

ADVANCING GROUP EFFICIENCY EVALUATION IN DEA WITH NEGATIVE DATA: AN EMPIRICAL APPLICATION IN THE BANKING INDUSTRY

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Abstract. Data Envelopment Analysis (DEA) plays a pivotal role in assessing production unit efficiency. This study extends group efficiency assessment within the banking sector by utilizing the Modified Semi-Oriented Radial Measure (MSORM) model, specifically designed to handle negative data. It introduces two distinct efficiency definitions and develops models for their evaluation within these groups. Focusing on banks as decision-making units, the MSORM model delves into the intricacies of group efficiency. By effectively addressing negative data complexities, it enables a comprehensive evaluation of bank efficiency across various group frameworks. The study further examines the efficacy of efficiency definitions based on average and weakest performances within the MSORM framework. Empirical findings reveal significant variations in group efficiency assessment under different paradigms, highlighting the impact of the evaluation approach. This research contributes valuable insights into performance variations within the banking industry and aids in enhancing efficiency evaluations in banking systems.

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1. INTRODUCTION

The evaluation of production units has long been a pivotal concern in the fields of management and economics. This pursuit has not only shaped the theoretical landscape but also played a critical role in practical decision-making processes across various industries. The foundations of efficiency measurement were initially established by Debreu in 1951, setting the stage for a new era of analytical insights into organizational performance [17]. However, it was Farrell's subsequent refinement of this concept in 1957 that introduced the seminal notion of technical efficiency, marking a watershed moment in the development of performance evaluation methodologies [23]. This refinement paved the way for practical applications that were further propelled by the contributions of Charnes *et al.*, culminating in the landmark creation of Data Envelopment Analysis (DEA) in 1978 – an innovative approach designed to assess relative efficiency [10].

Keywords. Data Envelopment Analysis, negative data, group efficiency, performance, GCC banking systems.

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DEA stands as a formidable non-parametric method that has revolutionized the realm of evaluating the relative performance of Decision-Making Units (DMUs). It does so by accommodating the consideration of multiple inputs and outputs, allowing for a comprehensive assessment of a DMU's productivity. What sets DEA apart is its capacity to empower DMUs to autonomously define their objectives while adhering to specific constraints, thereby encapsulating the real-world complexities of decision-making processes. This unique framework goes beyond merely pinpointing efficient DMUs; it quantifies their level of efficiency and prescribes actionable strategies for enhancement. Simultaneously, DEA identifies and quantifies inefficiencies, providing invaluable insights for improvement, and establishes benchmarks that facilitate the identification of optimal performance for units that fall short [9, 10, 24, 51].

In the contemporary landscape, the pursuit of economic growth through enhanced productivity remains a fundamental national and global aspiration. To realize this ambition, the optimization of production factors and the efficient management of resources are of paramount importance. However, practical production environments are rarely homogenous and frequently exhibit significant variance due to a plethora of factors – geographical, temporal, political, and more [18]. This variance underscores the complexity of achieving consistent levels of efficiency across seemingly comparable DMUs.

The evolution of the DEA framework over the years has been marked by the emergence of various extensions and generalizations that cater to an increasingly diverse array of applications while refining the core principles. Notable works by Liu *et al.* [30], Seiford [46], and Emrouznejad *et al.* [20, 21] stand as testaments to the adaptability and utility of the DEA approach. The foundational concepts put forth by Farrell and advanced by luminaries like Charnes, Cooper, and Rhodes have persisted, positioning DEA as a steadfast non-parametric technique for evaluating DMUs featuring multiple inputs and outputs. Crucially, Farrell's introduction of the Production Possibility Set (PPS) laid a conceptual cornerstone, elucidating the underlying mechanics of the production function and further enriching DEA's analytical framework [37].

Central to the DEA methodology is the process of efficiency assessment, a pivotal undertaking that hinges on the calculation of relative efficiency. This calculation is achieved through the division of weighted outputs by weighted inputs [31], offering a quantifiable measure of performance. The cornerstone of this process is the establishment of the Production Possibility Set (PPS), an encapsulation of the inputs and outputs of DMUs. This set's boundaries define the efficiency frontier – a conceptual demarcation that distinguishes efficient DMUs from their inefficient counterparts.

However, translating this conceptual framework into actionable insights entails addressing a mathematical programming challenge. This challenge involves the optimization of the output-to-input ratio across all units while maintaining the consistency of this ratio. This optimization problem finds its mathematical formulation in the realm of linear programming – a powerful mathematical tool that enables the derivation of optimal solutions. The crux of the matter lies in maximizing this ratio, a task that can be accomplished through both primal and dual forms of linear programming. The resultant optimal objective value offers a robust measure of efficiency for the DMU under examination.

A pivotal stride in the evolution of DEA occurred with the introduction of the BCC model [8]. This model extended the bounds of efficiency calculation to variable returns to scale conditions, expanding DEA's capacity to handle more intricate production scenarios. Importantly, this model retained the essence of the T_{CCR} model while eliminating the unbounded ray principle a key improvement that bolstered the model's practicality and applicability. Further contributions came to the fore, including the introduction of the Additive DEA model by Charnes *et al.*, in 1985 [11]. This model, a non-radial one featuring constant returns to scale, provided an alternative analytical approach to DEA's foundational principles. Contrasting with this, Tone's SBM model introduced a unit-invariant non-radial model, enriching the spectrum of analytical tools at the disposal of efficiency analysts [51].

DEA's versatile applications extend beyond theory, finding utility across sectors such as transportation, services, and education [42]. Notably, Mahmoudabadi *et al.* highlight in the financial sector that Islamic banks surpassed conventional counterparts during the early COVID-19 crisis due to superior pre-crisis efficiency levels, particularly evident through DEA analysis. However, as is often the case in the realm of modeling and analysis,

theoretical frameworks encounter challenges when confronted with the intricacies of real-world scenarios. This rings true for DEA, where a fundamental assumption – that input and output variables possess positive values – can clash with practical contexts. Real-world scenarios, such as instances of bank losses or negative profits, challenge this foundational premise [47]. To address this dissonance, a spectrum of methodologies has emerged, each seeking to reconcile the theoretical constructs with the practicalities of negative data. These methodologies encompass multi-objective networks [36], negative-data-inclusive slacks measurement [52], and enhancement of inefficient units [1]. The crux of this article embarks on tracing the historical trajectory of addressing this crucial issue within the DEA literature.

