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OPTIMIZING PRODUCTION PLANNING AND INVENTORY MANAGEMENT IN POST-PANDEMIC RECOVERY USING A MULTI-PERIOD HYBRID UNCERTAIN OPTIMIZATION MODEL

Purnawan Adi Wicaksono[1], Sutrisno Sutrisno[2,*], Solikhin Solikhin[2] and Abdul Aziz[2]

Abstract. During the post-COVID-19 pandemic recovery phase, decision-makers in the manufacturing and retail sectors are confronted with numerous uncertainties. These issues comprise various aspects of operations, including the acquisition of raw materials or components and planning production activities. Therefore, this research aimed to introduce an innovative dynamic hybrid optimization model that combined probabilistic and fuzzy techniques. The model would offer a solution for addressing the challenges posed by uncertain parameters, particularly in the context of post-pandemic scenarios for production planning and inventory management with multiple periods of observation. The model was designed to handle exceptional circumstances such as parameter uncertainties, augmented demand fluctuations, fuzzy variables, and probabilistic factors. The primary objective of the model was to maximize the expected total profit of the operational process. To achieve this aim, an uncertain programming algorithm based on the interior point method was used to compute the optimal decision for the problem at hand. Through the execution of simulations using randomly generated data, the proposed model was thoroughly evaluated and analyzed with six suppliers, three raw part types, three product types, and six periods. All six suppliers were selected to supply raw parts, however, not all suppliers were selected to supply particular raw part types. Furthermore, it was derived that the expectation of the maximum profit is 897 261.40; this is the best expected profit generated by the optimization model, meaning that other decisions may result in a smaller expectation of the profit. The results of these simulations unequivocally showed the effectiveness of the decision-making model in providing optimal solutions, specifically in terms of raw material procurement and production planning strategies. Subsequently, this model could serve as a valuable tool for decision-makers operating within the manufacturing and retail industries.

Mathematics Subject Classification. 90B05, 90B06, 90C11, 90C15, 90C70.

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[1] Department of Industrial Engineering, Diponegoro University, Semarang 50275, Indonesia.

[2] Department of Mathematics, Diponegoro University, Semarang 50275, Indonesia.

*Corresponding author: s.sutrisno@live.undip.ac.id

1. Introduction

The demand for certain products during the COVID-19 pandemic has surged, particularly medical supplies. As the world transitions into the post-pandemic era, increased demand is observed for other products and services including hospitality or tourism. Furthermore, factors such as pricing and quality become more uncertain due to various reasons including excess demands and restrictions [1]. Managers in the manufacturing and retail sectors also require new decision-making support systems customized to these conditions to optimize production and sales activities as well as maximize profits.

Two critical aspects demanding optimization in manufacturing and retail industries are raw material procurement and production. Operational expenses related to these activities significantly impact a company's profitability and should be optimized, including excess demand and uncertainties during recovery time after the COVID-19 pandemic. Therefore, managers require a new model to address the challenges effectively, which will motivate the development of a new decision-making tool proposed in this research.

Generally, a widely adopted method for process optimization includes the use of mathematical optimization models. Numerous companies across various industries depend on these models to optimize operations such as chemical product [2–5], portfolio management and accounting [6–9], as well as energy-related organizations [10–14]. The fundamental step in this method is to formulate the problem as a mathematical optimization model and then compute the optimal solution using an optimization algorithm.

Several mathematical optimization models have been proposed to tackle the specific issues of raw material procurement and production planning independently, customized to each problem's unique specifications. Examples include single supply chain [15], two-echelon [16], and periodic buyback networks [17]. However, these models typically operate in deterministic environments where all parameters are known with certainty. In more complex scenarios, advanced models have been developed to account for challenges such as product rejections [18–21] and risk management [22, 23].

The models remain problem-specific despite certain advanced frameworks handling uncertainties in certain parameters. Various examples can be found in the literature, such as energy-based [24], order acceptance schemes [25], and aggregate production models [26, 27], with documented applications in diverse sectors including crude oil refining [28], and textile industries [29].

A critical research gap still exists in integrating raw material procurement and production planning that incorporates excess demand and hybrid uncertainties, including fuzzy and probabilistic parameters. This research further aimed to bridge the gap by proposing a new mathematical optimization model that addresses these complexities, providing decision-making support for integrated raw material procurement and production planning in scenarios including excess demand and probabilistic-fuzzy parameters. Additionally, the model aims to generate optimal decisions for multiple observation periods, solving the problem dynamically. The research also includes numerical experiments to show how the proposed model effectively addresses the multifaceted problem.

2. Methodology and mathematical model

The problem and assumptions considered in this research were specified in the following subsection. Furthermore, the methodology and proposed model followed in the subsequent subsections.

