functions under consideration of reference price. The impact of (ρ, γ) on the different decision strategies are shown in Figures 11–13. From these figures, we observe the following remarks:

(i) From Figure 11a, it can be observed that the demand domination in the second period of strategy I shifted from retailer to manufacturer as ρ increases gradually. Figure 11b indicates that the retail demand in the second period of strategy II dominates across the region.



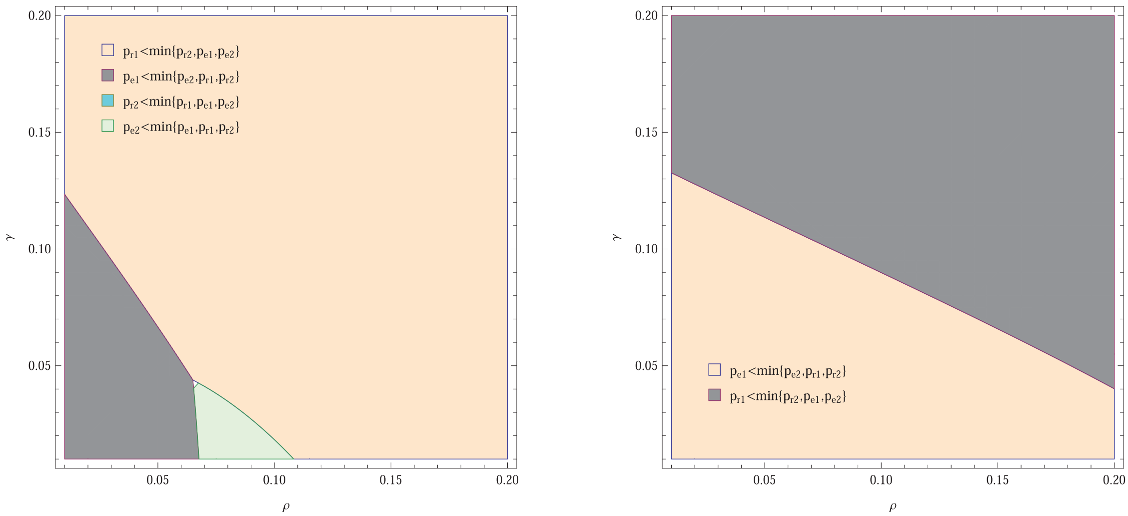
(a) (b)

FIGURE 11. Dominance of different strategies' demand rates over (ρ, γ) region. (a) Strategy I. (b) Strategy II.



(a) (b)

FIGURE 12. Dominance of different strategies' manufacturer's and retailer's profit over (ρ, γ) region. (a) Manufacturer's profits. (b) Retailer's profits.



(a) (b)

Figure 13. Dominance of different strategies' selling prices over (ρ, γ) region. (a) Strategy I. (b) Strategy II.

(ii) Figure 12a shows that the profit of the manufacturer in the second period of strategy I dominates across the domain. But, the high values of the two parameters (ρ, γ) can produce a higher profit in the second period under strategy II. In the case of retailer, the profit in the second period of strategy II prevails as seen in Figure 12b. This is an opposite occurrence as compared to the case of manufacturer.

(iii) In Figure 13, we depict the selling prices of different strategies of decentralized models over the region (ρ, γ). In Figure 13a, the strategy I is depicted, whereas in Figure 13b strategy II is depicted. In almost all regions of strategy I, the retailer offers the lowest selling price in her first period as seen in Figure 13a. Also, the manufacture offers lower prices when ρ and γ are in the region in $(0.01, 0.10)$ as seen in Figure 13a. But, with strategy II, the manufacturer offers a lower price in the first period when γ is low, as seen in Figure 13b. However, the retailer offers the lowest price when γ is high (*refer* Fig. 13b).

7. Managerial implications

This section illustrates the managerial insights for manufacturer and retailer in a two-period dual-channel supply chain influenced by online reviews and reference price. The findings of the present study have substantial consequences for various price decisions made by the several supply chain participants under numerous channel typologies. By analyzing the findings of the developed model, some management insights which are described below can be drawn:

(1) The derived sensitivity analysis proposes that the price ratio between the first and second periods is always less than 1 which shows that the second period has the higher prices across both the channels. This phenomenon indicates that the reference price has a great impact on the behavior of the consumer in the proposed dual-channel and two-period scenario. But, if the actual price of a product is higher than the reference price, the customer may feel that the product is too expensive and lose the interest to purchase the product. If the actual price is lower than the reference price, the customer will perceive the offer as a good deal and will be more inclined to buy it. This can affect both the demand and the sales volume for the manufacturer and for the retailer.