The initial explorations into handling negative data within the DEA literature were marked by seminal contributions from Pastor and Ruiz [39] and Seiford and Zhu [47]. These works, while pragmatic, often resorted to a straightforward replacement of negative outputs with minimal positive values. The rationale underlying this approach was grounded in the understanding that the impact of these minimal positive values on performance or efficiency rates would be negligible. This pragmatic workaround laid the foundation for subsequent theoretical developments that would offer more nuanced and accurate solutions. Scheel [44] tackled this issue by employing two fundamental DEA models under the assumption of variable returns to scale. However, their proposed model would only be valid when all units have negative values in the target variable. If one unit had a positive value in the output variable while others had negative values, the Scheel model [44] could not be executed, likely diminishing the desirability of its output level. Scheel [45] introduced a new model in which negative inputs and negative outputs were considered as inputs. However, their proposed model does not apply when both inputs and outputs of DMUs are negative.

Portela *et al.* [40] conducted a study to aid bank managers in monitoring the performance of their branches. They introduced a Range Directional Model (RDM), which utilizes the range values of the distances between initial evaluated values and the best observed value of the variable as a direction vector. It can work with inputs or outputs having positive or negative values. They also proposed another improvement path called the Inverse Range Directional Model (IRDM), through which a unit undergoing improvement is oriented towards the nearest point on the efficiency frontier. In 2005, Pastor and Ruiz [39], based on the RDM model's direction vectors, made modifications and introduced the Modified Slack Based Measure (MSBM) for dealing with negative data. This approach classifies the real negative data (measured variables in relative scale such as undesirable outputs) within the model, thus being more constrained compared to the RDM model.

Building on the idea of Portela *et al.*'s RDM model, Sharp *et al.* [49] presented a modified model known as the MSBM Model. It employs a classification approach in modeling with actual negative data (variables measured in relative scale, like non-desirable orders), making it more specialized than the RDM model. Kerstens and De Woestyne [29] published an article on measuring radial inputs and negative outputs. Kazemi Matin and Ghahfarokhi [33] introduced a modified slack-based fuzzy measurement approach in two phases for efficiency measurement and target setting in data envelopment analysis with negative data.

Recognizing the limitations of these initial approaches, the journey towards addressing negative data has evolved into embracing more sophisticated models. Notable among these models are the Variant of the Radial Measure (VRM) presented by Cheng *et al.* [12] and the Semi-Oriented Radial Measure (SORM) proposed by Emrouznejad *et al.* [22]. These models represent significant advancements, each offering unique insights into handling negative data within the DEA framework. As an advancement of the SORM model, Matin *et al.* [34] introduced the Modified Semi-Oriented Radial Measure (MSORM). This model showcases improved proficiency evaluation and target setting in handling the intricacies of negative data, exemplifying the innovative approach driving the progression of DEA methodologies.

Kaffash *et al.* [28] proposed an adapted SORM model, utilizing directional distance functions, to address negative data issues in the domain of banking. Their approach was accompanied by a super efficiency model, capable of managing negative data instances and interfacing with SBM super in such scenarios. In a separate investigation, Kaffash *et al.* [28], having assessed the performance of 30 electric machinery companies in Taiwan, suggested a model that accommodates negative data points and aligns with the model's compatibility.

Furthermore, Babaie Asli *et al.* in 2021 delved into the realm of modeling negative and random data within a framework where inputs and outputs can encompass negative values as random data points [7].

The trajectory of efficiency assessment has expanded to encompass group evaluations, an arena ripe with potential yet marked by its own set of challenges. Cook *et al.* [15] explored subgroup efficiency, delving into the collective performance of subsets within the broader DMU spectrum. Team-based DEA models were introduced by Xia *et al.* [53], and Ang *et al.* [6] pioneered self-evaluation models for groups. Nonetheless, integrating negative data into group efficiency evaluations remains uncharted territory, necessitating the formulation of innovative methodologies that bridge theoretical constructs with practical intricacies.

In many preferred practical settings where data envelopment analysis models are applied, decision making units are categorized into multiple groups, each characterized by the same technology derived from similar resources [3, 18]. Given the constraints of resources and the tensions between individual interests and general benefits, evaluating groups becomes essential for managers [14, 16].

Grouping DMUs brings forth two major issues in the context of DEA. First, how these groups should be treated as decision-making units, and second, how should different scores be allocated to each decision-making unit [6]. The initial concept of group formation was explored by Banker and Morey [8]. It allowed certain variables to enable comparison of each unit within its group and subgroup. Cook *et al.* [15] expanded this idea by introducing common weights to the entire group, determining efficiency scores for each subgroup within the group. Cook and Greene [13] examined efficiency differences among DMUs within a group and compared efficiencies among groups using the Malmquist index. Afsharian *et al.* [4] extended Cook and Greene's [13] approach, focusing on comparing the performance of managerial groups under the scenario of centralized management expansion using a DEA approach. Rezaee and Karimdadi [41] investigated 288 hospitals grouped by province, analyzing the impact of geographical location on their efficiency. Shahbazifar *et al.* [48] introduced a novel group assessment method for two-stage production systems and presented several new DEA network models for both average and weakest performance strategies.

A literature review on DEA-based bank evaluation indicates that existing models primarily emphasize individual bank assessment. In a country's banking system, subsidiary banks are established in various regions to offer more services to customers and generate greater benefits. Despite centralized management, each bank independently serves customers without relying on peers. As a result, each bank within a country is considered DMU, and then a banking chain within that country can be treated as a group consisting of multiple DMUs.

Xia *et al.* [53] explored a team-based DEA method, treating all DMUs as a team that might not individually achieve maximum efficiency but can optimize team-based indices. Ang *et al.* [6] developed self-assessment models for groups of Taiwanese hotels using two approaches: average performance, which considers group efficiency as the average of its members, and weakest performance, which highlights group efficiency using the poorest individual performance.

In most DEA models, the focus is on ranking individual DMUs, all of which utilize relative data with positive values. In other words, these approaches emphasize the performance of individual DMUs, disregarding the group to which the decision-making unit belongs. When comparing a group of production units with another group, considering negative data, and when all units within each group strive to improve, a new method for evaluating groups and calculating their efficiency scores individually must be devised.

