RAIRO-Oper. Res. 58 (2024) 5309–5339
https://doi.org/10.1051/ro/2024201

RAIRO Operations Research
www.rairo-ro.org

DEMAND INFORMATION SHARING STRATEGIES IN THE E-WASTE REVERSE SUPPLY CHAIN UNDER REMANUFACTURER ENCROACHMENT RISK

Xin Qi* and Tao Zhang

Abstract. Based on the background of the e-waste recycling, this study extends the remanufacturer channel invasion to a scenario in which recyclers might already have a greater grasp of recycling market demand than remanufacturers. In the scenario of demand information asymmetry, we use the contrasting signals game model to examine the interaction effect of recycler information distortion on the remanufacturer's belief criterion of recycling market scale and its response to the choice of whether or not to exploit the online channel. Recyclers attempt to distort recycling quantity to induce remanufacturers to make incorrect judgments about the recycling market scale and determine incorrect recycling prices. The discrepancy in recycling prices is then profited by recyclers. The study discovered that expanding online channels in a small market would result in costly signal behaviors by recyclers, such as downward distortion of the actual recycling quantity. This downward information distortion exacerbates the dual marginalization. In contrast to the "win–win" situation found in the online channel under information symmetry, the invasion of an online channel under information asymmetry may result in "double losses". Then we investigate the impact of other practical factors on channel expansion and information-sharing strategy. These conclusions can improve the efficiency of e-waste recycling and promote the process of waste recycling.

Mathematics Subject Classification. 91A40, 90B06.

Received October 7, 2023. Accepted October 22, 2024.

1. Introduction

The development of the Internet and e-commerce has greatly facilitated the rise of online transactions and rapidly transformed consumer buying habits [1]. An increasing number of consumers consider online channels as an important option for shopping or returning goods, and even prefer them over offline channels [2]. In the supply chain of retail, this combined online and offline channel distribution has gradually formed a system. In the electronic waste recycling supply chain, the trend of online transactions is also gradually emerging [3]. The concept of a dual-channel supply chain was proposed for the first time in the Journal of Marketing Science. The authors found that the combination of online and offline sales obviously promotes market sales [4]. With the development of internet technology, there have been significant changes in the ways of product sales, consumption, and recycling, making online channels an increasingly influential force that cannot be ignored [5].

Keywords. Information asymmetry, reverse supply chain, channel intrusion, signaling game, circular economy.

School of Economis and Management, China University of Petroleum (East China), 266580 Qingdao, P.R. China.
*Corresponding author: qxqixinchn@163.com

Currently, the rapid growth of socio-economic and electronic technologies has led people to use electronic and electrical devices more extensively. With the increasing demand for electronic products, more frequent updates and upgrades are inevitable, resulting in a growing number of discarded electronic and electrical equipment. The speed of recycling and disposal cannot keep up with the pace of electronic waste generation, and the accumulated electronic waste is also known as e-waste [6]. In April 2024, the China Association for Circular Economy reported that the average life of a mobile phone is currently about 2.2 years, resulting in over 400 million waste mobile phones annually and a stock of over 2 billion waste phones in China, with around 54.2% of discarded phones being retained by consumers while only about 5% are processed through professional recycling platforms, "old for new" activities, or other recycling channels[1]. This waste stream is becoming a significant source of pollution. The majority of discarded electronic and electrical equipment has yet to enter conventional recycling channels for recovery and reuse [7]. In recent years, as environmental issues caused by electronic waste worsen, considering the context of circular economy, low-carbon economy, and sustainable development, more attention has been paid to reverse channels [8].

The recycling and reuse of waste is an important approach to address the contradiction between economic development and resource scarcity. More and more companies are starting to realize that the recycling of used products can significantly reduce manufacturing costs and improve business efficiency [9]. For example, in the fiscal year 2008/2009, Fuji Xerox remanufactured about 230 000 equipment components in Australia alone, saving $6 million in raw material costs [10]. In 2011, remanufacturing totaled two percent of total goods sales in the US [11]. Since then, the government has tried continuously to promote remanufacturing [12]. The Volvo Group sold remanufactured items worth $1.017 billion in 2016 only [13]. There are also companies such as Philips, Chrysler, Xerox, and Kodak that have established their own reverse recycling networks and systems, actively engaging in waste collection and remanufacturing activities. The resource utilization of waste is gradually playing an important role in achieving sustainable development in the economic-environment-resource nexus [14]. However, the urgent challenge facing the current development of this sustainable industry is how to design effective reverse recycling channels to improve the efficiency of waste resource utilization.

Internet platforms can effectively improve the convenience and transparency of transactions in the recycling process of waste products [15], and they play a crucial role in accelerating the compliant recycling and disposal of e-waste. The widespread use of Internet technology in the supply chain has given rise to a new recycling model. The development of online recycling channels is a process in which recycling firms, remanufacturers, consumers, and other participants use Internet technology to conduct a series of transactions for waste [5]. Some companies acting as recyclable dealers previously collected e-wastes in the traditional recycling channel, such as Shanghai New Jin-qiao Environmental Protection Company Limited and Chang Hong Green Group Company Limited, and they have begun to collect e-wastes by utilizing an online recycling channel. These companies' recycling programs create a new pattern for the effective utilization of resources and environmental sustainability [7].

The dual-channel recycling model can provide more possibilities for the development of the reverse supply chain, but it also brings about some channel conflicts. On the one hand, the opening of online recycling channels brings competitive pressure to traditional recyclers. In the process of channel integration, remanufacturers and recyclers are no longer purely upstream and downstream, but also constitute horizontal competition, which inevitably leads to channel conflicts. On the other hand, the online recycling channel provides a new way for remanufacturers to obtain waste, enabling them to face consumers more directly. However, relying on online recycling channels alone, remanufacturers are unable to understand the individual needs of each consumer as directly as offline recyclers do, and need to speculate on the recycling market demand through the number of items recycled by offline recyclers, which causes asymmetric information on recycling demand between recyclers and remanufacturers. The conflict of interest under the asymmetry of recycling demand information makes it possible for recyclers to distort the recycling quantity for their own interests, pass false recycling demand information to remanufacturers, and induce remanufacturers to make wrong bulk recycling price decisions. The higher bulk recovery price may bring more profit to the recycler than the loss of profit caused by the reduction of

[1]Data from the Chinese association of circular economy (https://www.chinacace.org/news/fieldsview?id=15448).

recycling quantity, and the recycler will profit from it. Considering the distortion of deliverables by the recycler, the remanufacturer adjusts its beliefs about the recycler's recycling volume according to the recycler's recycling volume, which constitutes a signaling game between the recycler and the remanufacturer.

Most of the existing studies focus on forward or single-channel supply chains, and most of the studies on reverse supply chains assume symmetric information between remanufacturers and recyclers. Few studies have dealt with dual-channel reverse supply chain decision-making under asymmetric information. However, in real life, the information between remanufacturers and recyclers is often asymmetric [16]. Unlike previous studies, this study additionally considers the effects of the e-waste recycling rate and the remanufacturing pollution emission reduction cost coefficients on the profits of the remanufacturers and recyclers, which are seldom involved in the previous models at the same time. Based on the industrial practices and available literature, the motivation for the current paper is as follows.

This study confronts the status quo of information asymmetry in recycling demand and effectively resolves channel conflicts due to information asymmetry and the intrusion of online channels in order to better utilize the positive effects of online recycling channels and improve the efficiency of waste recycling and resourcing. This paper aims to address the following research questions:

(1) How does information on recycling market demand affect remanufacturers' judgments about market scale? And how does a remanufacturer's judgment of the recycling market affect whether or not to develop online recycling channels?
(2) How does a recycler's distortion of recycling quantities when sharing information affect the profit of both the recycler and the remanufacturer? What factors interfere with this process?
(3) How do practical parameters such as recycling market share, recycling reutilization rate, and low pollution remanufacturing cost factor affect the profitability of the two firms and the decision to explore new channels?

The rest of this article is structured as follows: Section 2 reviews the literature; Section 3 addresses problem formulation; Section 4 provides mathematical modeling of the proposed reverse supply chain; and Section 5 depicts a numerical experiment and provides sensitivity analysis for various parameters. Section 6 concludes the work and provides management insights and future extensions.

2. Literature review

A careful review of the recent literature reveals that this study is related to three literature streams and enriches and improves these three literature streams. Current research deals with dual-channel supply chains, information asymmetry, and reverse supply chains, respectively. This section compiles and summarize the literature on these three categories, and finds that research on dual-channel reverse supply chains in the asymmetric information context can be further deepened.

Most of the studies focus on forward supply chains, based on which scholars have examined the impact of many factors on dual-channel recycling supply chains and analyzed them using various methods such as Stackelberg or Bertrand games. In the research related to forward supply chain, fewer studies involve dual-channel supply chain. The establishment of online channels will inevitably increase market competition and thus generate conflicts with traditional recycling channels, so the first step to be taken in the study of dual-channel supply chains is to choose an appropriate channel structure. Regardless of the structure, pricing is an important issue in dual-channel supply chain management. All relevant partners in the supply chain need to set reasonable prices under the goal of profit maximization, and at the same time, minimize channel conflicts through reasonable concessions, profit sharing and cost sharing to maintain the continuity and stability of the supply chain. The main influencing factors affecting decision-making on price are market demand, channel preference, environmental protection preference, price sensitivity, market share, and personalized requirements. The complexity of dual-channel supply chain structure has attracted the attention of many scholars.

In dual-channel supply chain decision-making, many researchers have researched different recycling-channel choice strategies on recycling price [17, 18]. Chuang *et al.* [19] studied and compared three types of reverse channel structures to provide the theoretical basis for manufacturers by comparing three different circumstances. Jena *et al.* [20] studied price competition and coordination strategies in different situations for a closed-loop supply chain. Liu *et al.* [21] pioneered a study on both online and offline channels to investigate the optimal price acquisition strategy for dual channels. Likewise, some papers [22,23] explored a multi-echelon closed-loop supply chain and investigated the problem of coordination mechanisms. Xiong and Yan [24] suggested that the sale of remanufactured goods is influenced by a channel's structure. In addition, some papers [25–27] studied the influence of various factors in a closed-loop supply chain. Zou [28] further emphasized that environmental friendliness and convenient and safe recycling services are significant characteristics of an online recycling channel. Studies with respect to dual-channel reverse supply chains, most scholars focus on channel selection and contract coordination. Some scholars [29] found that the dual recycling channel has an advantage over single recycling channels in promoting the recovery of waste products and improving recovery efficiencies. Wu [30] found that income cost-sharing contracts, income-sharing contracts, cost-sharing contracts, and other contracts can effectively reduce efficiency loss and achieve supply-chain coordination.

The recycling price of online recycling channels is generally higher than that of offline channels. The main reason is that, on the one hand, recycling enterprises in online channels directly deal with consumers so as to avoid a situation in which offline recyclers earn the intermediate price difference. On the other hand, the recycling center can scientifically price the electronic waste products based on equipment conditions, such as the use time, standby status, and wear degree, reducing the occurrence of excessive prices in offline channels [31]. Zhang *et al.* [32] studied the impact of consumers' environmental protection preferences and retailers' fair preferences on enterprise pricing decisions for a supply chain, including the examination of one manufacturer and one retailer. Luo *et al.* [25] studied carbon tax policies in hybrid manufacturing–remanufacturing systems. Their results show that carbon tax discourages remanufacturing in hybrid systems, and may decrease environmental performance for products with low carbon savings during remanufacturing. Research on low-carbon supply chain focused on the importance of emissions reduction and the relationship between emissions and remanufacturing.

Most studies on reverse supply chains also assume symmetric information between remanufacturers and recyclers, and most of them focus on closed-loop supply chains, with few scholars studying reverse supply chains separately. In order to eliminate the profit loss caused by decentralized decision-making in a closed-loop supply chain system, Yuan *et al.* [3] used optimization analysis and game theory to analyze the necessary conditions for manufacturers, retailers and third-party online recyclers to achieve coordination of interests under equilibrium conditions. Chen *et al.* [33] studied the pricing strategy of closed-loop supply chains in which retailers are responsible for recycling processes and manufacturers are responsible for remanufacturing processes. Wang *et al.* [34] constructed an evolutionary game model including government, recyclers, and consumers, and analyzed the behavior evolution trajectory of participants in the e-waste recycling system. Most of these papers focus only on traditional recycling channels within a single recycling channel, and do not focus on mixed reverse dual-channel supply chains. We mainly examine the mixed recycling channel issue. In addition, the coordination of reverse supply chains, as an important emerging topic, has attracted explorations from different perspectives. Under this research category, some scholars [35] have further extended the issues of price competition and reverse channel selection by retailers. The reuse rate of recyclables is an important indicator for the waste resourcing process, which can significantly affect the process efficiency, but it has hardly been mentioned in previous studies. This study bridges the gap by adding the reuse rate of recyclables to the reverse supply chain model to analyze its impact on supply chain margins.

In an uncertain environment, the lack of intensive co-operation among supply chain members will lead to inefficiencies in the supply chain. For example, the bullwhip effect is a typical phenomenon, which refers to the fact that due to the uncertainty of market demand and the asymmetry of information, the downstream enterprises of the supply chain constantly distort and amplify the market demand to the upstream enterprises, resulting in the chaos of the product from supply, production, to the final sales segment, and thus the supply chain performance is often low. And demand information sharing is one of the effective methods to promote the co-operation of supply chain members, weaken the bullwhip effect, and achieve supply chain performance improvement. At present, scholars have paid attention to the diverse management problems faced by different enterprises in different industries under information asymmetry, and studied that the decision of supply chain affiliates to carry out information sharing is affected by a variety of factors. In the research on supply chain information sharing, some studies [36] have introduced retailers' personality information sharing into the model and demonstrated the importance of personality information sharing in the wholesale price contract of supply chain system. And some scholars [37] have studied the incentive mechanism for information sharing in dual-channel agricultural supply chains to provide new ideas for effective mobilization of information sharing. It has been suggested that information sharing, supply chain visibility, synergy and agility together have important direct and indirect impacts on supply chain performance, leading to better profitability, competitiveness and flexibility, and reveals the importance of the need for information sharing in improving supply chain performance [38].

Some scholars [39] focus on modeling the competition between two supply chains and discuss the impact of different operational management factors, such as supply chain competitive structure and information accuracy, on the sharing of demand information within the supply chain. A study [40] considers a situation where multiple retailers order from a single supplier and each retailer has private demand information, but one retailer has more accurate demand information than the others. The study concluded that such a competitive structure is prone to information leakage, which leads to retailers' reluctance to cooperate with suppliers for information sharing. Based on this, contractual schemes such as repurchase contracts and profit sharing contracts are designed to facilitate information sharing between suppliers and retailers in the supply chain on the one hand, and to prevent information leakage among retailers on the other hand [41]. When upstream manufacturers are at a demand information disadvantage, it has also been found that their time advantage can be utilized for strategic implementation [42]. Some scholars [43] have investigated the effect of incentives for suppliers to share demand information and performance evaluation as well as the impact of various parameters on the incentive effect of the supply chain in the exploration of the value of information sharing in a two-level supply chain, but it only involves aspects such as the measurement of the effect of sharing incentives and the impact of factors, and there are fewer studies of the specific value of information sharing and the impression of conceptual stocks. The current demand information has an important role in arranging the corresponding production and operation activities for each subject of the reverse supply chain, but the optimization of this demand information sharing related paths and decision-making is often neglected in the current research on reverse supply chain, and the current research involves less, and the role it plays and how it is considered in the perspective of all parties, and what kind of added value and problems it will bring to the reverse supply chain deserve to be It is worthwhile to further explore.