2.1. Problem setting

Suppose a scenario where various types of raw materials and parts were procured from multiple suppliers, and were used in the production of various product brands, as depicted in Figure 1. The suppliers differed in terms of the capacities, pricing structures, quality of the raw materials, and delivery reliability. Subsequently, the company would sell the manufactured products to customers. All products produced were sold to adapt to the post-pandemic landscape, characterized by excess or surplus demands.

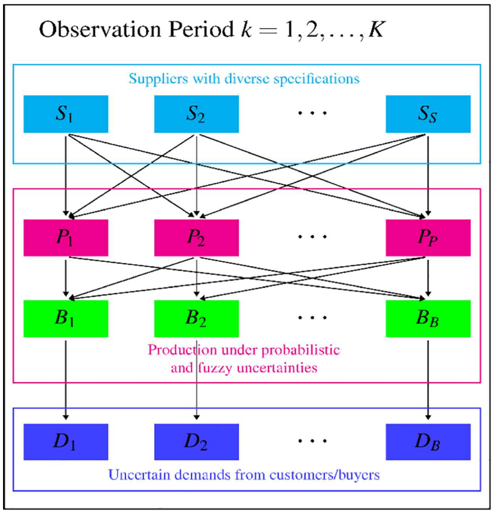

FIGURE 1. The layout of the supply chain.

The decision-maker aimed to maximize expected profit by optimizing the quantity of raw materials ordered from each supplier and the quantity of each product brand manufactured. This focus on maximizing expected profit was essential because the problem was tackled in an environment marked by uncertainty in certain parameters and further elucidated in the mathematical model.

The mathematical model formulated for this problem was supported by several assumptions. First, it was recognized that demands may not be completely met due to situations of excess or surplus demands in the pursuit of profit maximization. Second, raw materials delivered late were not used in the current production cycle but were incorporated into subsequent cycles. Third, any rejected raw materials and products were essentially discarded. Any remaining raw materials and products were disregarded as it did not contribute to the profit function.

All facilities had obvious limitations including the warehouses having capacity limits in storing raw materials or parts and products, suppliers having capacity limits in supplying raw materials or parts, and trucks used in deliveries having maximum capacity limits. Furthermore, procurement and production planning spanned multiple future observation periods known as the optimization time horizon with optimal decisions calculated in advance for the entire duration. This required a model that could handle the dynamic flow of the raw materials and products over observation periods.

2.2. Methodology

The methodology used in this research as shown in Figure 2 outlined the sequential steps engaged in solving the problem. Generally, the problem-solving process comprised four key stages. The first stage included problem definition, where the integrated challenge of raw parts procurement and production planning was precisely outlined and detailed in the preceding section. The second stage called mathematical modeling contained the conversion of the defined problem into a mathematical optimization model. This transformation was expounded upon in the subsequent subsection.

The optimal solution was computed using the formulated model which adopted an optimization algorithm for the third stage. For this purpose, an uncertain programming method based on the interior point algorithm was used with all computations executed within the LINGO 19.0 optimization software.

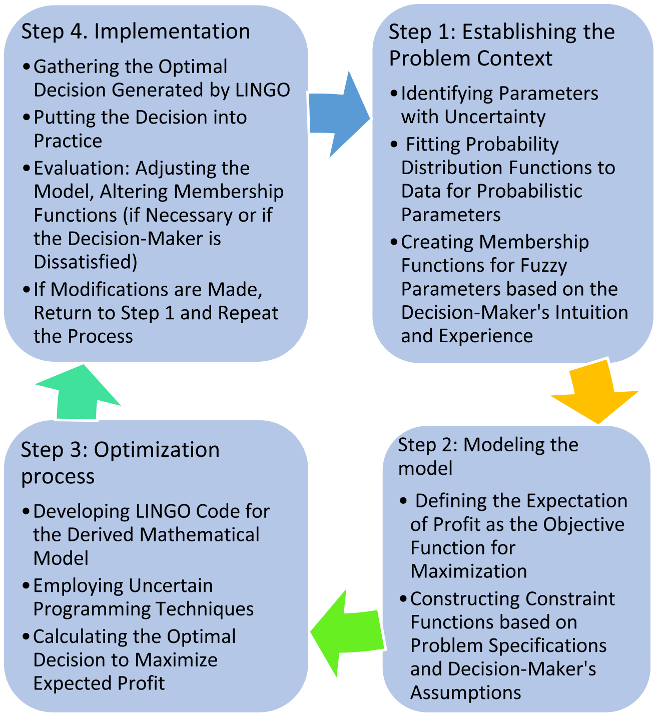

FIGURE 2. The problem-solving procedures.

Finally, the model experienced evaluation and potential adjustments in the fourth stage. Modifications were made to the model where necessary based on any alterations in the problem's parameters or conditions. Once the optimal decision was derived, it could then be put into action by the decision-maker.