This article embarks on a compelling exploration of the evaluation of DMUs group efficiency within the backdrop of negative data – a critical aspect that has remained underexplored, yet holds significant real-world implications. At its core, this investigation is grounded in the MSORM, a novel approach designed to effectively address the challenges posed by negative data. The exploration also encompasses the categorization of independent DMUs, harnessing negative data to elucidate two distinct group efficiency strategies: average performance and weakest performance. As we delve into the subsequent sections, the mechanics of the MSORM model take center stage, meticulously outlining both its linear and nonlinear forms, offering a comprehensive grasp of its underlying principles. Moreover, moving beyond theoretical discourse, the article delves into the practical application of the MSORM model, providing insights into the evaluation of group efficiency through the two proposed strategies.

The main contributions of this study are:

- Advanced Group Efficiency Evaluation: extending group efficiency evaluation within the GCC¹ banking sector, this research employs the innovative MSORM model. By effectively accommodating the complexities of negative data, the model enables a comprehensive assessment of bank efficiency across diverse group paradigms.
- New Efficiency Evaluation Models: this study introduces two novel efficiency definitions and develops models for self-evaluation within groups. These additions refine the approach to performance assessment, providing a more nuanced perspective on group efficiency.
- Empirical Insights into Evaluation Approaches: through empirical analysis, the study examines the efficacy of average and weakest performance efficiency definitions. This analysis sheds light on the varying impacts of different evaluation approaches on the understanding of group efficiency.
- Enriched Understanding of GCC Banking Efficiency: by delving into the intricate realm of group efficiency evaluation and thoughtfully considering real-world complexities, this research enhances the understanding of performance variations within GCC banking systems. These insights contribute valuable information to decision-making processes in the sector.

In conclusion, the landscape of efficiency evaluation within the realm of production units has undergone a remarkable evolution. Beginning from the conceptual foundations established by Debreu and Farrell, and culminating in the transformative contributions of Charnes *et al.* [10] DEA has emerged as a pivotal tool for evaluating relative efficiency. Its adaptability, demonstrated by its multifaceted applications and extensions, underscores its relevance across diverse industries. However, the practical complexities of real-world scenarios, particularly the presence of negative data, have necessitated the development of advanced methodologies like the MSORM model. Throughout this journey, the article not only traces the historical trajectory but also offers insights into the future of efficiency evaluation, laying the groundwork for a more nuanced and accurate understanding of organizational performance in complex and dynamic environments.

The subsequent sections of this paper are structured as follows. Section 2 delves into the MSORM model, encompassing the presentation of its standard form – specifically, the input-oriented multiplicative form – in both linear and fractional manifestations. In Section 3, an exploration of group efficiency evaluation unfolds through two distinct perspectives: average efficiency and weakest performance. This analysis takes into account negative data and leverages the application of the MSORM model. Moving forward to Section 4, the paper showcases an empirical application of the novel proposed model, employed to evaluate group efficiency within the banking industry of GCC members. Section 5 serves as the conclusion, summarizing the study’s findings and offering suggestions for potential directions of future research endeavors.

2. EFFICIENCY EVALUATION OF PRODUCTION UNITS CONSIDERING NEGATIVE DATA USING THE MSORM MODEL

The methodologies of data envelopment analysis often make use of non-negative input and output data for modeling purposes, despite the presence of negative data in real-world scenarios. The MSORM model, introduced by Matin *et al.* [34], offers a modified version of the SORM model that introduces a linear programming model for the efficiency evaluations of the production units with negative inputs and outputs.

Let’s consider a scenario with n observations, where $(\mathbf{x}_j, \mathbf{y}_j)$ denotes a typical observed DMU, with $j \in J = \{1, \dots, n\}$, each utilizing m inputs (x_{ij}) to generate s outputs (y_{rj}) . It is assumed that $\mathbf{x}_j \neq \mathbf{0}$ and $\mathbf{y}_j \neq \mathbf{0}$, where the bold symbol denotes a zero vector of suitable dimension. Inspired by our empirical application, we further assume that the outputs may contain negative values. The extension for the case of negative inputs is straightforward.

¹The Gulf Cooperation Council (GCC), is a trade bloc involving the six Arab states of the Persian Gulf with many economic and social objectives (for full details see www.gcc-sg.org).

The evaluation of DMU_k involves the input-output vector $(\mathbf{x}_k, \mathbf{y}_k = (\mathbf{y}_k^P, \mathbf{y}_k^N))$, where $\mathbf{y}_k^P$ represents the positive output values, and $\mathbf{y}_k^N$ contains negative outputs, which is decomposed into $\mathbf{y}_k^1$ and $\mathbf{y}_k^2$ parts as follows:

$$y_{rk}^1 = \begin{cases} y_{rk} & \text{if } y_{rk} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad y_{rk}^2 = \begin{cases} -y_{rk} & \text{if } y_{rk} < 0 \\ 0 & \text{otherwise.} \end{cases} \quad (2.1)$$

Building upon the Production Possibility Set (PPS) framework of the SORM model [22], T_{SORM} for $(\mathbf{x}, \mathbf{y}^P, \mathbf{y}^1, \mathbf{y}^2)$ is defined as follows:

$$T_{\text{SORM}} = \left\{ (\mathbf{x}, \mathbf{y}^P, \mathbf{y}^1, \mathbf{y}^2) \mid \sum_{j \in J} \mathbf{x}_j \lambda_j \leq \mathbf{x}, \sum_{j \in J} \mathbf{y}_j^P \lambda_j \geq \mathbf{y}^P, \sum_{j \in J} \mathbf{y}_j^1 \lambda_j \geq \mathbf{y}^1, \sum_{j \in J} \mathbf{y}_j^2 \lambda_j \leq \mathbf{y}^2, \lambda_j \geq 0 \forall j \in J \right\}. \quad (2.2)$$

Utilizing the proposed PPS, the input-oriented radial model for evaluating DMU_k is formulated as follows:

$$\begin{aligned} \min \quad & \theta_k \\ \text{s.t.} \quad & (\theta_k \mathbf{x}_k, \mathbf{y}_k^P, \mathbf{y}_k^1, \mathbf{y}_k^2) \in T_{\text{SORM}}. \end{aligned} \quad (2.3)$$