The current research seldom involves the dual-channel reverse supply chain decision-making research under the information asymmetry situation at the same time. However, in fact, in real life, the information between remanufacturers and recyclers is often asymmetric. In this study, we consider the changes in the decision-making of enterprises in a dual-channel reverse supply chain under information asymmetry, and take the reuse rate of waste recycling and the pollution emission cost coefficient of the remanufacturing process as well as the share of recycling market channels into account in the model, and analyze the impacts of the changes in these influencing factors on the profits of the remanufacturer and the recycler, so as to put forward the dual-channel reverse supply chain management strategy that better meets the real-world scenario to improve the efficiency of e-waste recycling.

3. The model description

3.1. Model background

In this paper, we study the signaling game problem of a dual-channel reverse supply chain in which a remanufacturer that expands its online direct recycling channel and a traditional offline recycler compete and cooperate with each other under the condition of asymmetric demand information. The following assumptions are made.

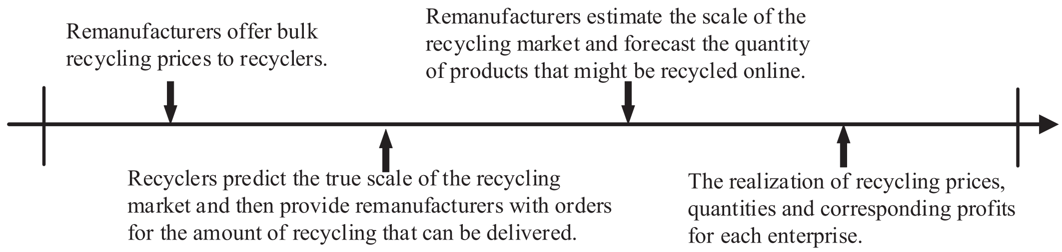

FIGURE 1. Description of the game process.

(1) The recycling quantity is a linear function of the recycling price, and the total share of the recycling market represents the quantity of waste products voluntarily provided by consumers when the recycling price is 0.
(2) Recyclers are closer to consumers involved in recycling than remanufacturers, and are able to grasp more information about the recycling market, and can more accurately predict the scale of the recycling market, which is the signal sender in the signaling game.
(3) Remanufacturers cannot observe the market directly, but can predict the demand and scale of the recycling market through the recycling quantities provided by recyclers. The remanufacturer has no other ways of obtaining market information except the recycling quantity from the recycler, that is, the remanufacturer is the signal receiver in the signaling game.
(4) To facilitate the calculations, this paper assumes that the other costs of the remanufacturer and the recovery costs of the recycler are fixed and set to 0. However, the remanufacturer incurs an additional environmental cost as a result of the low-pollution treatment process. The environmental costs are affected by the expenses they put into the low-pollution technology. Assume that $C_{(e_m)} = \frac{ke^2}{2}$ is the cost function of the remanufacturer's low-pollution operations, e denotes the pollution emission reduction level of low-pollution treatment and remanufactured products, and k is the cost coefficient of pollution emission reduction technology investment ($k > 0$). It is assumed that the remanufacturer needs to pay an additional online platform construction and recycling cost c when directly undertaking online recycling.
(5) The uncertainty of demand in the recycling market will exist in two scenarios, namely, a large market size and a small market size. It is assumed that a remanufacturer can generate η units of secondary marketable products per unit of recovered waste through processing and remanufacturing, and the reuse rate is η.
(6) The remanufacturers and recyclers are perfectly rational and pursue the maximization of their own interests. Consumers who participate in recycling are rational and have the tendency to implement environmentally friendly behaviors, and are willing to pay a corresponding price for low-pollution behaviors.

In addition, the decision-making procedure of the model is explained. First, the remanufacturer provides the bulk recycling price w to the recycler, who predicts the real recycling market scale (a_L or a_H), and after that provides the remanufacturer with a recycling quantity order Q_R that can be delivered, that is, sends a signal. After receiving the signal, the remanufacturer determines the recycling market size and predicts Q_D units of product that can potentially be recycled through the online platform, and allocates management and operational resources related to online recycling based on that recycling quantity, as shown in Figure 1.

3.2. Model parameters

Description of parameter symbols:

a_i The size of the recycling market, which represents the amount of waste voluntarily provided by consumers when the price of recycling is zero. i represents the type of "natural" choice in the signaling game. $i \in \{H, L\}$, $a_H > a_L > 0$.

μ Expectation of recovery market scale a_i. $\mu = \lambda a_H + (1 - \lambda)a_L$.

σ^2	Variance of recovery market scale a_i. $\sigma^2 = \lambda(a_H - \mu)^2 + (1-\lambda)(a_L - \mu)^2$.
λ	Probability that the recycling market scale is a_H.
θ	Potential recycling market share of online recycling channels. $0 < \theta < 1$.
γ	The competition level between offline recycling channels and online recycling channels, the competition degree can also be referred to as the degree of substitution, with larger values indicating more intense competition between the two recycling channels. Since the sensitivity coefficient of consumers to recycling prices is set to 1 in the assumptions, and considering that the recycling prices paid by recyclers themselves for waste electronics have a greater impact on their recycling quantities compared to the recycling prices of their competitors. $0 < \gamma < 1$.
p_R	The price paid by offline recyclers to acquire wastes from consumers.
p_M	The price paid by the remanufacturer in online channel to acquire wastes from consumers.
w	Bulk recycling prices when remanufacturers obtain e-waste through offline recyclers. Let $w > p_M$, otherwise the offline recyclers will sell their recyclables directly through the online recycling channels of the remanufacturers in bulk, and the market will become chaotic; let $w > p_R$, that is, the recycler's recycling price from the remanufacturer is higher than the recycler's recycling price from the consumer, otherwise the recycler will not be profitable.
c	Additional outlay costs (per unit) for remanufacturers to set up online platforms for recycling.
$C_{(e_m)}$	Cost function of technology inputs (per unit) by the remanufacturer for the purpose of low pollution operations. $C_{(e_m)} = \frac{ke^2}{2}$. e denotes the level of pollution reduction for low-pollution treatment and remanufactured products, and k is the technology input cost coefficient for pollution reduction ($k > 0$).
b	Environmental preferences of consumers participating in recycling. $0 < b < 1$.
η	Reutilization rate of recovered waste. The remanufacturer can produce η unit of remanufactured product by processing and remanufacturing each unit of waste product.
V	Profit per unit of product acquired by the remanufacturer through disposal-remanufacture-sale. The amount of saleable product R obtained through remanufacturing is a fraction of the amount of recycled waste Q. $R = \eta * Q = \eta * (q_R + q_M)$.

4. Equilibrium analysis

4.1. No online recycling channel (M1)

In order to better analyze and compare the impact of online recycling channels operated by remanufacturers on the recycling supply chain, the study first analyzes the scenario in which remanufacturers obtain e-waste only through traditional channels of direct recycling. It should be noted that all models, with or without online direct recycling channels, are based on the scenario of asymmetric recycling demand information, that is, the recycler has the demand information of the recycling market, and the remanufacturer only knows the type and probability distribution of the recycling market size, but not the real recycling market size.

The recycling quantity function of the offline recycler can be expressed as $q_R = a_i + p_R + be$. The profit function of the offline recycler is $\pi_R = (w - p_R)q_R$. The profit function of the remanufacturer is $\pi_M = VR - wq_R - C_{(e_m)}q_R = V\eta q_R - wq_R - C_{(e_m)}q_R$. In model M1, all members aim to maximize their respective profits, and the decision problem can be equated to solving the following planning problem.

$$\max_{w} \pi_M = \left(V\eta - w - C_{(e_m)}\right)(a_i + p_R + be) \tag{1}$$

$$\max_{p_R} \pi_R = (w - p_R)(a_i + p_R + be). \tag{2}$$

Proposition 1. *In the case of information asymmetry, when the remanufacturer does not open up online recycling channels, the operation strategy and profit expectation of the reverse supply chain members are w^*, $p_R{}^*$, $\pi_1{}^*$ and $\pi_M{}^*$ respectively, shown in appendix.*

4.2. With online recycling channels (M2)

The establishment of network recycling channels has, on the one hand, brought competitive pressure to traditional offline recyclers. In the process of channel integration, remanufacturers and offline recyclers are no longer purely upstream and downstream, but also constitute horizontal competition, which inevitably leads to recycling channel conflicts. On the other hand, remanufacturers are provided with a new channel for obtaining recyclables that can be utilized, so that they can face consumers who participate in recycling more directly. However, remanufacturers can only rely on online recycling channels to passively accept recycling orders that have already taken place, and are unable to get to know the direct recycling and remanufacturing needs of consumers involved in recycling and the actual scale of the recycling market like offline recyclers who visit the streets and alleys. At this point, the remanufacturer can only speculate the recycling market demand through the recyclables quantity contract issued by the offline recycler, which results in asymmetric demand information between the recycler and the remanufacturer. The conflict of benefits under the asymmetric demand information makes it possible for the offline recycler to distort the recycling quantity for its own benefit by delivering false recycling information to the remanufacturer, which induces the remanufacturer to make a wrong bulk recycling price decision w. And the growth of w is likely to bring more profit to recyclers than the loss of profit from the reduction of recycling quantity, from which recyclers will profit. Considering the recycler's behavior of distorting the recycling volume, the remanufacturer then continuously adjusts the belief criterion about the recycler's recycling quantity based on the amount of recyclable waste provided by the recycler, that is, a signaling game is constituted between the offline recycler and the remanufacturer.

Recyclers as signaling agents can adopt either a separation strategy or a mixing strategy. When the recycler adopts a segregation strategy, for different market scales, the recycler will sign contracts for different quantities of recycled products, reflecting the corresponding market scale; when the recycler adopts a pooling strategy, for different recycling market scales, the recycler will sign the same contract with the same quantities of recycled products, which is equivalent to not providing the recycling demand information to the remanufacturer. The former outcome is called a separating equilibrium, while the latter is called a pooling equilibrium. Typically, such signaling games may have multiple equilibrium depending on the belief norms of the players. However, this obstacle can be overcome by using the institution criterion, which is a classical equilibrium improvement developed by Cho and Kreps [16]. It can be shown that the mixed equilibrium cannot survive the intuitive criterion in our model, so this study focuses directly on the separated equilibrium. That is, all formulas and analyses given below are based on the assumption that remanufacturers can perfectly infer the recycling market scale (not necessarily the true market scale) from the quantity of orders from recyclers.

4.2.1. Demand function

The recycling demand functions for the traditional offline recycling channel and the remanufacturer's online channel are as follows.

$$\text{Offline channel: } q_R = (1-\theta)a_i + p_R - \gamma p_M + be \tag{3}$$

$$\text{Online channel: } q_M = \theta a_i + p_M - \gamma p_R + be. \tag{4}$$

The inverse demand function:

$$p_R = \frac{q_R + \gamma q_M}{1-\gamma^2} - \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i - \frac{be(1+\gamma)}{1-\gamma^2} \tag{5}$$

$$p_M = \frac{\gamma q_R + q_M}{1-\gamma^2} - \frac{\gamma-\gamma\theta+\theta}{1-\gamma^2}a_i - \frac{be(1+\gamma)}{1-\gamma^2}. \tag{6}$$

The profit functions for offline recyclers and remanufacturers are as follows.

$$\pi_R = (w - p_R)q_R \tag{7}$$

$$\pi_M = VR - wq_R - p_M q_M - C_{(e_m)}(q_R + q_M) - cq_M. \tag{8}$$

4.2.2. Recycling quantity decisions provided by recyclers to remanufacturers

First, the belief criterion of the remanufacturer is constructed. $a_{j(q_R)}$ denotes the presumed recycling market size after the remanufacturer receives orders for the quantity of recycled waste from offline recyclers. In general, when the true recycling market size is relatively small, the amount of waste offered by recyclers to remanufacturers will be smaller than if there were a large market size. Therefore, this paper sets that for a given bulk recycling price w, there exists a threshold $\widehat{q}_R(w)$ such that the following equation holds.

$$j(q_R) = \begin{cases} H & \text{otherwise} \\ L & \text{if } q_R < \widehat{q}_R(w). \end{cases} \tag{9}$$

If $q_R < \widehat{q}_R(w)$ in the recycler's recycling quantity contract, the remanufacturer will consider the recycling market to be small, and *vice versa*. After confirming the recycling quantity order q_R offered by the recycler, the remanufacturer decides the optimal amount of waste to be recycled through the online channel.

Profit of online channels:

$$\max_{q_M}[V\eta q_M - p_M q_M - c q_M] \to \max_{q_M}\left[\left(V\eta - \frac{\gamma q_R + q_M}{1-\gamma^2} + \frac{\gamma - \gamma\theta + \theta}{1-\gamma^2}a_j + \frac{be(1+\gamma)}{1-\gamma^2} - c\right)q_M\right] \tag{10}$$

$$q_M = \frac{(V\eta - c)\left(1-\gamma^2\right) + (\gamma - \gamma\theta + \theta)a_j + be(1+\gamma) - \gamma q_R}{2}. \tag{11}$$

Given the remanufacturer's belief criteria and possible options for action, a recycler who truly knows the size of the recycling market would need to solve for the following equation.

$$\max_{q_R}[(w - p_R)q_R] \to \max_{q_R}\left[\left(w - \frac{q_R + \gamma q_M}{1-\gamma^2} + \frac{1 - \theta + \gamma\theta}{1-\gamma^2}a_i + \frac{be(1+\gamma)}{1-\gamma^2}\right)q_R\right]. \tag{12}$$

$q_R(w, a_i)$ indicates the optimal solution of the above equation. Notice that for a given order quantity, the recycler would be better off if the remanufacturer believed the market size were small than if she believed it were large. Thus, the recycler may purposely order a lower quantity to induce the remanufacturer to believe the market size is small. The remanufacturer will adjust $\widehat{q}_R(w)$ taking the recycler's incentive into account. To solve this problem, we define the following equilibrium concept.

Definition 1. In this reverse supply chain, given any wholesale price w, a perfect Bayesian separating equilibrium is reached if $a_{j(q_R(w,a_i))} = a_i$ for each market size a_i, $i \in \{H, L\}$; that is, there exists a $\widehat{q}_R(w)$ such that $q_R(w, a_H) > \widehat{q}_R(w)$ while $q_R(w, a_L) \le \widehat{q}_R(w)$.