2.3. Nomenclature

Indices

$k \in \{1, 2, \ldots, K\}$	Observation time instants where K represented the optimization time horizon
$p \in \{1, 2, \ldots, P\}$	Raw material or part types
$s \in \{1, 2, \ldots, S\}$	Raw material suppliers
$b \in \{1, 2, \ldots, B\}$	Product types or service brands

Variables to decide

$Y_b(k)$	Decision variables for the number of product types or service brands b to be produced on the observation time instant k
$X_{sp}(k)$	Decision variables for the number of raw materials or part types p to be purchased from supplier s on the observation time instant k
$Z_s(k)$	Binary decision variables were used to suggest how the supplier s was selected on the observation time instant k
$S_s(k)$	Decision variables to calculate the number of deliveries for transporting raw materials or parts from suppliers s to the manufacturer on the observation time instant k
W_s	Binary decision variables that were used to suggest how supplier s was selected over the optimization time horizon

$IY_b(k)$	Decision variables to decide the number of product types or service brands b to be stored in the warehouse on the observation time instant k
$IX_p(k)$	Decision variables to decide the number of raw material or part type p to be stored in the warehouse on the observation time instant k

Uncertain parameters

$\widetilde{DE}_b(k)$	Fuzzy parameters representing demand values from buyers for product or service brand b on the observation time instant k
$\widetilde{BP}_b(k)$	Fuzzy parameters representing the unit prices of product or service brand b on the observation time instant k
$\widetilde{PP}_{sp}(k)$	Fuzzy parameters representing the unit prices of raw material or part p from supplier s on the observation time instant k
$\widetilde{TC}_s(k)$	Fuzzy parameters representing transportation costs for a single delivery of raw material or parts from supplier s on the observation time instant k
$\widetilde{SC}_{sp}(k)$	Fuzzy parameters representing the capacities of supplier s to supply raw material or part type p on the observation time instant k
$\overline{AC}_b(k)$	Probabilistic parameters representing unit costs to produce products or services of type b on the observation time intant k
$\overline{LR}_{sp}(k)$	Probabilistic parameters representing the percentages of late delivered raw materials or parts of type p from supplier s on the observation time instant k
$\overline{DR}_{sp}(k)$	Probabilistic parameters representing the percentages of rejected or damaged raw material or parts of type p purchased from supplier s on the observation time instant k
$\overline{DY}_b(k)$	Probabilistic parameters representing the percentages of products or services of type b not passing the quality standard from production on the observation time instant k

Deterministic or crisp parameters

RP_{pb}	Number of raw materials or parts of type b needed to produce a unit of product or service brand b
$DC(k)$	Capacity of raw material or part deliveries from suppliers to the manufacturer on the observation time instant k
$O_s(k)$	Costs for ordering raw materials or parts to supplier s on the observation time instant k
CC_s	Costs to make contracts with supplier s for supplying raw materials or parts for the observation time horizon $\{1, 2, \ldots, K\}$
$PD_p(k)$	Penalty costs were applied to penalize rejected or damaged raw materials or parts of type p on the observation time instant k
$PL_p(k)$	Penalty costs were applied to penalize late delivery of one raw material or part type p on the observation time instant k
$ICY_b(k)$	Holding or inventory costs applied to a single product of type b stored in the warehouse for an observation period over the observation time instant k
$ICX_p(k)$	Holding or inventory costs applied to a single raw material or part of type p stored in the warehouse for one observation period over the observation time instant k
$MIY_b(k)$	Capacities of the warehouse in storing products of type b on the observation time instant k
$MIX_p(k)$	Capacities of the warehouse in storing raw materials or parts of type p on the observation time instant k

Auxiliary notations

$\mathcal{E}[\,\cdot\,]$	Expectation value in the sense of hybrid probabilistic-fuzzy expectation

2.4. Mathematical model

The procurement and production planning problems described in the preceding section were modeled as a mathematical programming problem with hybrid probabilistic-fuzzy parameters, with the expectation of profit serving as the objective function. Various constraint functions were devised following the required conditions which were explained in the following mathematical modeling process.

First, the profit expectation function was defined as the expected income minus the expected total operational cost. Let F_1 be the income of the optimization time horizon $\{1, 2, \ldots, K\}$. It was further assumed that the problem included excess or surplus demands, suggesting that all the available products or services would be sold. Next, the unit price of products or services was formulated as $\widetilde{BP}_b(k)$ times the product or service demands. The formula was further formulated as follows.

$$F_1 = \sum_{k=1}^{K} \sum_{b=1}^{B} \left(\widetilde{BP}_b(k) \times \widetilde{DE}_b(k) \right). \tag{1}$$

Subsequently, the operational costs were modeled in the following modeling process. The initial component of operational cost pertained to the expense of manufacturing a single unit of the product over the optimization time horizon $\{1, 2, \ldots, K\}$, denoted as F_2 and was expressed as follows.

$$F_2 = \sum_{k=1}^{K} \sum_{b=1}^{B} \left(\overline{AC}_b(k) \times Y_b(k) \right). \tag{2}$$