The proportional multiplication of inputs in θ_k ensures a favorable approach to reduce input consumption while maintaining consistent output levels. The extended formulations of the above linear model can be represented as follows:

$$\begin{aligned} \theta_k^* &= \min_{\lambda, \theta_k} \theta_k \\ \text{s.t.} \quad & \sum_{j \in J} \lambda_j x_{ij} \leq \theta_k x_{ik} \quad \forall i \in I \\ & \sum_{j \in J} \lambda_j y_{rj}^P \geq y_{rk}^P \quad \forall r \in O' \\ & \sum_{j \in J} \lambda_j y_{rj}^1 \geq y_{rk}^1 \quad \forall r \in O'' \\ & \sum_{j \in J} \lambda_j y_{rj}^2 \leq y_{rk}^2 \quad \forall r \in O'' \\ & \lambda_j \geq 0 \quad \forall j \in J \\ & \theta_k \text{ free.} \end{aligned} \quad (2.4)$$

Here, I represents the set of input indicators, O' denotes the set of output indicators related to $\mathbf{y}_k^P$, and O'' corresponds to the set of negative component output indicators $(\mathbf{y}_k^1, \mathbf{y}_k^2)$. Note that O' and O'' are nonempty and disjoint. The optimal value of the objective function θ_k^* is constrained to be less than or equal to 1. As a result, $(\theta_k^* \mathbf{x}_k, \mathbf{y}_k^P, \mathbf{y}_k^1, \mathbf{y}_k^2)$ serves as the target point for the evaluated DMU_k , showcasing improvements in input values.

Given the linearity of model (2.4), deriving its dual yields the multiplicative input-oriented SORM model as an alternative standard form. This form allows for the consideration of efficiency by incorporating input and output weights:

$$\begin{aligned} \theta_k &= \max_{\mathbf{u}, \mathbf{v}} \sum_{r \in O'} u_r y_{rk}^P + \sum_{r \in O''} u_r^1 y_{rk}^1 - \sum_{r \in O''} u_r^2 y_{rk}^2 \\ \text{s.t.} \quad & \sum_{i \in I} v_i x_{ik} = 1 \end{aligned}$$

$$\begin{aligned} \sum_{r \in O'} u_r y_{rj}^P + \sum_{r \in O''} u_r^1 y_{rj}^1 - \sum_{r \in O''} u_r^2 y_{rj}^2 - \sum_{i \in I} v_i x_{ij} &\leq 0, \quad \forall j \in J \\ u_r &\geq 0, \quad \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \quad \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \end{aligned} \quad (2.5)$$

In this model, both input and output values are weighted. Notably, the weights assigned to the negative part of the output vector (O'') carry a negative sign. For enhanced economic interpretation, the fractional formulation of model (2.5) can be stated as follows. However, for computational purposes, we can apply the linear model (2.5):

$$\begin{aligned} \theta_k &= \max_{\mathbf{u}, \mathbf{v}} \frac{\sum_{r \in O'} u_r y_{rk}^P + \sum_{r \in O''} u_r^1 y_{rk}^1 - \sum_{r \in O''} u_r^2 y_{rk}^2}{\sum_{i \in I} v_i x_{ik}} \\ \text{s.t.} \quad &\frac{\sum_{r \in O'} u_r y_{rj}^P + \sum_{r \in O''} u_r^1 y_{rj}^1 - \sum_{r \in O''} u_r^2 y_{rj}^2}{\sum_{i \in I} v_i x_{ij}} \leq 1 \quad \forall j \in J \\ &u_r \geq 0, \quad \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \quad \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \end{aligned} \quad (2.6)$$

The objective function maximizes the ratio of weighted outputs to weighted inputs for DMU_k , balancing positive and negative outputs against inputs to find the most efficient weighting values. Constraints ensure that the efficiency of all observed units does not exceed a maximum value of unity.

3. GROUP EFFICIENCY EVALUATION USING THE MSORM MODEL

The seminal work by Cook *et al.* [14] marked the initial incorporation of the concept of group efficiency evaluation into the framework of DEA. Subsequently, Ang *et al.* [6] conducted a comprehensive investigation of this concept and formulated models for quantifying group efficiency. They introduced two distinct approaches: the average and weakest performance, aimed at assessing group efficiency. In this section, we delve into an exploration of both these approaches, while concurrently accounting for negative data within the MSORM model.

3.1. Group efficiency *via* average efficiency using the MSORM model

The work of Ang *et al.* [6] has directed its focus towards assessing the efficiency of hotel chains in Taiwan, specifically those operated under the management of a central team. The primary emphasis of the study revolves around evaluating the overall performance of hotel chains. In this context, two distinct definitions of group efficiency have been introduced, revolving around the measurement of average performance and weakest performance [6]. Notably, instead of aggregating individual output efficiencies into input efficiencies, the approach undertaken is to aggregate output efficiencies into input efficiencies. This strategic deviation serves to encapsulate the nuanced dynamics inherent in the operations of the entire hotel chain.

This study's analysis encompasses two primary dimensions: the consideration of negative data and the extension of models to encapsulate these intricacies. To delve into this extension, let us outline the methodology:

Let us consider n DMUs grouped into K categories, with D_k as the index set in each group k . For every group k , outputs and inputs are denoted by y_{rd_k} and x_{id_k} respectively. For each under evaluation group k ($k = 1, \dots, K$), the group efficiency is denoted by E_k^A , representing the measurement of relative performance. To further elaborate, the process for deriving the average efficiency for each group k is achieved through the introduction of the following fractional programming model:

$$\begin{aligned} E_k^A &= \max_{\mathbf{u}, \mathbf{v}} \frac{\sum_{r \in O'} \sum_{d_k \in D_k} u_r y_{rd_k}^P + \sum_{r \in O''} \sum_{d_k \in D_k} u_r^1 y_{rd_k}^1 - \sum_{r \in O''} \sum_{d_k \in D_k} u_r^2 y_{rd_k}^2}{\sum_{i \in I} \sum_{d_k \in D_k} v_i x_{id_k}} \\ \text{s.t.} \quad &\frac{\sum_{r \in O'} u_r y_{rd_j}^P + \sum_{r \in O''} u_r^1 y_{rd_j}^1 - \sum_{r \in O''} u_r^2 y_{rd_j}^2}{\sum_{i \in I} v_i x_{id_j}} \leq 1 \quad \forall d_j \in D_j, \forall j \in J \end{aligned}$$

$$\frac{\sum_{r \in o'} \sum_{d_k \in D_k} u_{rk} y_{rd_k}^P + \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^1 y_{rd_k}^1 - \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^2 y_{rd_k}^2}{\sum_{i \in I} \sum_{d_k \in D_k} v_{ik} x_{id_k}} \leq 1$$

$$u_r \geq 0, \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \quad (3.1)$$