To facilitate the characterization of the equilibrium, we define the following functions $V_{ij}(q_R)$ as the recycler's profit function. When the true recycling market scale is a_i, the remanufacturer speculates that the market scale is a_j based on the received recycling quantity signal from the recycler.

$$\begin{aligned} V_{ij}(q_R) &= \left[\left(w - \frac{q_R + \gamma q_M}{1-\gamma^2} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i + \frac{be(1+\gamma)}{1-\gamma^2}\right)q_R\right], \quad \forall i, j \epsilon \{H, L\} \\ &= \left[\left(w - \frac{(V\eta - c)\gamma}{2} + \frac{2be - be\gamma}{2(1-\gamma)} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i - \frac{(\gamma - \gamma\theta + \theta)\gamma a_j}{2(1-\gamma^2)}\right)q_R - \frac{\left(2-\gamma^2\right){q_R}^2}{2(1-\gamma^2)}\right]. \end{aligned} \tag{13}$$

Considering the complexity of the equation, it is brought to the numerical value for analysis. Parameters are set to: $\theta = 0.3$, $\gamma = 0.4$, $\lambda = 0.3$, $a_H = 3$, $a_L = 1$, $w = 1$, $\eta = 0.8$, $b = 0.6$, $e = 1$, $V = 3$, $c = 1.6$. Lemma 1 can be derived from Figure 2.

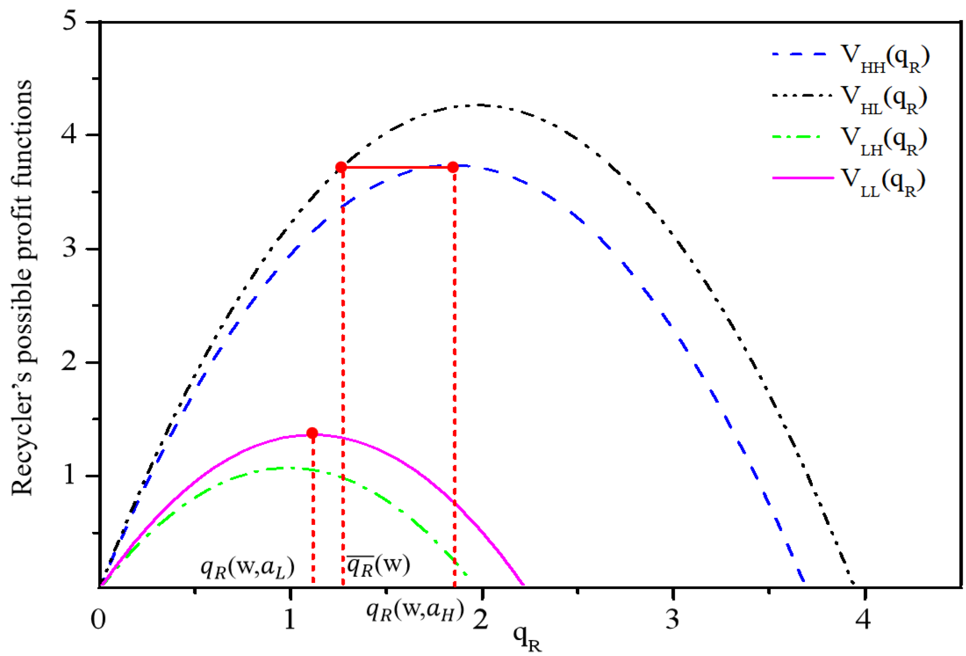

FIGURE 2. Recyclers' profit and thresholds $\overline{q}_R(w)$ under different market scales.

Lemma 1. *When the recycling market is large, recyclers will profit more if they distort the quantities recycled to convince the remanufacturer that the recycling market is smaller. Therefore, the recycler will deliberately contract for lower quantities of recyclables to be delivered to the remanufacturer, inducing the remanufacturer to believe that the recycling market is smaller, causing the remanufacturer to raise the bulk price of recycling, w. It is likely that the gains from higher w will outweigh the loss of profit from lower recycling quantities, and that the recycler will profit from the increase in operating efficiency. For this reason, the remanufacturer takes into account the incentive to distort the recycling volume downward and adjusts the belief criterion $\widehat{q}_R(w)$.*

First, assume that the remanufacturer is able to observe the market demand for recycling and thus accurately estimate the size of the market, that is, $a_{j(q_R(w,a_i))} = a_i$. Thus, $q_R(w,a_i)$ is solved for the case of information disclosure. The study will be conducted based on this to solve for the threshold of the recycling price when the recycler distorts the recycling quantity downward and the threshold of the quantity of deliverable recycling that the recycler provides to the remanufacturer, as shown in Figure 2.

Lemma 2. *The function $V_{ij}(q_R)$ is convex function in q_R for any $i,j\epsilon\{H,L\}$, and there is a unique maximizer of $V_{ii}(q_R)$; that is,*

$$\begin{aligned} q_R &= q_R^{\mathrm{SI}}(w^*,a_i) \\ &= \frac{(2w-(V\eta-c)\gamma)(1-\gamma^2)+(2be-be\gamma)(1+\gamma)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_i}{2(2-\gamma^2)}, \qquad \text{for each } i\epsilon\{H,L\}. \end{aligned}$$

If the remanufacturer is also able to observe the recycling market demand or is informed of the recycling market demand through information sharing so as to accurately estimate the scale of the recycling market ($a_{j(q_R(w^*,a_i))} = a_i$), for the recycling market scale a_i, the amount of recycling offered by the recycler to the remanufacturer is $q_R^{\mathrm{SI}}(w^*,a_i)$. But when this information is not available to the remanufacturer, the recycler has an incentive to distort the recycling quantity by selling the remanufacturer a quantity of recyclables smaller than

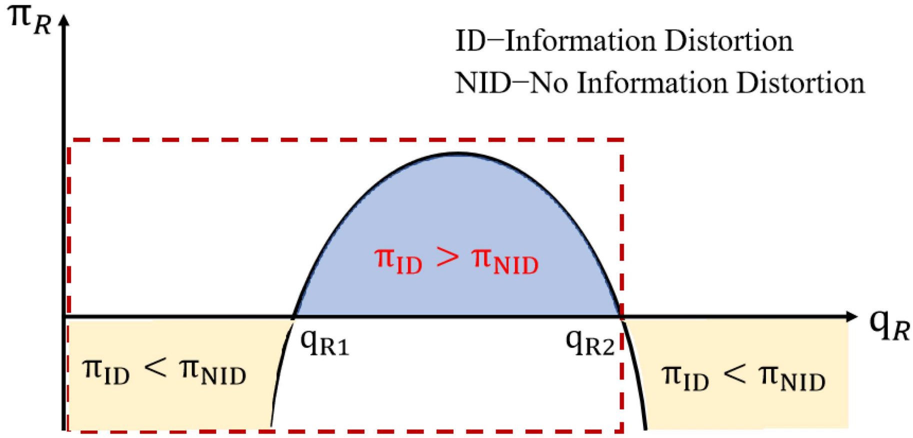

FIGURE 3. Information distortion trade-off process analysis.

$q_R(w, a_i)$. In addition, in order for the offline recycling channel to recycle more than zero, the remanufacturer's bulk recycling price should meet the following conditions.

$$w > \frac{(V\eta - c)\gamma}{2} - \frac{(2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_i}{2(1-\gamma^2)}.$$

Proposition 2. *There is a critical value of $\overline{w}$ (in appendix), at which point there will be no incentive for the recyclers to induce them to think the recycling market is small (the recyclers will not induce them to think the recycling market is large when the market is large).*

(1) *If $2a_L(1 - \theta - \gamma^2 + \gamma^2\theta) - 2a_H(1 + \theta - \gamma\theta) + 1 > 0$, the recycler will not distort the recovery amount when $w \le \overline{w}$. Conversely, if $w > \overline{w}$, recyclers observe that when the recycling market is large, recyclers may downward distort the amount of recycling delivered to re-manufacturers.*

(2) *If $2a_L(1 - \theta - \gamma^2 + \gamma^2\theta) - 2a_H(1 + \theta - \gamma\theta) + 1 < 0$, the recycler will not distort the recovery amount when $w \ge \overline{w}$. Conversely, if $w < \overline{w}$, the recyclers observe that when the recycling market is large, the recyclers may downward distort the amount of recovery that can be delivered to the remanufacturers. (The case of $2a_L(1 - \theta - \gamma^2 + \gamma^2\theta) - 2a_H(1 + \theta - \gamma\theta) + 1 = 0$ is not considered.)*

The significance of $\overline{w}$ is that if the recycler provides $q_R^{\mathrm{SI}}(w^*, a_L)$ when the market size is large, then the distortion of the deliverable recovery volume is unprofitable for the recycler when the recovery price is within the corresponding value range. That is, if $V_{HL}(q_R^{\mathrm{SI}}(w^*, a_L)) \le V_{HH}(q_R^{\mathrm{SI}}(w^*, a_H))$, for recyclers, the distorted recovery does not bring more profit.

In order to achieve separation equilibrium, recyclers observed the recycling market size, may twist downward recovery, makes the manufacturer believe that the size of the recycling market is small, there is a critical value, when the recycling market scale, recyclers delivered to the manufacturer of recycling quantity can twist down to the critical value, makes the manufacturer think recycling market size is small, and profitable. If the amount of recovery delivered is less than this critical value, the distortion of information will hurt its profit. Thus, there is a critical value of $\overline{q}_R(w)$, such that $V_{HL}(\overline{q}_R(w, a_L)) \le V_{HH}(q_R^{\mathrm{SI}}(w, a_H))$.

Consider that the recyclers will choose to distort the information only if the benefit is greater than the non-distorted information. Combined with Figure 3 analysis, only when the actual recycling market is large, recyclers choose to provide a smaller amount of recycling to remanufacturers to mislead the judgment that the recycling market is small.

Proposition 3. *The critical value of the recycler distortion information is $\overline{q}_R(w)$ (in appendix), and the trade-off process only exists when $q_R < q_{R2} = q_R^{\mathrm{SI}}(w, a_H)$. Based on this threshold, you can conclude that:*

$q_R(w) = \overline{q}_R(w)$, $V_{HL}(q_R(w, a_L)) = V_{HH}\big(q_R^{\mathrm{SI}}(w, a_H)\big)$;
$q_R(w) < \overline{q}_R(w)$, $V_{HL}(q_R(w, a_L)) < V_{HH}\big(q_R^{\mathrm{SI}}(w, a_H)\big)$;
$\overline{q}_R(w) < q_R(w) < q_R(w, a_H)$, $V_{HL}(q_R(w, a_L)) > V_{HH}\big(q_R^{\mathrm{SI}}(w, a_H)\big)$.

In addition, at $w = \overline{w}, \overline{q}_R(w) = q_R^{\mathrm{SI}}(w, a_L)$; at $w > \overline{w}$, $\overline{q}_R(w) > q_R^{\mathrm{SI}}(w, a_L)$; at $w < \overline{w}$, $\overline{q}_R(w) < q_R^{\mathrm{SI}}(w, a_L)$.

For manufacturers, in order not to be misled by the recycler signal, belief criterion $\widehat{q}_R(w)$ cannot be greater than $\overline{q}_R(w)$. Otherwise, when the recycler observed large recycling market size, can be delivered to the manufacturer less than $q_R^{\mathrm{SI}}(w, a_H)$ but greater than the number of $\overline{q}_R(w)$, so that the manufacturer think recycling market size is small. The resulting recycling amount of the online direct recycling channel will be smaller than the demand of the recycling market for the online recycling channel, and the remanufacturer will therefore increase the recycling price paid to consumers, and the recyclers will profit. Therefore, we can conclude as follows:

Lemma 3. *The signal game can reach equilibrium, when the amount of recovery given to the remanufacturer is*

$$q_R(w, a_i) = \begin{cases} q_R^{\mathrm{SI}}(w, a_H) & i = H \\ \widehat{q}_R(w) & i = L. \end{cases}$$

$\widehat{q}_R(w) = \min\{q_R^{\mathrm{SI}}(w, a_L), \overline{q}_R(w)\}$. The recycling quantity of the remanufacturer is: $q_M(w, a_i) = \frac{(V\eta - c)(1-\gamma^2) + (\gamma - \gamma\theta + \theta)a_i + be(1+\gamma) - \gamma q_R}{2}$.

For any given bulk recycling price and any recycling market size, if, then there exists $a_{j(q_R(w^*, a_i))} = a_i$ such that $q_R(w, a_H) > \widehat{q}_R(w)$, and $q_R(w, a_L) < \widehat{q}_R(w)$, at which point the signaling game will reach a separating equilibrium.

When the signaling game reaches a separating equilibrium, the recycler's recycling quantity to the remanufacturer when the recycling market size is large is $q_R^{\mathrm{SI}}(w, a_L)$. This is consistent with the recycling quantity when the remanufacturer also has access to information and observes the market scale. This implies that when the recycling market is large, the recycler has no incentive to distort the recycling quantity downward, even if it has private information. However, when the recycling market is small, the distortion of recycling quantities under asymmetric demand information will emerge. It follows from the above that when $w \geq \overline{w}$, $q_R(w, a_L) = \widehat{q}_R(w) = \min\{q_R^{\mathrm{SI}}(w, a_L), \overline{q}_R(w)\} = q_R^{\mathrm{SI}}(w, a_L)$; when $w < \overline{w}$, $q_R(w, a_L) = \widehat{q}_R(w) = \overline{q}_R(w) < q_R^{\mathrm{SI}}(w, a_L)$. When $w < \overline{w}$ and the recycling market scale is small, under information asymmetry, the recycler will offer the remanufacturer a smaller recycling quantity than it would if the remanufacturer also had access to information to observe the recycling market scale. This equilibrium result occurs mainly because the remanufacturer adjusts its belief criterion to take into account the recycler's incentive to distort the quantity of recycling downward. Thus, in any case, when the game reaches equilibrium, the remanufacturer is always correctly informed about the recycling market scale.

4.2.3. Bulk recycling price decisions for remanufacturers

In addition to adjusting their belief criteria to correctly perceive the scale of the recycling market, remanufacturers need to make decisions about the bulk recycling price for recovering e-waste from recyclers in order to maximize profit. Typically, when the recycling market size is large, the remanufacturer has a motivation to establish an online recycling channel regardless of the recycling cost of the direct online recycling channel, and at the same time, the recycler's recycling quantity is greater than zero. However, when the size of the recycling market is small, if the cost of the online channel is very low, the remanufacturer will be more inclined to obtain waste through the online channel. In this case, the bulk price paid by the remanufacturer for obtaining waste offline through the recycler will be lower, thus making it unprofitable for the recycler. On the contrary, if the

FIGURE 4. Diagram of the relationship between $\bar{w}, w^{\mathrm{ID}^*}$ and w^{NS^*}.

recycling cost of the online recycling channel is very high, the remanufacturer will prefer to obtain the usable waste through the offline recycler. In this section, we will assume that the other parameters are fixed and focus on analyzing the impact of the cost c of the online direct recycling channel of the remanufacturer on the offline bulk recycling price w.