The second element of operational cost pertained to the procurement cost for raw materials over the optimization time horizon $\{1, 2, \ldots, K\}$, represented as F_3 and was calculated as the unit price multiplied by the quantity ordered. It could then be formulated as follows.

$$F_3 = \sum_{k=1}^{K} \sum_{p=1}^{P} \sum_{s=1}^{S} \left(\widetilde{PP}_{sp}(k) \times X_{sp}(k) \right). \tag{3}$$

The subsequent operational cost factor integrated into the model accounted for the penalties associated with rejected and delayed raw materials or parts over the optimization time horizon $\{1, 2, \ldots, K\}$, designated as F_4 and was expressed in the following equation:

$$F_4 = \sum_{k=1}^{K} \sum_{p=1}^{P} \sum_{s=1}^{S} \left[\left(PD_p(k) \times \overline{DR}_{sp}(k) \times X_{sp}(k) \right) + \left(PL_p(k) \times \overline{LR}_{sp}(k) \times X_{sp}(k) \right) \right]. \tag{4}$$

The next component was the inventory costs for raw materials or parts and products or services over the optimization time horizon $\{1, 2, \ldots, K\}$, designated as F_5 and was expressed in the following formula.

$$F_5 = \sum_{k=1}^{K} \sum_{p=1}^{P} (IX_p(k) \times ICX_p(k)) + \sum_{k=1}^{K} \sum_{b=1}^{B} (IY_b(k) \times ICY_b(k)). \tag{5}$$

The subsequent operational cost element comprised expenses related to making contracts and ordering raw materials or parts over the optimization time horizon $\{1, 2, \ldots, K\}$, represented as F_6 and defined as follows.

$$F_6 = \sum_{s=1}^{S} CC_s \times W_s + \sum_{k=1}^{K} \sum_{s=1}^{S} \left(\overline{O}_s(k) \times Z_s(k) \right). \tag{6}$$

The last operational cost element covered expenses related to raw material or part transportation from suppliers to the manufacturer over the optimization time horizon $\{1, 2, \ldots, K\}$, denoted as F_7 and defined as follows.

$$F_7 = \sum_{k=1}^{K} \sum_{s=1}^{S} \left(\widetilde{TC}_s(k) \times S_s(k) \right). \tag{7}$$

Maximizing the actual profit function was not feasible until these uncertain parameters were realized due to the presence of uncertain parameters (with probabilistic and fuzzy characteristics described in the problem setup). Consequently, the decision-maker was only able to maximize the expected profit. The decision that achieved maximum expected profit had to adhere to the predefined problem conditions. This entire scenario was formalized as the following optimization problem with explanations for the constraint functions to be provided subsequently and subjected to the constraint functions explained as follows.

$$\max_{\left\{\substack{X_{sp}(k),\, Y_b(k),\\ IY_b(k),\, IX_p(k)}\right\}} \mathcal{E}[F_1 - F_2 - F_3 - F_4 - F_5 - F_6 - F_7]. \tag{8}$$

– Raw material or part demand satisfying:

$$\begin{aligned} IX_p(k-1) - IX_p(k) + \sum_{s=1}^{S} X_{sp}(k) - \sum_{s=1}^{S} \left[\left(\overline{LR}_{sp}(k) \times X_{sp}(k)\right) + \left(\overline{DR}_{sp}(k) \times X_{sp}(k)\right)\right] \\ \geq \sum_{b=1}^{B} (RP_{pb}(k) \times Y_b(k)) \qquad \forall p \in \{1, 2, \ldots, P\}, \quad \forall k \in \{1, \ldots, K\} \end{aligned} \tag{9}$$

where $IX_p(0)$ denoted the initial raw material inventory level, which was usually set as zero. The constraint function asserted that the quantity of non-rejected raw materials or parts received by the manufacturer should be sufficient for production. This was further defined as the raw materials ordered from suppliers (plus those in the inventory) minus the delayed or rejected raw materials and products stored for future observation periods, which had to be greater than or equal to the required quantity of raw materials or parts necessary for production.

– Product or service demand satisfying:

$$IY_b(k-1) + Y_b(k) - \overline{DY}_b(k) \times Y_b(k) - IY_b(k) \geq \widetilde{DE}_b(k), \qquad \forall b \in \{1, 2, \ldots, B\}, \quad \forall k \in \{1, 2, \ldots, K\} \tag{10}$$

where $IY_b(0)$ represented the initial inventory level of products on hand. This second constraint stipulated that the number of on-hand products (non-rejected products plus material in the warehouse) minus those to be stored in the warehouse satisfied the demand.

– Suppliers' capacity limitations:

$$X_{sp}(k) \leq \widetilde{SC}_{sp}(k), \qquad \forall p \in \{1, 2, \ldots P\}, \quad \forall s \in \{1, 2, \ldots, S\}. \tag{11}$$

This sets the upper limit on the quantity of raw materials or parts that were procured from suppliers.