The variables u_{rk} , u_{rk}^1 , u_{rk}^2 represent the weights assigned to the r -th output of the k -th group, while v_{ik} signifies the weight assigned to the i -th input of the DMUs within group k . It is worth noting that the constraint $\frac{\sum_{r \in o'} \sum_{d_k \in D_k} u_{rk} y_{rd_k}^P + \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^1 y_{rd_k}^1 - \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^2 y_{rd_k}^2}{\sum_{i \in I} \sum_{d_k \in D_k} v_{ik} x_{id_k}} \leq 1$ can be superfluous and may be omitted from calculations.

By incorporating this model, Ang (2008) navigates the complexities associated with group efficiency evaluation in the presence of negative data [6]. This approach not only contributes to a more holistic understanding of the hotel chain's performance but also showcases the adaptability of the methodology to account for the practical nuances of real-world scenarios.

The performance of a group is inherently influenced by its constituents. High-performing members contribute positively to the overall group performance; whereas the negative effects associated with underperforming members need to be offset by positive contributions. Additionally, the efficiency of individual constraints is constrained to be no greater than 1. To ensure linearity and mitigate non-linearity, we shall reformulate model (3.1) as follows:

$$E_k^A = \max_{\mathbf{u}, \mathbf{v}} \sum_{r \in o'} \sum_{d_k \in D_k} u_{rk} y_{rd_k}^P + \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^1 y_{rd_k}^1 - \sum_{r \in o''} \sum_{d_k \in D_k} u_{rk}^2 y_{rd_k}^2$$

s.t.

$$\sum_{r \in o'} u_{rk} y_{rd_j}^P + \sum_{r \in o''} u_{rk}^1 y_{rd_j}^1 - \sum_{r \in o''} u_{rk}^2 y_{rd_j}^2 - \sum_{i \in I} v_{ik} x_{id_j} \leq 0 \quad \forall d_j \in D_j, \forall j \in J$$

$$\sum_{i \in I} \sum_{d_k \in D_k} v_{ik} x_{id_k} = 1$$

$$u_r \geq 0, \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \quad (3.2)$$

Upon solving model (3.2) optimally, the average efficiency of each group can be articulated as follows:

$$E_k^{A*} = \sum_{r \in o'} \sum_{d_k=1}^{D_k} u_{rk}^* y_{rd_k}^P + \sum_{r \in o''} \sum_{d_k=1}^{D_k} u_{rk}^{1*} y_{rd_k}^1 - \sum_{r \in o''} \sum_{d_k=1}^{D_k} u_{rk}^{2*} y_{rd_k}^2. \quad (3.3)$$

Additionally, the efficiency of individual DMUs within group k , deemed optimal at the highest level, can be expressed as follows:

$$e_{d_k}^{A*} = \frac{\sum_{r \in o'} u_{rk}^* y_{rd_k}^P + \sum_{r \in o''} u_{rk}^{1*} y_{rd_k}^1 - \sum_{r \in o''} u_{rk}^{2*} y_{rd_k}^2}{\sum_{i \in I} v_{ik}^* x_{id_k}} \quad \forall d_k \in D_k. \quad (3.4)$$

3.2. Group efficiency using weakest performance approach with the MSORM model

The evaluation of group performance takes a distinctive route by identifying the member that exhibits the lowest performance within the operational context. The optimal efficiency for each group, denoted as E_k^{w*} , is ascertained using the weakest performer's approach, utilizing the assumptions outlined in model (3.1):

$$E_k^{w*} = \max_{\mathbf{u}, \mathbf{v}} E_k^w = \max_{\mathbf{u}, \mathbf{v}} \min_{d_k} \frac{\sum_{r \in o'} u_{rk} y_{rd_k}^P + \sum_{r \in o''} u_{rk}^1 y_{rd_k}^1 - \sum_{r \in o''} u_{rk}^2 y_{rd_k}^2}{\sum_{i \in I} v_{ik} x_{id_k}}$$

s.t.

$$\frac{\sum_{r \in O'} u_{rk} y_{rd_j}^P + \sum_{r \in O''} u_{rk}^1 y_{rd_j}^1 - \sum_{r \in O''} u_{rk}^2 y_{rd_j}^2}{\sum_{i \in I} v_{ik} x_{id_j}} \leq 1 \quad \forall d_j \in D_j, \forall j \in J$$

$$u_r \geq 0, \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \quad (3.5)$$

In model (3.5), the set of weights $(u_{rk}, u_{rk}^1, u_{rk}^2, v_{ik})$ constitutes a collective set utilized for calculating the efficiency of DMUs within group k . Given that DMUs within a group share uniform conditions, the weights within a group remain consistent. Consequently, the desired optimal group efficiency at the optimal solution is defined as follows:

$$E_k^{w*} = \min_{d_k} \frac{\sum_{r \in O'} u_{rk}^* y_{rd_k}^P + \sum_{r \in O''} u_{rk}^{*1} y_{rd_k}^1 - \sum_{r \in O''} u_{rk}^{*2} y_{rd_k}^2}{\sum_{i \in I} v_{ik}^* x_{id_k}}. \quad (3.6)$$

The computation of efficiency for individual DMUs within group k is as follows:

$$e_{d_k}^{w*} = \frac{\sum_{r \in O'} u_{rk}^* y_{rd_k}^P + \sum_{r \in O''} u_{rk}^{*1} y_{rd_k}^1 - \sum_{r \in O''} u_{rk}^{*2} y_{rd_k}^2}{\sum_{i \in I} v_{ik}^* x_{id_k}} \quad \forall d_k \in D_k. \quad (3.7)$$