The expected profit of the remanufacturer is $\pi_M(w)$ in the appendix. First, we discuss the case where the quantity recovered is greater than zero for both online and offline recycling channels. The recycler does not distort the recycled quantity when $w \leq \overline{w}$. At this point, the recycler's recycling quantity satisfies $q_R(w, a_i) = q_R^{\mathrm{SI}}(w, a_i)$. $\pi_M^{\mathrm{NS}}(w)$ indicates the expected profit of the remanufacturer in the absence of information distortion. When $w > \overline{w}$, the recycler distorts the recycling quantity downward to distort the recycling demand information when the recycling market is small. At this point, the recycling quantity delivered by the recycler to the remanufacturer satisfies $q_R(w, a_H) = q_R^{\mathrm{SI}}(w, a_H)$, $q_R(w, a_L) = \overline{q}_R(w, a_L)$. $\pi_M^{\mathrm{ID}}(w)$ (ID: information distortion) indicates the expected profit when information is distorted. When the recycling quantity of this recycling supply chain is greater than zero, the optimal bulk recycling price is determined by the two equations $\pi_M^{\mathrm{NS}}(w)$ and $\pi_M^{\mathrm{ID}}(w)$ together, and the optimal wholesale price w^{NS^*} can be obtained. The specific calculation process is shown in the appendix.

In this case, there may be a $w^{\mathrm{ID}^*} \epsilon (0, \overline{w})$, making $\frac{\mathrm{d}\pi_M^{\mathrm{ID}}(w)}{\mathrm{d}w} = 0$. $\pi_M^{\mathrm{NS}}(w)$ and $\pi_M^{\mathrm{ID}}(w)$ may reach equilibrium to obtain the optimal solution, or may have no optimal solution, but can achieve the maximum value in the range of values. In order to compare the profit states and preferred choices of the remanufacturer in different scenarios with and without information distortion, and to determine the possible combinations of decision states, this paper analyzes and derives Proposition 4. In the following section, we will also verify that there is only one optimal solution through numerical experiments.

Proposition 4. *In the case where two profit functions $\pi_M^{\mathrm{NS}}(w)$ and $\pi_M^{\mathrm{ID}}(w)$ have an optimal solution (considering the case where $\pi_M^{\mathrm{ID}}(w)$ only has an optimal solution), the possible results are as follows (shown in Fig. 4).*

(1) *$\pi_M^{\mathrm{NS}}(w)$ increases monotonically when $w < \overline{w}$, and the maximum value is $\pi_M^{\mathrm{NS}}(\overline{w})$; $\pi_M^{\mathrm{ID}}(w)$ is monotonically decreasing when $w > \overline{w}$, and the maximum value is $\pi_M^{\mathrm{ID}}(\overline{w})$. When $w = \overline{w}$, $\overline{q}_R(w, a_L) = q_M^{\mathrm{SI}}(w, a_L)$, and $\pi_M^{\mathrm{ID}}(\overline{w}) = \pi_M^{\mathrm{NS}}(\overline{w})$. Therefore, $\pi_M(w^*) = \max_w\{\pi_M^{\mathrm{ID}}(w), \pi_M^{\mathrm{NS}}(w)\} = \pi_M^{\mathrm{ID}}(\overline{w}) = \pi_M^{\mathrm{NS}}(\overline{w})$.*

(2) *$\pi_M^{\mathrm{NS}}(w)$ increases monotonically when $w < \overline{w}$, and the maximum value is $\pi_M^{\mathrm{NS}}(\overline{w})$; The maximum value of $\pi_M^{\mathrm{ID}}(w)$ is $\pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$, and $\pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*}) > \pi_M^{\mathrm{ID}}(\overline{w}) = \pi_M^{\mathrm{NS}}(\overline{w})$. When $w^* = w^{\mathrm{ID}^*}$, $\max_w \pi_M(w^*) = \pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$.*

(3) *The maximum value of $\pi_M^{\mathrm{NS}}(w)$ is $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$. $\pi_M^{\mathrm{ID}}(w)$ is monotonically decreasing when $w > \overline{w}$, and the maximum value is $\pi_M^{\mathrm{ID}}(\overline{w}) = \pi_M^{\mathrm{NS}}(\overline{w}) < \pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$. Thus, when $w^* = w^{\mathrm{NS}^*}$, the maximum value is $\pi_M(w^*) = \pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$.*

(4) *The maximum value of $\pi_M^{\mathrm{NS}}(w)$ is $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$. The maximum value of $\pi_M^{\mathrm{ID}}(w)$ is $\pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$. Thus it is necessary to determine $\pi_M(w^*)$ and the corresponding w^* by comparing the values of $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$ and $\pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$.*

Considering the scenario of a small recycling market, when the cost of the online recycling channel is low, the recycling quantity of the traditional recycling channel of the offline recycler may appear to be zero. At this time, the offline recycler's recycling quantity:

$$q_R(w, a_L) = 0$$

TABLE 1. Specific values for remanufacturers' profit and recycling prices when costs in online recycling channels take on different values.

c	$\pi_M^{\mathrm{NS}}\left(w^{\mathrm{NS}^*}\right)$	$\pi_M^{\mathrm{ID}}\left(w^{\mathrm{ID}^*}\right)$	$\pi_M(\bar{w})$	w^{NS^*}	w^{ID^*}	$\bar{w}$	c	$\pi_M^{\mathrm{NS}}\left(w^{\mathrm{NS}^*}\right)$	$\pi_M^{\mathrm{ID}}\left(w^{\mathrm{ID}^*}\right)$	$\pi_M(\bar{w})$	w^{NS^*}	w^{ID^*}	$\bar{w}$
0.1	16.8542	16.8698	16.8496	0.483542	0.308867	0.595127	1.1	13.8073	13.8098	13.8065	0.342252	0.180789	0.295127
0.2	16.5342	16.5488	16.5309	0.469413	0.296146	0.565127	1.2	13.5213	13.5222	13.5198	0.328123	0.167888	0.265127
0.3	16.2177	16.2311	16.2154	0.455284	0.283404	0.535127	1.3	13.2386	13.2379	13.2363	0.313994	0.154973	0.235127
0.4	15.9046	15.9167	15.9031	0.441155	0.270641	0.505127	1.4	12.9593	12.957	12.956	0.299865	0.142043	0.205127
0.5	15.5948	15.6057	15.5939	0.427026	0.257858	0.475127	1.5	12.6834	12.6794	12.6789	0.285736	0.1291	0.175127
0.6	15.2884	15.298	15.288	0.412897	0.245056	0.445127	1.6	12.4109	12.4052	12.405	0.271607	0.116143	0.145127
0.7	14.9854	14.9937	14.9853	0.398768	0.232237	0.415127	1.7	12.1418	12.1343	12.1343	0.257478	0.103174	0.115127
0.8	14.6858	14.6927	14.6858	0.384639	0.219399	0.385127	1.7702	11.9549	11.9462	11.9462	0.247559	0.094062	0.0940668
0.80307	14.6767	14.6835	14.6767	0.384205	0.219005	0.384206	1.7703	11.9546	11.9459	11.9459	0.247545	0.094049	0.0940368
0.80308	14.6767	14.6835	14.6767	0.384204	0.219004	0.384203	1.8	11.876	11.8668	11.8668	0.243349	0.090192	0.0851268
0.9	14.3896	14.3951	14.3895	0.37051	0.206545	0.355127	1.9	11.6137	11.6026	11.6025	0.22922	0.077199	0.0551268
1	14.0968	14.1008	14.0964	0.356381	0.193675	0.325127	2	11.3547	11.3418	11.3414	0.215091	0.064194	0.0251268

$$q_R(w, a_H) = q_R^{\mathrm{SI}}(w, a_H) = \frac{(2w - (V\eta - c)\gamma)\left(1 - \gamma^2\right) + be(2 - \gamma)(1 + \gamma) + \left(2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta\right)a_H}{2(2 - \gamma^2)}.$$

Then the manufacturer's expected profit at this point is $\pi_M^{\mathrm{RZ}}(w)$. Lemma 4 can be obtained by comparing the value of $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$ and $\pi_M^{\mathrm{RZ}}(w^{\mathrm{RZ}^*})$.

Lemma 4. *When the recycling cost of the online recycling channel is less than a certain critical value, the recycling quantity of the remanufacturer is zero when the scale of the recycling market is small. In this case, the optimal solution for the recycling price is w^{RZ^*}. The corresponding offline recycling quantity and the recycling quantity of the online recycling channel of the remanufacturer are $q_R(w^{\mathrm{RZ}^*}, a_i)$ and $q_M(w, a_i)$. The profit of the remanufacturer is maximized as $\pi_M^{\mathrm{RZ}}(w^{\mathrm{RZ}^*})$. Specific definitions can be found in the appendix.*

4.2.4. Analysis of numerical experiments

In order to gain a more intuitive view of the results of the recycling price decision, as well as to support the results of the model analysis more significantly, numerical experiments with visual images of the recycling price decision will be conducted in this study. The first and second scenarios are used as the basis for illustration. The relevant parameters are set as follows: $a_H = 2.5$, $a_L = 1.5$, $\theta = 0.8$, $\gamma = 0.6$, $\lambda = 0.35$, $\eta = 0.6$, $b = 0.6$, $e = 1$, $k = 2$, $V = 12$.

(1) The better solution for the remanufacturer when the recycler is distorting *versus* not distorting the information when the boundary condition $\overline{w}$ is not limited.
The specific values of the remanufacturer's profit and recovery price when the recovery cost of the online recycling channel takes different values are shown in Table 1.

(2) The overall maximum value of the remanufacturer's profit function and the corresponding recovery price.
When $w \leq \overline{w}$, the recycler does not distort the recycled quantity; when $w > \overline{w}$, the recycler distorts the recycled quantity downward to disrupt the judgment of the remanufacturer when the recycling market is small. Combined with the previous analysis of the four possible scenarios, it is possible to derive the value of the remanufacturer's return when the recycling price is limited to the boundary condition $\overline{w}$, as shown in Table 2.
By calculating and comparing the data in Tables 1 and 2, it can be found that when the remanufacturer's recovery cost in the online channel is $0 < c \leq 0.80307$, $w^{\mathrm{ID}^*} < w^{\mathrm{NS}^*} < \overline{w}$, and the maximum is $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$. When $0.80308 \leq c \leq 1.7702$, $w^{\mathrm{ID}^*} < \overline{w} < w^{\mathrm{NS}^*}$ and the maximum is $\pi_M(\overline{w})$. When $1.7703 \leq c \leq 2$, $\overline{w} < w^{\mathrm{ID}^*} < w^{\mathrm{NS}^*}$ and the maximum is $\pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$. Observation of the resulting diagram provides a more intuitive view of this pattern (as shown in Fig. 5).
The above three results correspond to cases (3), (1), and (2) shown previously.
When $w^{\mathrm{ID}^*} < w^{\mathrm{NS}^*} < \overline{w}$, $\pi_M(w^*) = \pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$;

TABLE 2. Recycling prices of remanufacturers when costs of online channels take different values.

c	π_M	w^*	c	π_M	w^*
0.1	16.8542	0.483542	1.1	13.8065	0.295127
0.2	16.5342	0.469413	1.2	13.5198	0.265127
0.3	16.2177	0.455284	1.3	13.2363	0.235127
0.4	15.9046	0.441155	1.4	12.956	0.205127
0.5	15.5948	0.427026	1.5	12.6789	0.175127
0.6	15.2884	0.412897	1.6	12.405	0.145127
0.7	14.9854	0.398768	1.7	12.1343	0.115127
0.8	14.6858	0.384639	1.7702	11.9462	0.0940668
0.80307	14.6767	0.384205	1.7703	11.9459	0.094049
0.80308	14.6767	0.384203	1.8	11.8668	0.090192
0.9	14.3895	0.355127	1.9	11.6026	0.077199
1	14.0964	0.325127	2.0	11.3418	0.064194

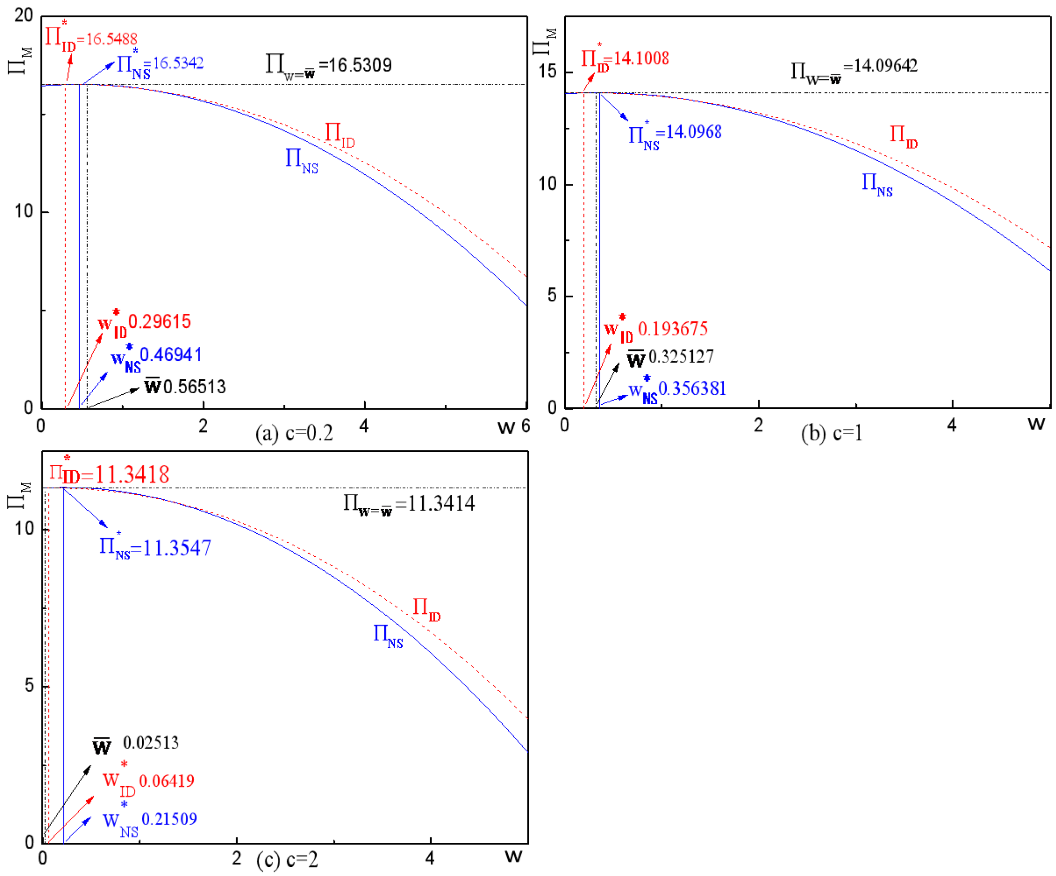

FIGURE 5. The impact of online channel costs on bulk recycling prices.

When $w^{\mathrm{ID}^*} < \overline{w} < w^{\mathrm{NS}^*}$, $\pi_M(w^*) = \pi_M^{\mathrm{NS}}(\overline{w}) = \pi_M^{\mathrm{ID}}(\overline{w})$;

When $\overline{w} < w^{\mathrm{ID}^*} < w^{\mathrm{NS}^*}$, $\pi_M(w^*) = \pi_M^{\mathrm{ID}}(w^{\mathrm{ID}^*})$.

(3) Boundary conditions for the cost of online recycling channels.
Determine boundary conditions for the cost of the online channel ($\overline{c}$) that makes the recycler's recycling quantity zero when the recycling market scale is small. When $w^{\mathrm{NS}^*} \le \overline{w}$, there exist $\overline{c}$ such that $c \le \overline{c}$, and the maximum value of $\pi_M(w^*)$ is $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$. At this point, the maximum value of $\pi_M^{\mathrm{RZ}}(w)$ is $\pi_M^{\mathrm{RZ}}(w^{\mathrm{RZ}^*})$. It can be found that $\pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*}) < \pi_M^{\mathrm{RZ}}(w^{\mathrm{RZ}^*})$.