– Calculations for the number of raw material or part deliveries from suppliers to the manufacturer were calculated as follows.

$$\sum_{p=1}^{P} X_{sp}(k) \leq C(k) \times S_s(k), \qquad \forall s \in \{1, 2, \ldots, S\}, \quad \forall k \in \{1, 2, \ldots, K\}. \tag{12}$$

This constraint function was used to determine the number of deliveries or trucks required for transporting raw materials or parts from each supplier.

– Raw material or part inventory capacity limits:

$$IX_p(k) \leq MIX_p(k), \qquad \forall p \in \{1, 2, \ldots P\}, \quad \forall k \in \{1, 2, \ldots, K\}. \tag{13}$$

This suggested that the quantity of raw materials or parts stored in the inventory should not exceed the capacities of the allocated warehouse.

– Product inventory capacity limits:

$$IY_b(k) \leq MIY_b(k), \qquad \forall b \in \{1, 2, \ldots B\}, \quad \forall k \in \{1, 2, \ldots, K\}. \tag{14}$$

This implied that the quantity of products stored in the inventory should not exceed the capacities of the allocated warehouse.

– Binary functions for the indicator variables for suppliers that were selected to supply raw materials or parts at a particular observation period k:

$$Z_s(k) = \begin{cases} 1, & \text{if } \sum_{p=1}^{P} X_{sp}(k) > 0 \\ 0, & \text{others} \end{cases} \qquad \forall s \in \{1, 2, \ldots, S\}, \quad \forall k \in \{1, 2, \ldots, K\}. \tag{15}$$

The model was designed to show how $Z_s(k)$ should be 1 when the corresponding supplier was selected or 0 when not selected. This was used to calculate the order cost in the cost function.

– Binary functions for the indicator variables for suppliers that were selected to supply raw materials or parts during the contract period (optimization horizon period):

$$W_s = \begin{cases} 1, & \text{if } \sum_{k=1}^{K} Z_s(k) > 0, \\ 0, & \text{others,} \end{cases} \qquad \forall s \in \{1, 2, \ldots, S\}. \tag{16}$$

This implied that W_s would be equal to one when supplier s was selected and zero otherwise.

– Product or service demand satisfying:

$$\begin{aligned} &X_{sp}(k), Y_b(k), ICY_b(k), ICX_p(k) \geq 0 \;\; \text{(and integer)} \\ &\quad \forall s \in \{1, 2, \ldots, S\}, \quad \forall p \in \{1, 2, \ldots, P\}, \quad \forall k \in \{1, 2, \ldots, K\}. \end{aligned} \tag{17}$$

This model enforced non-negativity values for all decision variables, including the number of raw materials to be procured from each supplier, the number of products to be produced during production, and those to be stored in the inventory.

The programming model presented in equation (8) and the associated constraints incorporated elements characterized by both probabilistic and fuzzy parameters, rendering the framework an instance of uncertain programming. The research used the uncertain programming method to determine the optimal decision. In calculating the expectation value of a probabilistic parameter, conventional formulas derived from probability theory were applied, and for more detailed technical information, refer to [30]. Furthermore, discrete membership functions were used for fuzzy parameters included in the model and the weighting schemes were used to calculate the expectation values of the fuzzy parameters. For technical details regarding this assertion, refer to [31].

In particular, the proposed optimization model belongs to nonlinear uncertain programming class; the nonlinearity was due to the presence of the piecewise functions (15) and (16). The integer constraints for the decision variables are optional depending on the context of the application. When all decision variables must be integer, it belongs to the pure integer nonlinear uncertain programming class. When only some of them must be integer, it belongs to the mixed-integer nonlinear uncertain programming. Based on the sturcture of the objective function and the constraint functions where most of them are linear functions, the complexity of the optimization model can be categorized as medium. This is due to the massive indices especially when the number of suppliers, the number of raw part types and the number of product brands are sufficiently large, which will lead to a massive number of decision variables; note that in this case, the number of decision variables and then number of scenarios generated by the uncertain parameters grow exponentially. Furthermore, the model proposed above has a number of superiority compared to existing models mentioned in the introduction. First, it can handle two types of uncertain parameters (probabilistic parameters and fuzzy parameters) in one model, which is suitable for situations under uncertainties such as the time during recovery time after a pandemic. Second, the model

TABLE 1. Quantity of each raw material or part type needed to make one unit product or service ($RP_{pb}(k)$) for all observation time instants k.

Raw material or part	Product or service		
	$B1$	$B2$	$B3$
$P1$	1	1	2
$P2$	1	3	3
$P3$	3	1	1

TABLE 2. Cost to penalize a unit reject raw material or part ($PD_p(k)$) for all observation time instants k.

Supplier	Part type		
	$P1$	$P2$	$P3$
$S1$	2.00	1.00	1.00
$S2$	1.00	0.50	1.00
$S3$	2.00	0.50	0.50

TABLE 3. Cost to penalize a unit late delivered raw material or part ($PL_{sp}(k)$) for all observation time instants k.