Note that the minimax model (3.5) is nonlinear. To linearize the objective function, an auxiliary variable δ_k can be introduced for group k :

$$\delta_k = \min_{d_k} \frac{\sum_{r \in O'} u_{rk} y_{rd_k}^P + \sum_{r \in O''} u_{rk}^1 y_{rd_k}^1 - \sum_{r \in O''} u_{rk}^2 y_{rd_k}^2}{\sum_{i \in I} v_{ik} x_{id_k}}. \quad (3.8)$$

Consequently, the formulation of model (3.5) can be expressed as follows:

$$\begin{aligned} E_k^w &= \max_{\mathbf{u}, \mathbf{v}} \delta_k \\ \text{s.t} \quad & \sum_{r \in O'} u_{rk} y_{rd_j}^P + \sum_{r \in O''} u_{rk}^1 y_{rd_j}^1 - \sum_{r \in O''} u_{rk}^2 y_{rd_j}^2 - \delta_k \sum_{i \in I} v_{ik} x_{id_j} \geq 0 \\ & \sum_{r \in O'} u_{rk} y_{rd_j}^P + \sum_{r \in O''} u_{rk}^1 y_{rd_j}^1 - \sum_{r \in O''} u_{rk}^2 y_{rd_j}^2 - \sum_{i \in I} v_{ik} x_{id_j} \leq 0 \quad \forall d_j \in D_j, \forall j \in J \\ & \sum_{r \in O'} u_{rk} + \sum_{r \in O''} u_{rk}^1 + \sum_{r \in O''} u_{rk}^2 = 1 \\ & u_r \geq 0, \forall r \in O', \quad u_r^1, u_r^2 \geq 0, \forall r \in O'', \quad v_i \geq 0, \quad \forall i \in I. \end{aligned} \quad (3.9)$$

Given the unit invariance property of the multiplier model (3.5), we can introduce the third constraint (3.8) to normalize the output weights. This normalization not only facilitates computational efficiency but also mitigates potential weight discrepancies [6]. It is worth noting that various nonlinear constrained optimization algorithms are applicable to solve this model. Specifically, the iterative bisection search optimization method outlined in [6] emerges as a viable option for computing optimal input and output weights. Notably, this method incorporates δ_k constrained within the range $0 \leq \delta_k \leq 1$ as parameter.

4. EMPIRICAL STUDY ON THE GCC BANKING SYSTEM

In the current economic landscape, the role of financial systems, money and capital markets, and consequently, financial and credit institutions led by banks, holds significant importance. Faced with intensifying competition among financial institutions, enhancing competitiveness necessitates the execution of performance evaluation and operational efficiency analysis for each branch. Scholars have been concentrating on the assessment of bank performance from the past until the present [34].

TABLE 1. Descriptions of variables research.

Variables	Type	Descriptions	Reference
Assets	Positive	All properties and rights that have monetary value.	Mohamed Shahwan and Kaba [35]; Henriques <i>et al.</i> [27]
Equity	Positive	The financial value remaining for the owner post-asset sale and debt settlement.	Matin <i>et al.</i> [34]; Aghimien <i>et al.</i> [2]; Kaffash <i>et al.</i> [28]
Deposit	Positive	A type of bank account, such as current or savings, where upon opening, the customer deposits a sum of money with the bank.	Emrouznejad and Anouze [19]; Saleh <i>et al.</i> [43]
Loan	Positive	Money provided by a bank to a customer, received with interest after a certain period.	Paradi and Zhu [38]; Fukuyama <i>et al.</i> [25, 26]
Net Income	Positive and Negative	It is the residual profit for the bank after subtracting the total cost of goods sold, operational expenses, and taxes from the total revenues during a specific period.	Matin <i>et al.</i> [34]; Sillah <i>et al.</i> [50]; Kaffash <i>et al.</i> [28]

TABLE 2. Descriptive statistics of inputs and outputs (in million USD).

Variable (million \$)	Min	Max	Mean	Median	St. dev
<i>Input</i>					
Assets	291.26	2 454 060.65	163 943.08	68 078.94	310 777.72
Equity	50 296.43	12 862 923.3	2 446 943.3	1 089 121.33	297 023.97
Deposit	21 328.60	6 166 019.6	10 740 279.7	4 226 310.11	14 510 075.7
<i>Outputs</i>					
Loan	2184.49	49 306 629.2	8 035 207.75	3 275 514.69	11 010 079.2
Net Income	−188 453.94	1 849 893.18	229 876.44	78 347.83	387 881.83

In recent years, GCC banks have undergone consistent evaluations utilizing diverse data envelopment analysis (DEA) models. Significant contributions have been made by researchers such as Emrouznejad and Anouze [19], Mohamed Shahwan, and Kaba [35], Sillah and Harrathi [50], Aghimien *et al.* [2], and Alsharif [5]. These studies have considered multiple inputs and outputs for efficiency assessment.

Expanding upon these works and drawing upon available data, this section incorporates three inputs – Assets, Equity, and Deposits – along with two outputs – Loans and Net Income – for the efficiency assessment. Additionally, Table 1 provides the background information and interpretation of these variables.

Applying the methodologies outlined in models (3.2) and (3.9), we employ them to assess the performance of banks in the GCC member countries for the year 2011. The dataset encompasses 6 countries and 82 affiliated banks. Additionally, we employ the results obtained from the traditional model (2.4) for comparative analysis.

For the United Arab Emirates, there are 22 banks, and for both Oman and Saudi Arabia, there are 11 bank branches each. Bahrain boasts 18 banks, Kuwait has 13 branches, and Qatar encompasses 7 affiliated banks. These banks are primarily situated in the major cities of their respective countries. Table 2 presents the descriptive statistics of the dataset.

It is worth noting that the last output variable Net Income can encompass both positive and negative values across branches. These variables reflect the bank's quality and influence its banking advantage, a factor of particular concern to managerial decision-making.

Table 3 displays the group efficiency outcomes for banks in six GCC countries using both the average performance and weakest performance approaches. As evident, these two approaches yield distinct results. According to the average performance approach, Qatar's banks achieve relatively high efficiency, exceeding 0.80, making it

TABLE 3. Group efficiencies of GCC banks average, weakest and traditional approaches.