In addition, considering the fact that when the cost of online recycling is very large, the quantity recycled by the remanufacturer through the online channel may be zero, notate this boundary condition as c_G, which indicates that the remanufacturer's recycling quantity through the online recycling channel is zero when the demand in the recycling market is minimized. The following provides subgame equilibriums for the recycler for different costs of online recycling.

(1) When $0 < c \le \min(\overline{c}, c_G)$, $\pi_M(w^*) = \pi_M^{\mathrm{NS}}(w^{\mathrm{NS}^*})$.

$$q_R(w^*, a_H) = \frac{(2w^* - (V\eta - c)\gamma)\big(1-\gamma^2\big) + (2be - be\gamma)(1+\gamma) + \big(2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta\big)a_H}{2(2-\gamma^2)}$$

$$q_R(w^*, a_L) = \frac{(2w^* - (V\eta - c)\gamma)\big(1-\gamma^2\big) + (2be - be\gamma)(1+\gamma) + \big(2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta\big)a_L}{2(2-\gamma^2)}.$$

(2) When $\min(\overline{c}, c_G) < c < c_G$, $w^* = \min\big(w^{\mathrm{ID}^*}, \overline{w}\big)$.

$$q_R(w^*, a_H) = \frac{(2w^* - (V\eta - c)\gamma)(1-\gamma^2) + (2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_H}{2(2-\gamma^2)}$$

$$q_R(w^*, a_L) = \overline{q}_R(w^*, a_L).$$

5. Factor analysis

Market share, channel substitution coefficients, and environmental abatement costs also affect the decision of the remanufacturer to invade the recycling channel as well as the earnings of the firms involved. For ease of analysis, only the scenario when information is not distorted is discussed ($w^* = w^{\mathrm{NS}^*}$). In this case, the profit function of the remanufacturer is $\pi_M^{\mathrm{NS}}\big(w^{\mathrm{NS}^*}\big)$ and the profit function of the recycler is $\pi_R^{\mathrm{NS}}\big(w^{\mathrm{NS}^*}\big)$.

5.1. Impact of online recycling channel market share

The related parameters are set as follows:

$$a_H = 2.5,\ a_L = 1.2,\ \gamma = 0.6,\ \lambda = 0.4,\ \eta = 0.5,\ b = 0.55,\ e = 1,\ V = 10,\ k = 1.5.$$

This section discusses only the case of $0 < c \le \min(\overline{c}, c_G)$ at $w^* = w^{\mathrm{NS}^*}$. The results are shown in Figure 6.

As seen in Figure 6, regardless of the cost of the online channel, as its market share increases, the benefits to the recycler are monotonically decreasing and the benefits to the remanufacturer are monotonically increasing. In addition, observing the profit curve of remanufacturer expanding and not expanding online direct recovery channel, it can be found that when the network direct recovery cost c is small, when the cost of the online channel c is small, there exists θ_1 such that, when $\theta\epsilon(0, \theta_1)$, the benefits to both remanufacturers and recyclers are greater than in the case of not developing the online channel. When $\theta\epsilon(\theta_1, 1)$, invading the online recycling channel benefits only the remanufacturer and is detrimental to the recycler. When the cost of the online channel is large, there exists θ_2 such that, when $\theta\epsilon(0, \theta_2)$, the gains from invading the online recycling channel for the remanufacturer are smaller than the gains without invasion, and θ_2 tends to increase as the cost of online recycling increases. When $\theta\epsilon(0, \theta_3)$, the recycler's profit is better when the remanufacturer invades the recycling

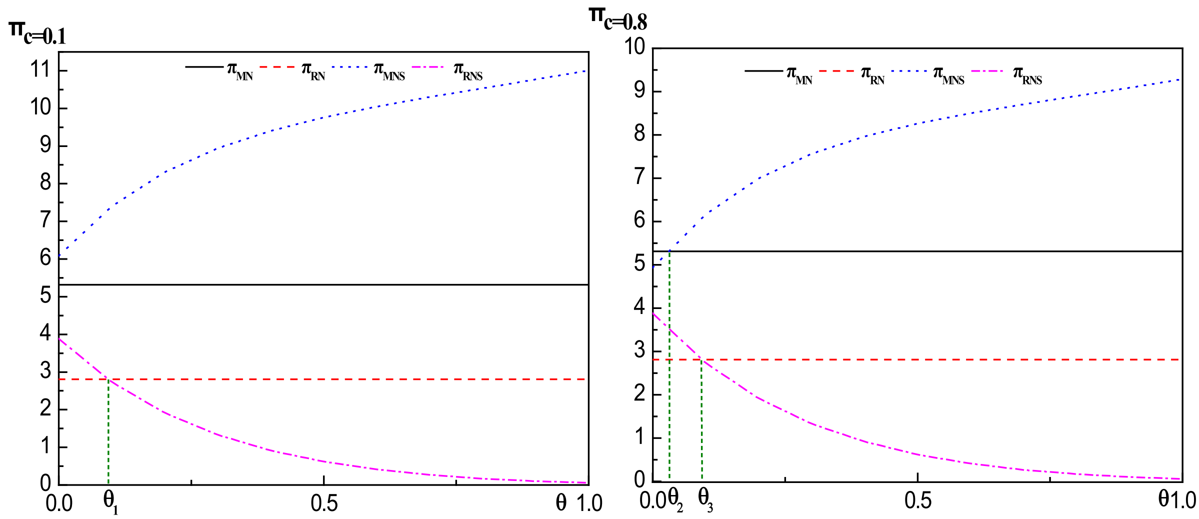

FIGURE 6. The impact of online recycling market share on remanufacturers' and recyclers' profits ($c = 0.1$ & $c = 0.8$).

channel than without invasion. When $\theta\epsilon(\theta_2, \theta_3)$, the decision to develop an online recycling channel benefits both the remanufacturer and the recycler. However, when $\theta\epsilon(\theta_3, 1)$, the development of an online recycling channel can only benefit the remanufacturer and is detrimental to the recycler.

As a result, there is a critical threshold of market share for the online recycling channel, making the establishment of an online recycling channel by the remanufacturer a "win–win", "win–loss", or "win–loss" scenario for both parties. And the cost of online recycling will have a greater impact on the final outcome of the game. In deciding whether to expand online direct recycling channels and making decisions such as recycling prices, remanufacturers need to consider the cost of recycling and the possible scale of market share of online recycling channels.

5.2. Impact of waste reutilization rate

The main purpose of recycling electronic waste is to reuse and secondary manufacturing to achieve the resourcefulness of waste and the reuse of resources. However, the current production of electronic products, end-of-life recycling, and other pre-processes of reuse do not consider the feasibility of reuse in advance. Because there is no uniform production standard, the feasibility and utilization rate of recycling is low. Considering the reality of the low utilization rate of e-waste recycling, the post-recycling treatment and reproduction of e-waste, known as "urban minerals", is also an important factor hindering the process of waste recycling.

Therefore, the reutilization rate of e-waste is an important parameter that affects the profitability of remanufacturers and recyclers. Exploring the impact of e-waste reusability on the profitability of remanufacturers and recyclers is important for remanufacturers and recyclers to optimize their operations and decision-making. The following will be analyzed through an arithmetic example. The relevant parameters are set as follows:

$$a_H = 2.5,\ a_L = 1.2,\ \gamma = 0.6,\ \lambda = 0.4,\ \eta = 0.5,\ b = 0.55,\ e = 1,\ V = 10,\ k = 1.5.$$

According to Figure 7, the reutilization rate of recycled e-waste has a more significant impact on the profits of both remanufacturers and recyclers. Both recycler and remanufacturer profits are significantly higher as the recycled e-waste reutilization rate increases. This tendency diminishes for remanufacturers as the cost of recycling increases in the online recycling channel. When the reusability of recyclables is low, the overall profit of the supply chain is lower. Especially when online recycling channels are not developed, the low reusability

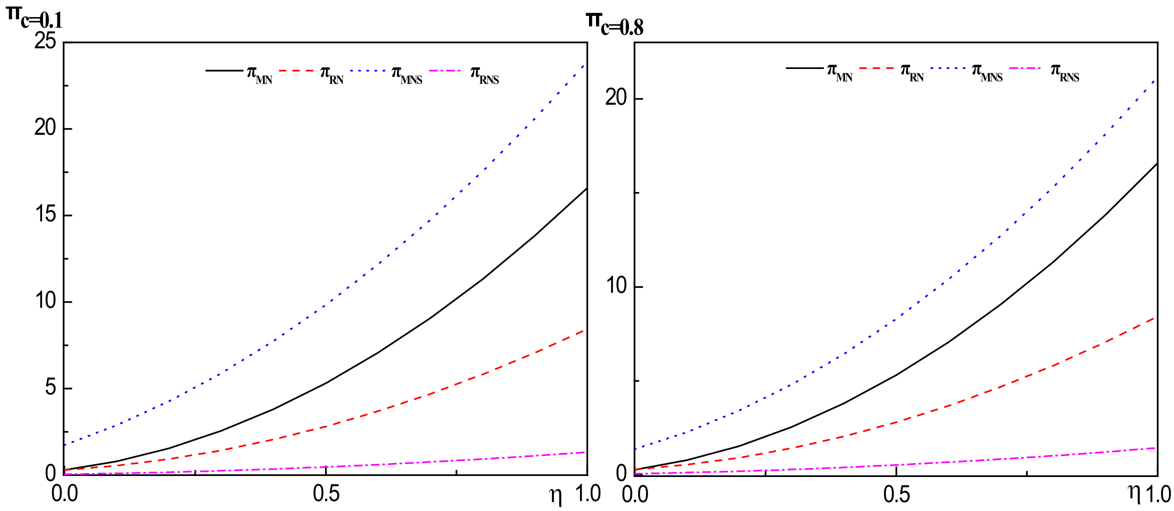

FIGURE 7. Impact of waste reutilization rate η on profits of remanufacturers and recyclers.

rate makes the profit of the remanufacturer almost lower than that of the recycler, and the ineffective labor increases. Therefore, one of the effective ways to improve the efficiency of waste recycling is to enhance the utilization rate of recycled e-waste. The actual production can be upgraded by increasing the technical input of remanufacturing and reuse technology. Remanufacturers can coordinate with the production chain to carry out unified parts transformation and design to enhance the reuse rate of end-of-life electronic products and the matching degree of new and old products, and prepare for remanufacturing from the design stage to improve the reuse rate of waste.

5.3. Impact of pollution emission reduction input cost coefficients

With the emphasis on pollution reduction and net-zero carbon emissions in the regions, more and more consumers are inclined to buy environmentally friendly products, which in turn incentivizes corporations to take the initiative to adopt low-pollution production procedures and build a responsible social profile. More and more companies, such as the Walmart, IBM and Toshiba, have adopted low-carbon strategies and have seen significant benefits. At the same time, more and more consumers are willing to pay more for environmentally friendly products than ordinary products. Consumers' purchasing preferences, in turn, influence companies' environmentally friendly production procedures. The key to the implementation of low-pollution strategies lies in low-pollution technology, which is often directly related to the cost of low-pollution renovation, and can be reflected in the pollution reduction cost input coefficient k in the assessment of the actual effect. Research on the impact of pollution emission reduction cost input coefficient on the profit relationship between remanufacturers and recyclers is meaningful to the decision-making and operation management of enterprises. The following will be analyzed through the arithmetic example. The relevant parameters are set as follows:

$$a_H = 2.5,\ a_L = 1.2,\ \gamma = 0.6,\ \lambda = 0.4, \theta = 0.5,\ b = 0.55,\ e = 1,\ V = 8,\ \eta = 0.5.$$

From Figure 8, it can be seen that the larger k is, that is, when more cost is needed to raise the pollution reduction e per unit ($C_{(e_m)} = \frac{ke^2}{2}$), the profits of remanufacturers and recyclers show a decreasing trend. The profit curve shows the same trend when the recycling cost of the online recycling channel is lower or higher. When $k < k_1$, the remanufacturer opens the online recycling channel can bring higher profit, but cannot bring higher profit to the recycler, then the remanufacturer usually chooses to open the recycling channel. When $k > k_1$, the remanufacturer's establishment of a recycling channel jeopardizes the interests of its company and

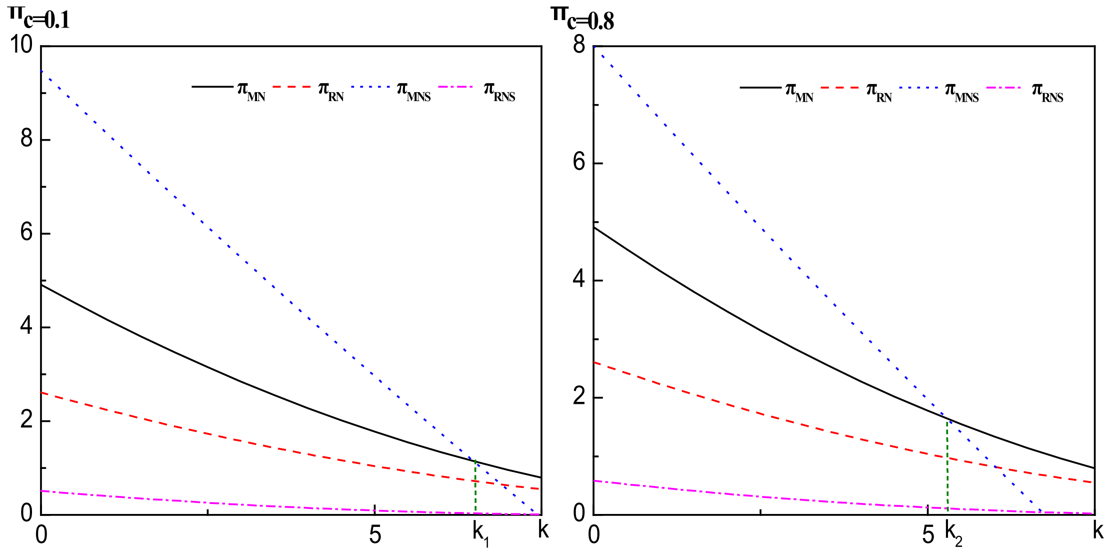

FIGURE 8. Impact of the coefficient k on the profits of remanufacturers and recyclers.

recyclers, and the remanufacturer generally chooses not to establish an online recycling channel. Overall, it is not in the interest of the recycler for the remanufacturer to develop an online recycling channel, and as the cost coefficient of pollution reduction increases, the remanufacturer's profitability will be jeopardized by the establishment of an online recycling channel.