Supplier	Raw material/Part type		
	$P1$	$P2$	$P3$
$S1$	0.90	1.10	0.70
$S2$	1.00	1.10	0.80
$S3$	1.20	1.10	1.00

TABLE 4. Raw material or part delivery ($TC_s(k)$), order ($O_s(k)$), contract costs ($CC_s(k)$), and the maximum capacity of trucks ($C(k)$) for all observation time instants k.

Supplier	$TC_s(k)$	$O_s(k)$	$CC_s(k)$	$C(k)$
$S1$	50	250	20	
$S2$	60	300	15	100
$S3$	55	240	25	

is comprehensive in the sense that it integrates many aspects including costs of all operational activities, capacities of all parties involved in the problems and many requirements that are commonly used in applications. Third, the model is straighforward to be implementated in an optimization software such as LINGO due to its summation notations. Basic features such as @SUM and @FOR in LINGO can be easily implemented to solve the proposed optimization model. Moreover, the stochastic programming feature in LINGO can be used directly with effortless code writing.

TABLE 5. Parameters related to inventory of raw materials or parts for all observation time instants k.

Raw materials or parts	$ICX_p(k)$	$MIX_p(k)$
$P1$	1	50
$P2$	2	75
$P3$	3	70

TABLE 6. Parameters related to inventory of products and production capacities for all observation time instants k.

Products	$ICY_b(k)$	$MIY_b(k)$	Production capacities
$B1$	1	50	500
$B2$	2	25	450
$B3$	3	75	540

TABLE 7. Mean (μ) and variance (σ) for normally distributed parameters.

k	Parameter	μ	σ
1 to 3	$\overline{AC}_b(k)$	10	2
	$\overline{LR}_{sp}(k)$	0.05	0.01
	$\overline{DR}_{sp}(k)$	0.05	0.01
	$\overline{DY}_b(k)$	0.05	0.02
4 to 6	$\overline{AC}_b(k)$	12	5
	$\overline{LR}_{sp}(k)$	0.04	0.02
	$\overline{DR}_{sp}(k)$	0.04	0.02
	$\overline{DY}_b(k)$	0.06	0.01

TABLE 8. Values membership functions of fuzzy parameter $\widetilde{DE}_b(k)$ for all observation time instants k.

i	$\widetilde{DE}_b^{(i)}(k)$	$\mu_{\widetilde{DE}_b^{(i)}}(k)$	$w^{(i)}_{\widetilde{DE}_b}(k)$
1	50	0.50	0.250
2	55	0.80	0.150
3	60	1.00	0.275
4	65	0.65	0.170
5	70	0.31	0.155

TABLE 9. Values membership functions of fuzzy parameter $\widetilde{BP}_b(k)$ for all observation time instants k.

i	$\widetilde{BP}_b^{(i)}(k)$	$\mu_{\widetilde{BP}_b^{(i)}}(k)$	$w^{(i)}_{\widetilde{BP}_b}(k)$
1	1000	0.25	0.125
2	1010	0.77	0.260
3	1020	1.00	0.190
4	1030	0.85	0.150
5	1040	0.55	0.275

TABLE 10. Values membership functions of fuzzy parameter $\widetilde{PP}_{sp}(k)$ for all observation time instants k

i	$\widetilde{PP}_{sp}^{(i)}(k)$	$\mu_{\widetilde{PP}_{sp}^{(i)}}(k)$	$w^{(i)}_{\widetilde{PP}_{sp}}(k)$
1	25	0.10	0.050
2	26	0.20	0.050
3	32	1.00	0.700
4	33	0.40	0.050
5	34	0.30	0.150

TABLE 11. Values membership functions of fuzzy parameter $\widetilde{TC}_s(k)$ for all observation time instants k.

i	$\widetilde{TC}_s^{(i)}(k)$	$\mu_{\widetilde{TC}_s^{(i)}}(k)$	$w^{(i)}_{\widetilde{TC}_s}(k)$
1	110	0.10	0.050
2	120	0.30	0.100
3	180	1.00	0.575
4	190	0.55	0.165
5	200	0.22	0.110

TABLE 12. Values membership functions of fuzzy parameter $\widetilde{SC}_{sp}(k)$ for all observation time instants k.

i	$\widetilde{SC}_{sp}^{(i)}(k)$	$\mu_{\widetilde{SC}_{sp}^{(i)}}(k)$	$w^{(i)}_{\widetilde{SC}_{sp}}(k)$
1	450	0.40	0.200
2	500	0.87	0.235
3	550	1.00	0.185
4	600	0.76	0.280
5	650	0.20	0.100