Group	DMUs	EFF	AGE	EFF	WGE	MODEL (2.5)	AVERAGE
AE	1	0.96	0.78	0.81	0.66	1	0.71
	2	0.12		0.13		0.2	
	3	0.84		0.71		0.88	
	4	0.37		0.32		0.44	
	5	0.56		0.50		0.57	
	6	0.75		0.56		0.76	
	7	0.78		0.74		0.84	
	8	0.77		0.60		0.82	
	9	0.01		0.13		0.88	
	10	0.76		0.47		0.79	
	11	0.90		0.66		0.93	
	12	0.31		0.41		0.5	
	13	0.73		0.69		0.87	
	14	0.53		0.48		0.56	
	15	0.86		0.70		0.95	
	16	0.87		0.76		0.93	
	17	0.47		0.56		0.62	
	18	0.46		0.50		0.55	
	19	0.14		0.13		0.21	
	20	0.95		0.80		0.95	
	21	0.79		0.69		0.88	
	22	0.05		0.24		0.53	
BH	23	1	0.56	1	0.55	1	0.55
	24	0.70		0.72		0.79	
	25	0.70		0.72		0.81	
	26	0.24		0.27		0.35	
	27	0.34		0.23		0.62	
	28	0.53		0.53		0.57	
	29	0.64		0.48		0.88	
	30	0.41		0.45		0.45	
	31	0.57		0.66		0.66	
	32	0.35		0.28		0.45	
	33	0.55		0.58		0.71	
	34	0.29		0.05		1	
	35	0.33		0.37		0.37	
	36	0.37		0.38		0.38	
	37	0.06		0.06		0.06	
	38	0.53		0.46		0.85	
	39	0.06		0.07		0.07	
	40	0.01		0.01		0.01	

the top-performing group in terms of average performance during 2011. Oman's banks secure the second rank with a negligible difference of 0.86. Therefore, Qatar and Oman emerge as the top two groups throughout the banking period, with Qatar slightly ahead.

On the other hand, Bahrain demonstrates the lowest group efficiency in terms of the average performance approach. Kuwait is another underperforming group, standing only slightly better than Bahrain. Saudi Arabia and the United Arab Emirates, with a minor margin separating them, occupy intermediate positions. The

TABLE 3. continued.

Group	DMUs	EFF	AGE	EFF	WGE	MODEL (2.5)	AVERAGE
KW	41	0.16	0.60	0.17	0.57	0.19	0.53
	42	0.77		0.80		0.8	
	43	0.72		0.69		0.74	
	44	0.71		0.65		0.75	
	45	0.60		0.61		0.64	
	46	0.65		0.60		0.66	
	47	0.05		0.05		0.05	
	48	0.01		0.05		0.55	
	49	0.45		0.34		0.53	
	50	0.10		0.07		0.09	
	51	0.72		0.70		0.73	
	52	0.48		0.44		0.5	
	53	0.62		0.61		0.73	
OM	54	1	0.86	0.64	0.52	1	0.92
	55	1		0.24		1	
	56	0.85		0.56		0.88	
	57	0.93		0.38		1	
	58	0.69		0.55		0.89	
	59	0.96		0.93		1	
	60	0.91		0.50		0.92	
	61	0.83		0.63		0.88	
	62	0.62		0.64		1	
	63	0.69		0.51		0.79	
	64	0.53		0.57		0.77	
QA	65	0.94	0.87	0.76	0.73	0.95	0.85
	66	0.84		0.75		0.88	
	67	0.82		0.62		0.82	
	68	0.94		0.74		1	
	69	0.58		0.52		0.58	
	70	1		0.78		1	
	71	0.72		0.72		0.75	
SA	72	1	0.82	1	0.66	1	0.82
	73	0.77		0.57		0.77	
	74	0.63		0.39		0.63	
	75	0.57		0.25		0.73	
	76	0.90		0.65		0.93	
	77	0.86		0.67		0.99	
	78	0.65		0.49		0.69	
	79	0.77		0.68		0.86	
	80	0.99		0.70		1	
	81	0.87		0.59		0.87	
	82	0.50		0.39		0.57	

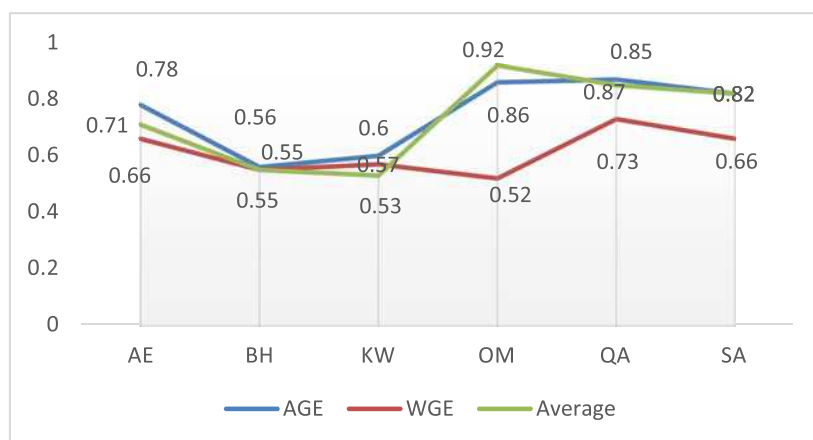


FIGURE 1. GCC banks by group efficiencies on average, weakest and traditional performances.

weakest performance approach identifies Qatar as the best-performing group among these six countries. Saudi Arabia and the UAE rank second with a group efficiency of 0.66 based on the weakest performance criterion.

By comparing the group efficiency outcomes based on the two definitions, Qatar emerges as the leading group. Oman ranks among the top groups in terms of average performance, while being among the weaker performers in terms of the weakest performance approach. Thus, Oman's banks are positioned favorably in the context of average performance evaluation, whereas they exhibit lower efficiency when considered under the weakest performance perspective. Figure 1 graphically illustrates the group efficiency of banks in the six GCC countries based on both average and weakest performance criteria.

In Figure 1, there is a significant difference in group efficiency scores for some countries between two approaches. For instance, the group efficiency of banks in Oman, using the average performance approach, is 0.86, securing the second rank among the GCC countries. However, under the weakest performance approach, this figure decreases to 0.52, ranking Oman last among GCC countries. This difference can be attributed to the dispersion in efficiency scores of Omani banks, as several banks in the country achieved the highest and lowest efficiency scores, a pattern reflected in both modeling approaches.