6. Discussion

6.1. Remanufacturers' belief criteria for recyclers' distortion of recycled quantities

On the one hand, this study derives the recovery prices of recyclers and remanufacturers in the case of asymmetric information on recycling demand, as well as an analysis of the decision of remanufacturers to take into account the downward distortion of recycling quantities by recyclers and thus the adjustment of the belief criteria. The study also discusses the boundary conditions for the remanufacturer's bulk recovery price when the recycler generates the behavior of distorting the recycled quantity downward, when the recycling market scale is large. And it derives the boundary condition that when the market scale is large, it is profitable for the recycler to distort the recycling quantity to a critical value. It is found that there is a relationship between the boundary conditions for the bulk recycling price and the boundary conditions for the recycling quantity when information distortion occurs. It is also pointed out that the remanufacturer can better hedge the risk of information distortion from recyclers by adjusting the belief criterion to $\widehat{q_R}(w)$.

6.2. Impact of offline recycling quantities on bulk recycling prices of remanufacturers

This study also analyzes and discusses the influences on the price paid for recycling by remanufacturers and evaluates the influence of the relevant parameter variations in the model on the profitability and decision-making of remanufacturers and recyclers. First, the effect of recycler's recovery quantity on the remanufacturer's bulk recycling price when the recycling market scale is small is briefly discussed, and it is found that when the bulk recycling price is greater than $\overline{w}$, the recycler will distort the recycling quantity downward in order to distort the information of the recycling market to interfere with the decision-making of the remanufacturer. At this point, the remanufacturer perceives the current recycling market to be smaller, which increases its willingness to increase the price it pays for recycling and the price it pays for bulk recycling, and the remanufacturer's gains from the online recycling channel will be jeopardized. Considering a situation where a recycler recovers a large amount of waste but delivers only a portion of it to a remanufacturer, there is a trade-off between the loss of profit from the backlog of inventory and lower quantities delivered, and the increase in profit from higher bulk recovery prices, and there is a critical threshold that makes the remanufacturer and the recycler both lose out, increasing the double marginalization.

6.3. The impact of the cost of online recycling channels on remanufacturers' bulk recycling prices

This study also analyzes the bulk recycling prices of remanufacturers when the recycling costs of the online recycling channel are in different value ranges, and calculates the critical values for changing different states. It is also found that when the online channel is smaller than the critical value $\overline{c}$, the remanufacturer has the highest profit in the case of no information distortion because in this situation, the remanufacturer is able to accurately judge the scale of the current recycling market and reasonably determine the recycling price and manage the online channel with transparent information to obtain higher profit. On the contrary, when the cost of online recycling channels is large and exceeds a critical value c_G, the remanufacturer is able to obtain higher earnings in an information-distorting scenario. When the cost of the online channel is very high, the quantity recycled by the remanufacturer through the online channel may be zero. Remanufacturers can maximize profits when the cost of recycling through the online channel is in the middle of the two thresholds, and when the bulk price of recycling is set at a threshold where the recycler does not distort the demand information in the recycling market.

6.4. Impact of fluctuations in relevant parameters on the recycling supply chain

(1) This study considers the impact of market share of online and offline recycling channels on the profits of remanufacturers and recyclers. Regardless of the cost of the online channel, as the market share of the online channel increases, the recycler's profit decreases while the remanufacturer's profit increases. This study analyzes the impact of market share changes on firms' profits and gives a schematic diagram of the critical point through numerical simulation to make the results clearer. Under the scenario of the online channel with low cost, when the market share of the online recycling channel is small, it is beneficial to both the remanufacturer and the recycler to explore the online channel; when the market share of the online recycling channel is large, it is beneficial to the remanufacturer and unfavorable to the recycler to explore the online recycling channel. When the online channel is costly and the market share of the online channel is small, expanding the online channel is beneficial to recyclers but not to remanufacturers. When the market share of the online channel is within a certain range, the expansion of the online recycling channel is beneficial to both companies. However, when the online recycling channel gains a large market share, the exploitation of the channel can only benefit the remanufacturer but not the recycler.

(2) The study examines the impact of waste reuse rates on the profits of remanufacturers and recyclers. The destination of e-waste after recycling is generally for remanufacturing, and a few recyclables that cannot be used for remanufacturing are sold at the value of the raw materials, but such treatment leads to a secondary depreciation of the waste compared to remanufacturing. Since not all recycled e-waste can be used for remanufacturing, the benefits of the reuse process are generally closely associated with the reuse rate of recycled waste. Therefore, it is of practical significance to study the impact of e-waste reuse rate on the reverse supply chain. It is found that as the waste reuse rate increases, there is a significant increase in the benefits for both remanufacturers and recyclers. Recyclers can control the quality of recycling by increasing the threshold and standard of the recycling procedure. The remanufacturers can increase the reuse rate of waste by increasing the investment in technological development of the remanufacturing process and upgrading the technology to improve the efficiency of waste recycling, which will in turn increase the profit of the supply chain overall.

(3) This study also discusses the impact of environmental input cost coefficients in the remanufacturing process on the profits of remanufacturers and recyclers. In recent years, various countries and regions have been paying more and more attention to green and sustainable development, and more consumers have begun to pay attention to and tend to buy environmentally friendly products, so the emission reduction treatment of the remanufacturing process is imperative. Green production process can often attract more consumers to buy environmentally friendly products or services, but the realization of low-pollution production needs to invest a high amount of low-pollution technology transformation costs. The green production technology input cost coefficient is important in decision making, and an increase in this coefficient means that higher cost inputs are required to reduce pollution emissions per unit of production. As the coefficient increases, the profitability of remanufacturers and recyclers decreases, and this trend becomes more pronounced in a dual-channel recycling model. Therefore, technological innovation for emission reduction in the remanufacturing process is also an aspect of waste reuse that needs to be continuously enhanced and optimized so as to improve the efficiency of the reverse supply chain and waste resourcing from remanufacturing.

The reduction of e-waste and waste resourcing requires the cooperation of multiple links, and the continuous optimization of each link can guarantee the efficient operation of the reverse supply chain. This study involves the recycling, disposal, and remanufacturing processes, and tries to optimize the key segments of the reverse supply chain to improve the efficiency and accelerate the process of waste resourcing, and to promote the sustainable development of the economy and the environment.

7. Conclusion

Based on the background of e-waste recycling, the study explores the impacts of information asymmetry on remanufacturers' judgment of the recycling market and recycling price decisions by the truthfulness of recyclers' information sharing, and explores the impacts of remanufacturers' decisions on the exploration of online recycling channels by recyclers' information sharing. We further analyze the trend of the influence of realistic factors on the profitability of remanufacturers and recyclers when online recycling channels exist, and based on this, we propose effective management strategies.

7.1. Main findings and managerial insights

The process of e-waste reduction treatment and waste recycling requires close connection and cooperation of all links, and the continuous optimization of each link can ensure the efficient operation of the reverse supply chain. By studying the interactive relationship between recycling cost and online channel expansion on the profits of remanufacturers and recyclers, this paper finds that when other factors remain unchanged, with the expansion of online channels on a single platform (the increase in the market share of online channels), the more extensive the coverage of online channels is, the more beneficial it is to remanufacturers but not to recyclers. However, compared with the situation where there is no online recycling channel, the impact of the decision to open online channels on the development of both enterprises is uncertain and will be affected by the online recycling cost, and this chapter gives the influencing law. Therefore, whether to open an online channel or not requires the remanufacturer to make a better decision based on multiple factors such as the cost of online channel recovery. In addition, at the end of this chapter, the influence of technical factors in the processing and remanufacturing process of electronic waste after recycling is discussed through numerical experiments, mainly involving the recycling rate and environmental protection factors in the processing process, and the influence law is obtained. Remanufacturers should strive to improve the level of disassembly and processing and pay attention to environmentally friendly production at the same time, through the improvement of environmentally friendly disassembly and processing technology, to increase profits. According to the conclusions of this paper, in the dual-channel recycling supply chain, the remanufacturer should predict the demand information distortion behavior of recyclers, and make decisions on recycling price and channel expansion under the premise of fully considering this factor, so as to avoid the risk of revenue loss. Since the conclusion of this chapter finds that the recyclers' behavior of distorted market demand information is also affected by the bulk recycling price of upstream remanufacturers, and this influence process is interfered by the recycling cost, remanufacturers can properly consider the recycling cost and market environment when making price decisions, and then make recycling price decisions closer to the critical point, so as to obtain higher profits. The research conclusions can provide some management insights and theoretical support for the management of recycling supply chain and the coordination of competition and cooperation between enterprises, and then contribute to the acceleration and efficiency improvement of waste recycling development, and promote the sustainable development of economy and environment.

7.2. Limitations and future research

First, in order to simplify the model, this study assumes that there are no additional inventory costs, storage costs, and wastage of e-waste collected by recyclers when not all of it is passed on to the remanufacturers, which does not correspond to the actual situation. Most recyclers have the issues of backlogging inventory and waiting for the right time to sell, and this inventory issue is extremely uncertain, which may bring losses to the recycler's profitability, and future research can consider the study of the related issues in the uncertainty scenarios. The calculation of the quantity of waste in this study does not take into account individual differences, product weight, and the severity of damages, while in reality, the residual value of electronic and electrical products is not uniformly measured due to their different performance and deterioration, and there is a great deal of heterogeneity, and future research can consider these factors to conduct a finer classification study.

Second, in the demand information asymmetry scenario discussed in this paper, only the recyclers have private information about the scale of the recycling market, and the remanufacturers do not have any acquisition of market information, but can only guess the scale of the recycling market from the quantities of recycling supplied by the recyclers. In reality, once remanufacturers establish online direct recycling channels, they have their own personal access to information. And with the development of information technology, remanufacturers are gradually able to make more realistic bulk recycling price decisions and recycling quantity orders based on the information they acquire. However, as long as the remanufacturer has not obtained all the information, the signaling game between the recycler and the remanufacturer still exists. There also exists a more special case where both companies have incomplete information, which may lead to the two being signal receivers and senders of each other. All these deserve further research to achieve a more accurate replication of the real situation with modeling to address the real problem of waste resourcing.

Appendix A.

A.1. Proof of Proposition 1

In model M1, all members aim to maximize their respective profits, and the decision problem can be equated to solving the following planning problem.

$$\max_{w} \pi_M = (V\eta - w - C_{(e_m)})(a_i + p_R + be)$$
$$\max_{p_R} \pi_R = (w - p_R)(a_i + p_R + be).$$

First, the remanufacturer decides w; subsequently, the offline recycler decides p_R. This reverse supply chain is a Stackelberg game among its members and is analyzed using backward projection. The profit function of the recycler is derived:

$$\frac{\partial \pi_R}{\partial p_R} = -2p_R + w - a_i - be = 0 \to p_R = \frac{w - a_i - be}{2}.$$

Based on this, the benefit function of the remanufacturer is: $\pi_M = \left(a_i + \frac{w-a_i-be}{2} + be\right)(V\eta - w - C_{(e_m)})$.

$$\frac{\partial \pi_M}{\partial w} = \frac{1}{2}\left(V\eta - 2w - a_i - be - C_{(e_m)}\right) = 0 \to \; w^* = \frac{V\eta - \mu - be - C_{(e_m)}}{2}$$

$$\pi_M{}^* = \frac{1}{8}\left(V\eta - \frac{ke^2}{2} + \mu + be\right)^2$$
$$p_R{}^* = \frac{V\eta - \frac{ke^2}{2} - 3\mu - 3be}{4}$$
$$\pi_R{}^* = \frac{\left(V\eta - \frac{ke^2}{2} - \mu + be\right)^2 + 4\left(V\eta - \frac{ke^2}{2} + be\right)\mu + 4\sigma^2}{16}.$$

A.2. Proof of Lemma 2

$$\begin{aligned} V_{ij}(q_R) &= \left[\left(w - \frac{q_R + \gamma q_M}{1-\gamma^2} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i + \frac{be(1+\gamma)}{1-\gamma^2}\right)q_R\right] \\ &= \left[\left(w - \frac{(V\eta - c)\gamma}{2} + \frac{2be - be\gamma}{2(1-\gamma)} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i - \frac{(\gamma-\gamma\theta+\theta)\gamma a_j}{2(1-\gamma^2)}\right)q_R - \frac{(2-\gamma^2){q_R}^2}{2(1-\gamma^2)}\right], \\ &\quad \forall i, j \epsilon \{H, L\} \\ \frac{\mathrm{d}V_{ii}(q_R)}{\mathrm{d}q_R} &= w - \frac{(V\eta - c)\gamma}{2} + \frac{2be - be\gamma}{2(1-\gamma)} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_i - \frac{(\gamma-\gamma\theta+\theta)\gamma a_i}{2(1-\gamma^2)} - \frac{(2-\gamma^2)q_R}{1-\gamma^2} \\ \frac{\mathrm{d}^2V_{ii}(q_R)}{\mathrm{d}{q_R}^2} &= -\frac{(2-\gamma^2)}{1-\gamma^2} < 0. \end{aligned}$$

The second-order derivative of $V_{ii}(q_R)$ with respect to q_R is less than zero, and thus the function is concave, such that $\frac{\mathrm{d}V_{ij}(q_R)}{\mathrm{d}q_R} = 0$, the optimal solution in the case of full information sharing (SI) is obtained:

$$q_R^{\mathrm{SI}}(w^*, a_i) = \frac{(2w - (V\eta - c)\gamma)(1-\gamma^2) + (2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_i}{2(2-\gamma^2)}, \qquad i\epsilon\{H, L\}.$$

A.3. Proof of Proposition 2

$$\begin{aligned} V_{HL}\left(q_R^{\mathrm{SI}}(w^*, a_L)\right) &= \left(w - \frac{(V\eta - c)\gamma}{2} + \frac{2be - be\gamma}{2(1-\gamma)} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_H - \frac{(\gamma-\gamma\theta+\theta)\gamma a_L}{2(1-\gamma^2)}\right) \\ &\quad \times \frac{(2w - (V\eta - c)\gamma)(1-\gamma^2) + (2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_L}{2(2-\gamma^2)} \\ &\quad - \frac{(2-\gamma^2)}{2(1-\gamma^2)}\left(\frac{(2w - (V\eta - c)\gamma)(1-\gamma^2) + (2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_L}{2(2-\gamma^2)}\right)^2 \\ V_{HH}\left(q_R^{\mathrm{SI}}(w^*, a_H)\right) &= \left(w - \frac{(V\eta - c)\gamma}{2} + \frac{2be - be\gamma}{2(1-\gamma)} + \frac{1-\theta+\gamma\theta}{1-\gamma^2}a_H - \frac{(\gamma-\gamma\theta+\theta)\gamma a_H}{2(1-\gamma^2)}\right) \\ &\quad \times \frac{(2w - (V\eta - c)\gamma)\left(1-\gamma^2\right) + (2be - be\gamma)(1+\gamma) + \left(2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta\right)a_H}{2(2-\gamma^2)} \\ &\quad - \frac{(2-\gamma^2)}{2(1-\gamma^2)}\left(\frac{(2w - (V\eta - c)\gamma)(1-\gamma^2) + (2be - be\gamma)(1+\gamma) + (2 - 2\theta + \gamma\theta - \gamma^2 + \gamma^2\theta)a_H}{2(2-\gamma^2)}\right)^2. \end{aligned}$$