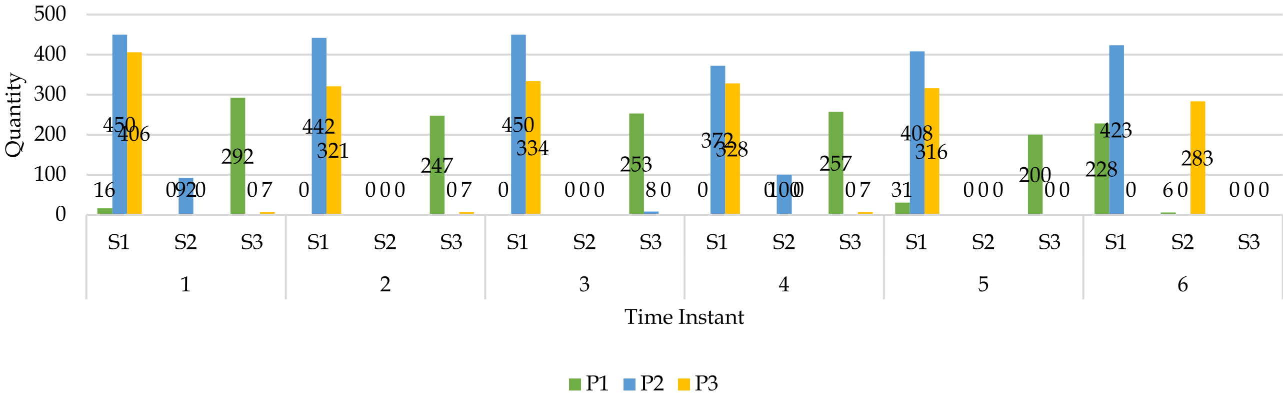

FIGURE 3. The optimal raw part procurement.

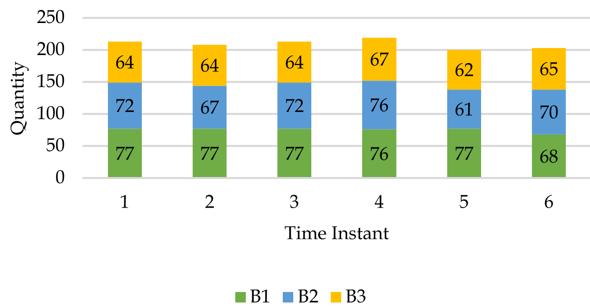

FIGURE 4. The optimal decision regarding the production.

3. Numerical analysis and discussion

Laboratory simulations were conducted using randomly generated data to evaluate the model presented in this research. These experiments included five suppliers, types of raw materials, and product brands each. Specific values for deterministic parameters were depicted in Tables 1–6. For probabilistic parameters, normal probability distributions were assumed with the mean and standard deviation detailed in Table 7. Additionally, each fuzzy parameter η was characterized by a discrete membership function in the following form.

$$\mu_\eta = \mu_{\eta^{(i)}} \text{ if } \eta = \eta^{(i)}, \qquad i = 1, 2, \ldots, 5$$

where $\eta^{(i)}$ of each fuzzy parameter were shown in Tables 8–12. The optimization software LINGO 20.0 software was also used to solve equation (8) while the uncertain programming was adopted to calculate the optimal decision, and the outcomes were depicted in Figures 3–5.

Figure 3 depicted the optimal procurement planning leading to the selection of all suppliers for the supply of raw material parts. The selection of suppliers varied across different observation time instants, with certain suppliers supplying specific raw part types at particular time points. This decision was primarily influenced by the diverse performance levels of the suppliers. For instance, at the observation time instant $k = 2$, only suppliers $S1$ and $S3$ were selected. Supplier $S1$ provided only raw part types $P2$ and $P3$, while supplier $S3$ supplied raw part type $P1$. This strategic method aimed to maximize profit, showcasing the nuanced outcomes attainable through the application of mathematical optimization models.

The optimal production decision was depicted in Figure 4 and the quantities shown were not the actual quantities of the on-hand products due to the uncertain rejection rates. The actual quantities could only be

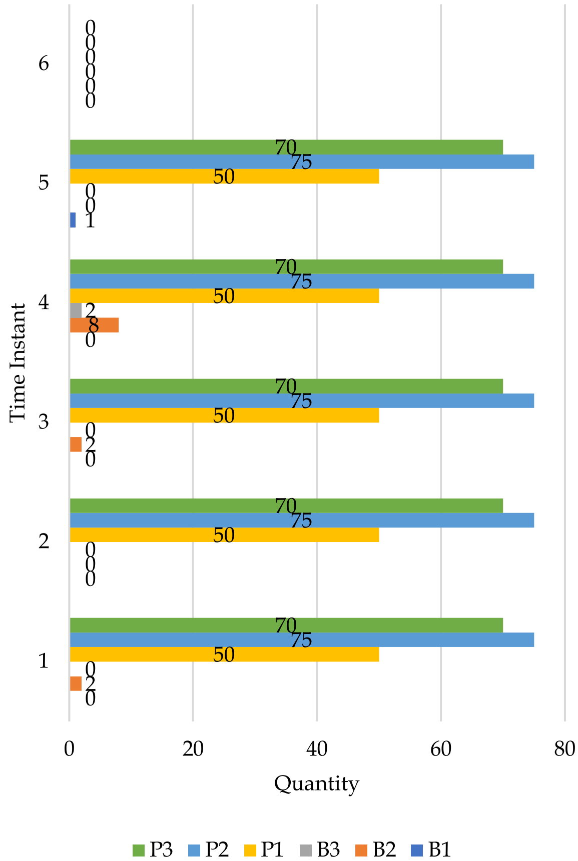

FIGURE 5. The optimal decision regarding the inventory.

determined after the rejection rates were discovered. Significantly, the expected profit from this production decision was 897 261.40, underscoring the effectiveness of the optimized allocation strategy compared to non-optimized decisions.