Generally, the group efficiency score obtained from the weakest performance approach is less than or equal to the average performance approach. Considering the optimistic perspective in data envelopment analysis, the weakest performance approach could yield better evaluation results.

It is noteworthy that Banks 23 (First Bank of Bahrain) and 72 (First Bank of Saudi Arabia) were identified as efficient using all three methods, indicating their superior performance among GCC banks. Additionally, two Omani banks (Banks 54 and 55) and one Qatari bank (Bank 70) were identified as efficient using both the foundational model (2.5) and the average performance model. However, employing the weakest performance approach, these banks exhibit inefficiency, indicating inferior performance compared to Banks 23 and 72. The greater number of efficient banks in model (2.5) compared to other models suggests that this model has less discriminatory power.

Moreover, the traditional efficiency computation using the foundational DEA model (2.5) for banks, as illustrated in the last two columns of Table 3, has introduced a noteworthy departure. In this method, the group efficiency score is determined by utilizing the average efficiency score computed for banks within each country.

Comparative analysis of the rankings derived from the new methodologies with those obtained through the traditional approach reveals significant disparities in the assessment of bank group efficiency. In the ranking by the model (2.5), for most banking groups (excluding Kuwait branches), at least one efficient unit has been iden-

tified, a feature absent in group-based approaches. This phenomenon may stem from the higher discriminatory power of the group-based approach, given that if the group efficiency score is 1 in either the average or weakest performance approach, then all branches within the group will receive a score of 1. Consequently, in contrast to the group-based approach, the traditional approach utilizing model (2.5) indicates an overestimation error.

As evident in the results and also shown in Figure 1, the relative ranking of country groupings remains preserved in the average and weakest performance approaches, yet experiences fluctuations under the traditional model (2.5). For instance, for Oman, the model (2.5) suggests a score of 0.92 for the banking sector, surpassing both the average group efficiency score of 0.86 and the weakest group efficiency score of 0.52, indicating a higher efficiency level. Conversely, for Kuwait, the traditional approach yields a lower score of 0.53 compared to both the average group efficiency of 0.6 and the weakest group efficiency of 0.57.

Overall, this study advocates for the stability of group-based ranking and proposes it as a better foundation for decision-making and the adoption of performance enhancement strategies within the banking sector.

The comparison accentuates the necessity of employing diverse evaluation methodologies to comprehensively capture the multifaceted aspects of bank performance and resilience. While the traditional approach furnishes a fundamental assessment, the average efficiency and weakest performance methods provide supplementary insights, enabling strategic decision-making and policy development within the GCC banking landscape. Leveraging insights from varied evaluation frameworks, policymakers, regulators, and banking stakeholders can enact targeted interventions and strategies aimed at nurturing sustainable growth and stability across the financial domain of the region.

5. CONCLUSION

In this study, we embarked on the development of a robust group efficiency assessment framework through the lens of Data Envelopment Analysis (DEA), employing the innovative Modified Semi-Oriented Radial Measure (MSORM) model. Our focus was on evaluating the efficiency of banks across the six member nations of the GCC for the year 2011. Within the realm of group evaluation, we introduced two distinct definitions of group efficiency and extended our self-formulated group evaluation models, thereby expanding the applicability of the MSORM model beyond individual Decision Making Units (DMUs).

The two efficiency definitions – average performance and weakest performance – provided nuanced perspectives on group efficiency, offering insights into the dynamics of the GCC banking sector. The average performance definition gauged group efficiency as the amalgamation of individual member performances, while the weakest performance definition accentuated the impact of the weakest individual member's performance on the overall efficiency assessment. By introducing these definitions, we transcended the realm of traditional individual-centric evaluations and ventured into the unexplored territory of holistic group efficiency analysis.

As we delved into the application of our newly formulated models, rooted in the MSORM framework, we unearthed compelling findings that underscore the importance of diverse efficiency assessment approaches. These findings can be summarized as follows. (i) Comprehensive Group Insight: The integration of the MSORM model within the group efficiency framework allowed for a comprehensive understanding of how different banks collectively contribute to the efficiency landscape. The consideration of both average and weakest performance definitions provided a multi-dimensional view that takes into account both higher and lower-performing members. (ii) Identification of Disparities: Through the distinct lenses of average and weakest performance, we unveiled significant disparities in the efficiency rankings of GCC member banks. This granular insight contributes to a more precise identification of areas requiring improvement, enabling tailored strategies for enhancing overall group efficiency. (iii) Strategic Decision Support: By offering a holistic evaluation of group efficiency, our approach equips decision-makers with a richer toolkit for strategic planning. The nuanced perspectives provided by the two efficiency definitions empower stakeholders to make more informed choices, whether in terms of resource allocation, performance benchmarking, or collaborative initiatives.

Building on the foundation laid by this study, several avenues for future research present themselves:

Exploring Imprecise Data: consider extending the group efficiency evaluation framework to accommodate imprecise data types, such as interval or ratio data. This expansion would align the model with real-world scenarios where uncertainties and variability often play a significant role. Exploring how imprecision affects group efficiency rankings could offer novel insights.

Network Production Systems: extend the developed group efficiency assessment methodology to network production systems, where the interactions and interdependencies between units are more complex. This could involve adapting the MSORM model to account for the intricate relationships between different DMUs within the network, leading to a more comprehensive evaluation of group efficiency.

Comparative Study Across Industries: apply the MSORM-based group efficiency assessment across different industries to discern patterns and variations in efficiency dynamics. Comparative studies could shed light on sector-specific challenges and opportunities, contributing to a deeper understanding of how the approach performs in diverse contexts.

Dynamic Analysis: consider incorporating temporal dimensions into the group efficiency evaluation, enabling the analysis of efficiency trends over time. Dynamic assessments could unveil evolving patterns of efficiency within the GCC banking sector and offer insights into the sector's adaptability and resilience.

In conclusion, this study marks a pivotal step forward in the realm of group efficiency evaluation using the MSORM model. By introducing innovative definitions of group efficiency and extending the model's application to collective performance assessments, we have enriched the landscape of decision-making tools available to the banking sector. The advantages of our approach and the directions for future research underscore the ongoing evolution of efficiency evaluation methodologies, positioning them as vital instruments for understanding and enhancing organizational performance in complex and dynamic environments.

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