According to $V_{HL}\left(q_R^{\mathrm{SI}}(w^*, a_L)\right) \leq V_{HH}\left(q_R^{\mathrm{SI}}(w^*, a_H)\right)$:

$$\overline{w} = \frac{A + B + C}{D + E + F}$$

$s=\theta,\ r=\gamma,\ t=\eta,\ a_H=H,\ a_L=L$

$$A=\frac{\big(\big(sr^2-r^2+sr-2s+2\big)H-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)\left(\frac{(r-2)be}{2r-2}+\frac{Hr(-sr+r+s)}{2r^2-2}+\frac{1}{2}r(c-tv)-\frac{H(rs-s+1)}{r^2-1}\right)}{2r^2-4}$$

$$B=-\frac{\big(\big(sr^2-r^2+sr-2s+2\big)L-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)\left(\frac{(r-2)be}{2r-2}+\frac{Lr(-sr+r+s)}{2r^2-2}+\frac{1}{2}r(c-tv)-\frac{H(rs-s+1)}{r^2-1}\right)}{2r^2-4}$$

$$C=\frac{\big(r^2-2\big)\big(\big(sr^2-r^2+sr-2s+2\big)H-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)^2}{(2r^2-2)(2r^2-4)^2}-\frac{\big(r^2-2\big)\big(\big(sr^2-r^2+sr-2s+2\big)L-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)^2}{(2r^2-2)(2r^2-4)^2}$$

$$D=\frac{\big(2r^2-2\big)\left(\frac{(r-2)be}{2r-2}+\frac{Hr(-sr+r+s)}{2r^2-2}+\frac{1}{2}r(c-tv)-\frac{H(rs-s+1)}{r^2-1}\right)}{2r^2-4}+\frac{2\big(r^2-2\big)\big(\big(sr^2-r^2+sr-2s+2\big)H-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)}{(2r^2-4)^2}$$

$$E=\frac{\big(sr^2-r^2+sr-2s+2\big)L-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)}{2r^2-4}-\frac{\big(2r^2-2\big)\left(\frac{(r-2)be}{2r-2}+\frac{Lr(-sr+r+s)}{2r^2-2}+\frac{1}{2}r(c-tv)-\frac{H(rs-s+1)}{r^2-1}\right)}{2r^2-4}$$

$$F=-\frac{\big(sr^2-r^2+sr-2s+2\big)H-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)}{2r^2-4}-\frac{2\big(r^2-2\big)\big(\big(sr^2-r^2+sr-2s+2\big)L-r(c-tv)\big(r^2-1\big)+(r+1)(2be-ber)\big)}{(2r^2-4)^2}.$$

A.4. Proof of Proposition 3

There exists a critical value $\overline{q}_R(w)$ such that $V_{HL}(\overline{q}_R(w,a_L))\le V_{HH}(q_R^{\mathrm{SI}}(w,a_H))$.

$$V_{HL}(\overline{q}_R(w,a_L))=\left[\left(w-\frac{(V\eta-c)\gamma}{2}+\frac{2be-be\gamma}{2(1-\gamma)}+\frac{1-\theta+\gamma\theta}{1-\gamma^2}a_H-\frac{(\gamma-\gamma\theta+\theta)\gamma a_L}{2(1-\gamma^2)}\right)q_R-\frac{(2-\gamma^2){q_R}^2}{2(1-\gamma^2)}\right]$$

$$\begin{aligned}V_{HH}\Big(q_R^{\mathrm{SI}}(w,a_H)\Big)&=\left(w-\frac{(V\eta-c)\gamma}{2}+\frac{2be-be\gamma}{2(1-\gamma)}+\frac{1-\theta+\gamma\theta}{1-\gamma^2}a_H-\frac{(\gamma-\gamma\theta+\theta)\gamma a_H}{2(1-\gamma^2)}\right)\\&\times\frac{(2w-(V\eta-c)\gamma)\big(1-\gamma^2\big)+(2be-be\gamma)(1+\gamma)+\big(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\big)a_H}{2(2-\gamma^2)}\\&-\frac{\big(2-\gamma^2\big)}{2(1-\gamma^2)}\left(\frac{(2w-(V\eta-c)\gamma)(1-\gamma^2)+(2be-be\gamma)(1+\gamma)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H}{2(2-\gamma^2)}\right)^2\end{aligned}$$

$$V_{HL}(\overline{q}_R(w,a_L))\le V_{HH}\Big(q_R^{\mathrm{SI}}(w,a_H)\Big)$$

$$\begin{aligned}&V_{HL}(\overline{q}_R(w,a_L))-V_{HH}\Big(q_R^{\mathrm{SI}}(w,a_H)\Big)\\&=-{q_R}^2+\frac{(2w-(V\eta-c)\gamma)\big(1-\gamma^2\big)+be(2-\gamma)(1+\gamma)+2a_H(1-\theta+\gamma\theta)-(\gamma-\gamma\theta+\theta)\gamma a_L}{2-\gamma^2}q_R\\&-\left[\frac{(2w-(V\eta-c)\gamma)\big(1-\gamma^2\big)+be(2-\gamma)(1+\gamma)+\big(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\big)a_H}{2(2-\gamma^2)}\right]^2\le 0\end{aligned}$$

$$q_{R1,2}(w) = \frac{2(1-\theta+\gamma\theta)a_H - \gamma(\gamma-\gamma\theta+\theta)a_L + (2w-(V\eta-c)\gamma)(1-\gamma^2) + be(2-\gamma)(1+\gamma)}{2(2-\gamma^2)}$$
$$\mp \frac{1}{2(2-\gamma^2)}\sqrt{\begin{array}{c}\gamma(\theta+\gamma-\gamma\theta)(a_H-a_L)\Big[\big((4-\gamma^2)(1-\theta)+3\gamma\theta\big)a_H \\ -\gamma(\gamma(1-\theta)+\theta)a_L + 2(2w-(V\eta-c)\gamma)(1-\gamma^2) + 2be(2-\gamma)(1+\gamma)\Big]\end{array}}.$$

Combined with Figure 3 to get:

$$\overline{q}_R(w) = \frac{2(1-\theta+\gamma\theta)a_H - \gamma(\gamma-\gamma\theta+\theta)a_L + (2w-(V\eta-c)\gamma)(1-\gamma^2) + be(2-\gamma)(1+\gamma)}{2(2-\gamma^2)}$$
$$- \frac{\sqrt{\begin{array}{c}\gamma(\theta+\gamma-\gamma\theta)(a_H-a_L)\Big[\big((4-\gamma^2)(1-\theta)+3\gamma\theta\big)a_H \\ -\gamma(\gamma(1-\theta)+\theta)a_L + 2(2w-(V\eta-c)\gamma)(1-\gamma^2) + 2be(2-\gamma)(1+\gamma)\Big]\end{array}}}{2(2-\gamma^2)}.$$

A.5. Proof of Proposition 4

Based on the previous analysis, it can be seen that the remanufacturer's bulk recovery price decision can be derived from the following equation π_M, which can be written:

$$\max_w \Bigg\{\left(\frac{(V\eta-\frac{ke^2}{2})(2-\gamma)}{2} - w\right)q_R(w,a) + \frac{[(\gamma-\gamma\theta+\theta)a_i + be(1+\gamma) - \gamma q_R(w,a)]^2 - \big[(V\eta-c)(1-\gamma^2)\big]^2}{4(1-\gamma^2)}$$
$$+ \frac{\left(V\eta-\frac{ke^2}{2}-c\right)\big[(V\eta-c)(1-\gamma^2) + (\gamma-\gamma\theta+\theta)a_i + be(1+\gamma)\big]}{2}\Bigg\}$$

$$\pi_M(w) = \left(\frac{(V\eta-\frac{ke^2}{2})(2-\gamma)}{2} - w\right)[\lambda q_R(w,a_H) + (1-\lambda)q_R(w,a_L)] + \frac{\lambda[(\gamma-\gamma\theta+\theta)a_H + be(1+\gamma) - \gamma q_R(w,a_H)]^2}{4(1-\gamma^2)}$$
$$+ \frac{(1-\lambda)[(\gamma-\gamma\theta+\theta)a_L + be(1+\gamma) - \gamma q_R(w,a_L)]^2}{4(1-\gamma^2)}$$
$$+ \frac{(V\eta-\frac{ke^2}{2}-c)\big[(V\eta-c)(1-\gamma^2) + \lambda(\gamma-\gamma\theta+\theta)a_H + (1-\lambda)(\gamma-\gamma\theta+\theta)a_L + be(1+\gamma)\big]}{2}$$
$$- \frac{(V\eta-c)^2(1-\gamma^2)}{4}.$$

First, discuss the case where, regardless of the recycling market scale condition, the recycling quantity is greater than zero for both the remanufacturer's online direct recycling channel and the recycler's classic recycling channel. The remanufacturer does not distort the recycling quantity when $w \leq \overline{w}$, at which point the remanufacturer's recycling quantity satisfies $q_R(w,a_i) = q_R^{\mathrm{SI}}(w,a_i)$. Denoting the remanufacturer's expected profit in the absence of information distortion by $\pi_M^{\mathrm{NS}}(w)$, it follows:

$$\pi_M^{\mathrm{NS}}(w) = \frac{(V\eta-\frac{ke^2}{2})(2-\gamma)}{4(2-\gamma^2)}\Big\{[(2w-(V\eta-c)\gamma)(1-\gamma^2)\lambda + (2be-be\gamma)(1+\gamma)\lambda + (2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H\lambda]$$
$$+ [(2w-(V\eta-c)\gamma)(1-\gamma^2)(1-\lambda) + (2be-be\gamma)(1+\gamma)(1-\lambda) + (2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_L(1-\lambda)]\Big\}$$
$$- \frac{\begin{array}{c}(2ww-(V\eta-c)\gamma w)(1-\gamma^2) + [(2be-be\gamma)(1+\gamma)(1-\lambda) + (2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_L(1-\lambda) \\ +(2be-be\gamma)(1+\gamma)\lambda + (2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H\lambda]w\end{array}}{2(2-\gamma^2)}$$
$$+ \frac{\lambda}{16(1-\gamma^2)(2-\gamma^2)^2}\Big\{(2w-(V\eta-c)\gamma)^2(\gamma-\gamma^3)^2 - 2[(2-\gamma^2)(\gamma(1-\theta)+2\theta-\gamma^2\theta)a_H$$

$$+ be(1+\gamma)(4-2\gamma-\gamma^2)][(2w-(V\eta-c)\gamma)(1-\gamma^2)\gamma] + [(2-\gamma^2)(\gamma(1-\theta)+2\theta-\gamma^2\theta)a_H$$
$$+ be(1+\gamma)(4-2\gamma-\gamma^2)]^2\Big\} + \frac{1-\lambda}{16(1-\gamma^2)(2-\gamma^2)^2}\Big\{(2w-(V\eta-c)\gamma)^2(\gamma-\gamma^3)^2 - 2[a_L(\gamma(1-\theta)(2-\gamma^2)$$
$$+\theta(4-3\gamma^2)) + be(1+\gamma)(4-2\gamma-\gamma^2)][(2w-(V\eta-c)\gamma)(1-\gamma^2)\gamma] + [a_L(\gamma(1-\theta)(2-\gamma^2)+\theta(4-3\gamma^2))$$
$$+ be(1+\gamma)(4-2\gamma-\gamma^2)]^2\Big\}$$
$$+ \frac{(V\eta-\frac{ke^2}{2}-c)\big[(V\eta-c)(1-\gamma^2)+\lambda(\gamma-\gamma\theta+\theta)a_H+(1-\lambda)(\gamma-\gamma\theta+\theta)a_L+be(1+\gamma)\big]}{2}$$
$$- \frac{(V\eta-c)^2(1-\gamma^2)}{4}.$$

When $w > \overline{w}$, the recycler distorts the recycling quantity downward to distort the information on recycling demand when the recycling market is small. In this case, the recycling quantity given by the recycler to the remanufacturer satisfies $q_R(w,a_H) = q_R^{\mathrm{SI}}(w,a_H)$ and $q_R(w,a_L) = \overline{q}_R(w,a_L)$. $\pi_M^{\mathrm{ID}}(w)$ (ID: information distortion) indicates the expected profit when the information is distorted.

$$\pi_M^{\mathrm{ID}}(w) = \left(\frac{(V\eta-\frac{ke^2}{2})(2-\gamma)}{2}-w\right)[\lambda q_R(w,a_H)+(1-\lambda)q_R(w,a_L)] + \frac{\lambda[(\gamma-\gamma\theta+\theta)a_H+be(1+\gamma)-\gamma q_R(w,a_H)]^2}{4(1-\gamma^2)}$$
$$+ \frac{(1-\lambda)[(\gamma-\gamma\theta+\theta)a_L+be(1+\gamma)-\gamma q_R(w,a_L)]^2}{4(1-\gamma^2)}$$
$$+ \frac{\left(V\eta-\frac{ke^2}{2}-c\right)[(V\eta-c)(1-\gamma^2)+\lambda(\gamma-\gamma\theta+\theta)a_H+(1-\lambda)(\gamma-\gamma\theta+\theta)a_L+be(1+\gamma)]}{2}$$
$$- \frac{(V\eta-c)^2(1-\gamma^2)}{4}$$
$$q_R^{\mathrm{SI}}(w,a_H) = \frac{(2w-(V\eta-c)\gamma)(1-\gamma^2)+be(2-\gamma)(1+\gamma)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H}{2(2-\gamma^2)}$$
$$\overline{q}_R(w,a_L) = \frac{2(1-\theta+\gamma\theta)a_H-\gamma(\gamma-\gamma\theta+\theta)a_L+(2w-(V\eta-c)\gamma)(1-\gamma^2)+be(2-\gamma)(1+\gamma)}{2(2-\gamma^2)}$$
$$- \frac{\sqrt{\begin{array}{c}\gamma(\theta+\gamma-\gamma\theta)(a_H-a_L)[((4-\gamma^2)(1-\theta)+3\gamma\theta)a_H-\gamma(\gamma(1-\theta)+\theta)a_L\\ +2(2w-(V\eta-c)\gamma)(1-\gamma^2)+2be(2-\gamma)(1+\gamma)]\end{array}}}{2(2-\gamma^2)}.$$

When the recycling quantities in this recycling supply chain are all greater than zero, the optimal bulk recycling price is mutually determined by $\pi_M^{\mathrm{NS}}(w)$ and $\pi_M^{\mathrm{ID}}(w)$.