Based on the numerical example discussed above, the following observations are derived. In the literature addressing the optimization of supplier selection, production planning and inventory management problems, the most common approach is based on mathematical optimizations [32]. Every mathematical model suits a particular problem depending on its specifications (see *e.g.* the models in [33–41]) and furthermore, on the context of the applications, see *e.g.* the models in [42–51]. Special attention may be addressed to the models developed in [27, 52, 53] in which some other aspects are included in the model, such as the following three objective functions: operational cost, purchase resiliency and environmental pollution. However, those aspects may not be needed to be included when their contributions are sufficiently small and thus may be omitted. Nevertheless, based on the optimization theory, all of those optimization models as well as the model proposed in this study provides optimal decisions for the given problem. This justifies the completeness of the proposed decision-making support. Note that every mathematical optimization model is flexible in the sense that when the existing specifications are changed or new specifications are added, the model can be modified by changing the objective function and the constraint functions and adding constraint functions based on the new specifications. As an illustration, if the aspect of green economy in [54] is added in which the decision-makers want to minimize the carbon emission in their supply chain activities, then cost terms that correspond to this specification can be added to the objective function. In the end, it is the decision-makers' policy to select the factors that are considered to be included in the model. Nevertheless, note that too many aspects may lead to a large-scale optimization problem, and more resource may be needed to find the optimal decision, see the forthcoming Section 4 for more managerial insights.

4. Managerial insights

Based on the mathematical optimization model outlined in the preceding section and the outcomes of the laboratory-based numerical experiments, decision-makers can consider the following managerial insights for implementation. These insights aim to enhance decision-making processes and optimize operational efficiency in complex supply chain environments.

First, the mathematical model can be customized to correlate with the specific needs and the unique problem specifications. This can include incorporating additional operational costs and introducing various constraint functions such as limitations on production and transportation costs. The model introduced in this research is versatile and applicable to a wide range of raw materials, products, or services. However, slight adjustments may be necessary when decision-makers encounter different specifications. For instance, essential constraints on decision variables may not be required when measurements follow real numbers.

Second, alternative probability distribution functions such as the Poisson distribution may be explored based on the results of probability tests conducted on the data. Similarly, decision-makers can experiment with different types of membership functions for fuzzy parameters. These can include triangular or trapezoidal functions, based on intuition and prior experience with similar problems. The flexibility is crucial given the absence of complete data.

Third, sensitivity analysis can be further performed, although decision-makers with ample computation time can conduct multiple runs using the same or different values for uncertain parameters and different membership functions. These variations may yield different decisions but should provide the same expected profit. Different values for uncertain parameters can also yield varying profit expectations due to the inherent uncertainty. Eventually, it falls upon the decision-maker to select the decision to implement.

Fourth, it is crucial to recognize that the actual profit realized by the manufacturer may deviate from the profit expectation provided by the mathematical model. This divergence occurs because activities are carried out in an environment rife with uncertainties. Consequently, optimizing activities using mathematical models represents the best method available to decision-makers.

Lastly, this research used a standard computer with typical specifications regarding computational resources, accomplishing computations within minutes. However, computational time can extend to hours or days for larger problem sizes including hundreds of suppliers and dozens of product brands. When the computational time exceeds a time instant, the equipment used becomes unreliable. In these cases, decision-makers can consider adopting high-performance computing or heuristic algorithms despite moving beyond the scope of the research.

5. Conclusion and future works

In conclusion, this research introduced an innovative mathematical model in the form of dynamic hybrid probabilistic-fuzzy programming. This model offered valuable capabilities for optimizing procurement and production planning with multiple observation times, particularly in cases of excess demand and significant uncertainties treated with probability and fuzzy theory. The framework was well-suited for tackling extraordinary situations such as post-pandemic recovery, where demand outpaced supply and many parameters remained uncertain. Numerical experiments were also conducted to assess the effectiveness of the proposed model. The results affirmed that the framework consistently delivered optimal decisions that maximized profit expectations. This showed the model's applicability to industry managers and decision-makers.

As a promising avenue for future publication, it was suggested to expand the model to comprise additional stakeholders not currently considered such as third-party carriers and distributors. Furthermore, the model could be enhanced to accommodate other aspects not related to profit such as sustainability and environmental aspects.

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