(2) Online reviews can reduce the consumer uncertainty about the online direct channel, thus improving the manufacturer's channel advantages. When the consumers are affected by online reviews, the retailer should strategically adjust their price strategies to improve channel position in a two-period dual-channel supply chain.
(3) Impacts of reference prices on consumers suggest that the retailer may not earn more profit if they always consider the ideal pricing strategy. A moderately high positive reference effect is beneficial for the traders.
(4) Channel efficiency of our developed model reveals that the responsive pricing approach dominates the maximum range of the selected set of parameters. Hence, in the real-world situation, any company may optimize their pricing strategy by modifying their prices based on the demand *via* responsive pricing strategy. This strategy enables the managers to maximize their sales by raising prices at the peak periods or at the time of high demand. The responsive pricing strategy also assists the organizations to survive in the business competition by monitoring and adjusting the market conditions. The price adjustments, in response to competitive pricing and market changes, can attract the customers, which provides a competitive edge. This will help the companies to vary the prices of their products with demand variation and with other unanticipated events like economic downturns or supply chain interruptions or COVID-19 situation, etc.
(5) The quick responsive choices can bring higher benefits to the companies. After generating the brand value of a high-quality product, they can increase their selling price.
(6) Finally, the pre-announced price fosters openness and client expectations, whereas the responsive pricing strategy maximizes the revenue and gives the flexibility in volatile markets. Both tactics have some advantages and some disadvantages, and can be employed strategically based on one's own business goals and market conditions. This observation is also supported by the numerical and sensitivity study of our developed model.

8. Conclusions and future research directions

Many companies have adopted a dual-channel supply chain comprising online and offline channels. Although the market related issues in a two-period problem have been researched thoroughly, still there was a lack of knowledge on dual-channel pricing strategy with two-period. To fill this gap, this article analyzed the problem involving a two-period dual-channel supply chain, including the effects of online reviews and reference pricing. The impacts of online consumer reviews and reference pricing on the profit of both manufacturer and retailer are illustrated under different decision strategies. The proposed model is optimized in a centralized scenario and two decentralized scenarios under the manufacturer led Stackelberg game. The summary of the present study is as follows:

- First four demand functions are considered for a dual-channel two-period supply chain by incorporating the consumers reviews, reference price and selling price of the current period. Then a centralized model, which one is considered as a benchmark case, and two decentralized models with pre-announced and responsive pricing strategies are developed. The optimal price strategies for both channels and both periods are determined.
- Both players decide to sell their products at low prices in a highly competitive market, and also to instill the confidence of their products.
- The reference price which is a reflection of the previous period, affects the selling price of both the channels. Using the feedback of the previous periods, the company can improve their pricing decision for the next period.
- It is also observed that the online reviews play an important role not only in the direct channel but also in the retail channel. Online reviews can reduce the consumer uncertainty which improves the advantages of the direct channel. These online reviews also help the retailer to adopt their price strategies to improve their profit.
- The optimal results for both players in different pricing strategies are graphically investigated. These investigations reveal both players can make higher revenue with some limited conditions. But, both players can benefit from a quick responsive strategy in the later part (second period) of their selling season.

– The numerical and sensitivity analysis suggest that the responsive pricing strategy provides some additional benefits to both the players.
– One of the most important management ideas that can be considered from the present study is a vertical and horizontal competition environment under different decision strategies. This idea brings some new aspects to the management which are already discussed in the earlier section.

The current study has also some limitations which are described below with some future avenues:

– The online reviews and reference price can be treated as decision variables which are constant in the present study.
– The online reviews have a direct impact on the quality of products [9, 27]. The inclusion of product quality, which is related with online reviews, will be an interesting addition.
– The present study considers only a retailer and a manufacturer in the system. However, it can be extended to a closed loop supply chain [10, 35] scenario by introducing a third party logistics who collects the used products.
– The developed model can be extended by considering the impact of various coordination contracts in a closed loop supply chain. Inclusion of a risk-averse supplier or manufacturer [32] can also be an interesting addition for future research.
– The developed model can be generalized for multi-period situations instead of two-period. Therefore, the present study can be extended to the multi-goal [12,13,17], multi-period [1] model with a few caveats.

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