$$\pi_M^{\mathrm{NS}}(w) = \frac{\left(V\eta-\frac{ke^2}{2}\right)(2-\gamma)}{4(2-\gamma^2)}\Big\{[(2w-(V\eta-c)\gamma)(1-\gamma^2)\lambda+(2be-be\gamma)(1+\gamma)\lambda+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H\lambda]$$
$$+[(2w-(V\eta-c)\gamma)(1-\gamma^2)(1-\lambda)+(2be-be\gamma)(1+\gamma)(1-\lambda)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_L(1-\lambda)]\Big\}$$
$$- \frac{\begin{array}{c}(2ww-(V\eta-c)\gamma w)(1-\gamma^2)+[(2be-be\gamma)(1+\gamma)(1-\lambda)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_L(1-\lambda)\\ +(2be-be\gamma)(1+\gamma)\lambda+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H\lambda]w\end{array}}{2(2-\gamma^2)}$$
$$+ \frac{\lambda}{16(1-\gamma^2)(2-\gamma^2)^2}\Big\{(2w-(V\eta-c)\gamma)^2(\gamma-\gamma^3)^2 - 2[(2-\gamma^2)(\gamma(1-\theta)+2\theta-\gamma^2\theta)a_H$$
$$+ be(1+\gamma)(4-2\gamma-\gamma^2)][(2w-(V\eta-c)\gamma)(1-\gamma^2)\gamma] + [(2-\gamma^2)(\gamma(1-\theta)+2\theta-\gamma^2\theta)a_H$$
$$+ be(1+\gamma)(4-2\gamma-\gamma^2)]^2\Big\} + \frac{1-\lambda}{16(1-\gamma^2)(2-\gamma^2)^2}\Big\{(2w-(V\eta-c)\gamma)^2(\gamma-\gamma^3)^2 - 2[a_L(\gamma(1-\theta)(2-\gamma^2)$$

$$+\theta(4-3\gamma^2))+be(1+\gamma)(4-2\gamma-\gamma^2)][(2w-(V\eta-c)\gamma)(1-\gamma^2)\gamma]+[a_L(\gamma(1-\theta)(2-\gamma^2)+\theta(4-3\gamma^2))$$
$$+be(1+\gamma)(4-2\gamma-\gamma^2)]^2\Big\}$$
$$+\frac{\left(V\eta-\frac{ke^2}{2}-c\right)\left[(V\eta-c)\left(1-\gamma^2\right)+\lambda(\gamma-\gamma\theta+\theta)a_H+(1-\lambda)(\gamma-\gamma\theta+\theta)a_L+be(1+\gamma)\right]}{2}$$
$$-\frac{(V\eta-c)^2\left(1-\gamma^2\right)}{4}$$

$$\frac{\mathrm{d}\pi_M^{\mathrm{NS}}(w)}{\mathrm{d}w}=(2V\eta-ke^2)(2-\gamma)(1-\gamma^2)(2-\gamma^2)-(1-\gamma^2)\gamma(3\gamma^2-4)(V\eta-c)-be(1+\gamma)(8-6\gamma^2+\gamma^3)-\lambda(1-\theta+\gamma\theta)$$
$$\times\left(\gamma^4+8-6\gamma^2\right)a_H+(1-\lambda)[5\gamma^3\theta-8(1+\gamma\theta-\theta)+\gamma^2(1-\theta)(6-\gamma^2)]a_L-2(1-\gamma^2)(8-5\gamma^2)w.$$

The optimum wholesale price w^{NS^*} is obtained.

$$w^{\mathrm{NS}^*}=\frac{(2V\eta-ke^2)(2-\gamma)(2-\gamma^2)-\gamma(3\gamma^2-4)(V\eta-c)}{2(8-5\gamma^2)}$$
$$+\frac{-be(1+\gamma)(8-6\gamma^2+\gamma^3)-\lambda(1-\theta+\gamma\theta)\left(\gamma^4+8-6\gamma^2\right)a_H+(1-\lambda)[5\gamma^3\theta-8(1+\gamma\theta-\theta)+\gamma^2(1-\theta)(6-\gamma^2)]a_L}{2(1-\gamma^2)(8-5\gamma^2)}.$$

Let $a_H(\gamma-\gamma\theta+\theta)+be(1+\gamma)=A>0, a_L(\gamma-\gamma\theta+\theta)+be(1+\gamma)=B>0, be(2-\gamma)(1+\gamma)+(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta)a_H=C, \frac{(V\eta-\frac{ke^2}{2}-c)[(V\eta-c)(1-\gamma^2)+\lambda(\gamma-\gamma\theta+\theta)a_H+(1-\lambda)(\gamma-\gamma\theta+\theta)a_L+be(1+\gamma)]}{2}-\frac{(V\eta-c)^2(1-\gamma^2)}{4}=D, \gamma(\theta+\gamma-\gamma\theta)(a_H-a_L)[((4-\gamma^2)(1-\theta)+3\gamma\theta)a_H-\gamma(\gamma(1-\theta)+\theta)a_L+2be(2-\gamma)(1+\gamma)]=E>0, 2\gamma(1-\gamma^2)(\theta+\gamma-\gamma\theta)(a_H-a_L)=F>0, 2(1-\theta+\gamma\theta)a_H-\gamma(\gamma-\gamma\theta+\theta)a_L+be(2-\gamma)(1+\gamma)=G>0.$

$$q_R^{\mathrm{SI}}(w,a_H)=\frac{(2w-(V\eta-c)\gamma)\left(1-\gamma^2\right)+C}{2(2-\gamma^2)};\quad \overline{q}_R(w,a_L)=\frac{G+(2w-(V\eta-c)\gamma)\left(1-\gamma^2\right)-\sqrt{F(2w-(V\eta-c)\gamma)}}{2(2-\gamma^2)};$$

$$\pi_M^{\mathrm{ID}}(w)=\left(\frac{\left(V\eta-\frac{ke^2}{2}\right)(2-\gamma)\lambda}{2}-\frac{2\lambda A\gamma}{4(1-\gamma^2)}\right)q_R(w,a_H)-\lambda wq_R(w,a_H)+\frac{\lambda\gamma^2}{4(1-\gamma^2)}q_R(w,a_H)^2$$
$$+\left(\frac{\left(V\eta-\frac{ke^2}{2}\right)(2-\gamma)(1-\lambda)}{2}-\frac{2(1-\lambda)B\gamma}{4(1-\gamma^2)}\right)q_R(w,a_L)-(1-\lambda)wq_R(w,a_L)+\frac{(1-\lambda)\gamma^2}{4(1-\gamma^2)}q_R(w,a_L)^2$$
$$+\frac{\lambda A^2+(1-\lambda)B^2}{4(1-\gamma^2)}+D$$
$$=\left(\frac{(V\eta-\frac{ke^2}{2})(2-\gamma)\lambda}{2}-\frac{2\lambda A\gamma}{4(1-\gamma^2)}\right)\frac{(2w-(V\eta-c)\gamma)(1-\gamma^2)+C}{2(2-\gamma^2)}-\lambda\frac{(2w^2-(V\eta-c)\gamma w)(1-\gamma^2)+C}{2(2-\gamma^2)}$$
$$+\frac{\lambda\gamma^2}{4(1-\gamma^2)}\frac{(4w^2+(V\eta-c)^2\gamma^2-4w(V\eta-c)\gamma)(1-\gamma^2)^2+C^2+2C(2w-(V\eta-c)\gamma)(1-\gamma^2)}{4(2-\gamma^2)^2}$$
$$+\left(\frac{(V\eta-\frac{ke^2}{2})(2-\gamma)(1-\lambda)}{2}-\frac{2(1-\lambda)B\gamma}{4(1-\gamma^2)}\right)\frac{G+(2w-(V\eta-c)\gamma)\left(1-\gamma^2\right)-\sqrt{F(2w-(V\eta-c)\gamma)}}{2(2-\gamma^2)}$$
$$-(1-\lambda)\frac{Gw+(2ww-(V\eta-c)\gamma w)\left(1-\gamma^2\right)-w\sqrt{F(2w-(V\eta-c)\gamma)}}{2(2-\gamma^2)}+\frac{(1-\lambda)\gamma^2}{4(1-\gamma^2)4(2-\gamma^2)^2}$$
$$\times[G^2+\left(4w^2+(V\eta-c)^2\gamma^2-4w(V\eta-c)\gamma\right)\left(1-\gamma^2\right)^2+2G(2w-(V\eta-c)\gamma)\left(1-\gamma^2\right)$$
$$+F(2w-(V\eta-c)\gamma)-2(G+(2w-(V\eta-c)\gamma)\left(1-\gamma^2\right))\sqrt{F(2w-(V\eta-c)\gamma)}]+\frac{\lambda A^2+(1-\lambda)B^2}{4(1-\gamma^2)}+D;$$

$$\frac{\mathrm{d}\pi_M^{\mathrm{ID}}(w)}{\mathrm{d}w}=\frac{\left(V\eta-\frac{ke^2}{2}\right)(2-\gamma)(1-\gamma^2)-\lambda A\gamma}{2(2-\gamma^2)}+\frac{2\left(1-\gamma^2\right)-F(2wF-V\eta\gamma F+c\gamma F)^{-\frac{1}{2}}}{2(2-\gamma^2)}$$

$$- (1-\lambda)\frac{G + (4w - (V\eta - c)\gamma)\left(1-\gamma^2\right) - (2wF - V\eta\gamma F + c\gamma F)^{\frac{1}{2}} - wF(2wF - V\eta\gamma F + c\gamma F)^{-\frac{1}{2}}}{2(2-\gamma^2)}$$
$$+ \frac{G(1-\lambda)\gamma^2 + C\lambda\gamma^2 + 2w\lambda\left(-8+12\gamma^2-3\gamma^4\right) + 2w\gamma^2\left(1-\gamma^2\right) + \left((4\lambda-\gamma^2)\left(1-\gamma^2\right)\gamma + \lambda\gamma^5\right)(V\eta - c)}{4(2-\gamma^2)^2}$$
$$+ \frac{(1-\lambda)\gamma^2 F}{8(1-\gamma^2)(2-\gamma^2)^2} + (1-\lambda)\gamma^2\frac{-3(2wF - V\eta\gamma F + c\gamma F)^{\frac{1}{2}}}{8(2-\gamma^2)^2} + (1-\lambda)\gamma^2 F\frac{1 - G(2wF - V\eta\gamma F + c\gamma F)^{-\frac{1}{2}}}{8(1-\gamma^2)(2-\gamma^2)^2}$$
$$- (1-\lambda)B\gamma\frac{2\left(1-\gamma^2\right) - F(2wF - V\eta\gamma F + c\gamma F)^{-\frac{1}{2}}}{4(1-\gamma^2)(2-\gamma^2)}.$$

In this case, there may exist $w^{\mathrm{ID}^*}\epsilon(0,\overline{w})$, such that $\frac{\mathrm{d}\pi_M^{\mathrm{ID}}(w)}{\mathrm{d}w} = 0$. It can be seen that $\pi_M^{\mathrm{NS}}(w)$ and $\pi_M^{\mathrm{ID}}(w)$ may reach equilibrium to achieve an optimal solution, or there may be no optimal solution, but a maximum can be achieved within the range of values.

A.6. Proof of Lemma 4

$$\pi_M^{\mathrm{RZ}}(w) = \frac{\left(V\eta - \frac{ke^2}{2}\right)(2-\gamma)}{2}\lambda q_R(w,a_H) - w\lambda q_R(w,a_H)$$
$$+ \lambda\frac{(a_H(\gamma-\gamma\theta+\theta) + be(1+\gamma))^2 - 2(a_H(\gamma-\gamma\theta+\theta) + be(1+\gamma))\gamma q_R(w,a_H) + \gamma^2 q_R(w,a_H)^2}{4(1-\gamma^2)}$$
$$+ \frac{(1-\lambda)[a_L(\gamma-\gamma\theta+\theta) + be(1+\gamma)]^2}{4(1-\gamma^2)}$$
$$+ \frac{\left(V\eta - \frac{ke^2}{2} - c\right)\left[(V\eta - c)\left(1-\gamma^2\right) + \lambda(\gamma-\gamma\theta+\theta)a_H + (1-\lambda)(\gamma-\gamma\theta+\theta)a_L + be(1+\gamma)\right]}{2}$$
$$- \frac{(V\eta - c)^2\left(1-\gamma^2\right)}{4}$$

$$\pi_M^{\mathrm{RZ}}(w) = \lambda\left[\frac{(V\eta - \frac{ke^2}{2})(2-\gamma)}{2} - w\right]\frac{(2w - (V\eta - c)\gamma)\left(1-\gamma^2\right) + be(2-\gamma)(1+\gamma) + \left(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\right)a_H}{2(2-\gamma^2)}$$
$$+ \frac{\lambda}{4(1-\gamma^2)}\Bigg\{[a_H(\gamma-\gamma\theta+\theta) + be(1+\gamma)]^2 - 2[a_H(\gamma-\gamma\theta+\theta) + be(1+\gamma)]\gamma$$
$$\times \frac{(2w - (V\eta - c)\gamma)\left(1-\gamma^2\right) + be(2-\gamma)(1+\gamma) + \left(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\right)a_H}{2(2-\gamma^2)}$$
$$+ \frac{\gamma^2}{4(2-\gamma^2)^2}\Big\{(2w - (V\eta - c)\gamma)^2\left(1-\gamma^2\right)^2 + [be(2-\gamma)(1+\gamma) + \left(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\right)a_H]^2$$
$$+ 2(2w - (V\eta - c)\gamma)\left(1-\gamma^2\right)[be(2-\gamma)(1+\gamma) + \left(2-2\theta+\gamma\theta-\gamma^2+\gamma^2\theta\right)a_H]\Big\}\Bigg\}$$
$$+ \frac{(1-\lambda)[a_L(\gamma-\gamma\theta+\theta) + be(1+\gamma)]^2}{4(1-\gamma^2)}$$
$$+ \frac{\left(V\eta - \frac{ke^2}{2} - c\right)\left[(V\eta - c)\left(1-\gamma^2\right) + \lambda(\gamma-\gamma\theta+\theta)a_H + (1-\lambda)(\gamma-\gamma\theta+\theta)a_L + be(1+\gamma)\right]}{2}$$
$$- \frac{(V\eta - c)^2\left(1-\gamma^2\right)}{4}$$

$$w^{\mathrm{RZ}^*} = \frac{\begin{array}{c}(2V\eta - ke^2)(2-\gamma)\left(1-\gamma^2\right)\left(2-\gamma^2\right) - (V\eta - c)(3\gamma^3 - 4\gamma)\left(1-\gamma^2\right) + be(1+\gamma)(6\gamma^2 - \gamma^3 - 8)\\ +a_H(-8 + 8\theta - 8\gamma\theta + 6\gamma^2 - 6\theta\gamma^2 + 5\gamma^3\theta - \gamma^4 + \gamma^4\theta)\end{array}}{2(1-\gamma^2)(8-5\gamma^2)}.$$

At this point, the corresponding offline recycling quantities and the recycling quantities from the remanufacturer's online recycling channel are as follows.

$$q_R\left(w^{\mathrm{RZ}^*}, a_H\right) = \frac{\begin{array}{c}\left(1-\gamma^2\right)\left(2-\gamma^2\right)[4(1-\gamma)V\eta - (2-\gamma)ke^2 + 2\gamma c] + 2be(1+\gamma)(8\gamma^2 - 8 - 3\gamma^3 + 4\gamma - 4)\\ +a_H[24(\gamma^2 + \theta - 1) - 16\gamma\theta(1+\gamma) + 6\gamma^4(\theta - 1) + 2\gamma^2\theta(5\gamma - 4)]\end{array}}{2(2-\gamma^2)(8-5\gamma^2)}$$

$$q_M\left(w^{\mathrm{RZ}^*}, a_H\right) = \left(1-\gamma^2\right)\frac{2(V\eta - c)\gamma(8-5\gamma^2) - 4(1-\gamma)V\eta + (2-\gamma)ke^2 - 2\gamma c}{4(8-5\gamma^2)} + \frac{be(1+\gamma)[8 + 4\gamma^4 - 4\gamma^3 - 11\gamma^2 + 6\gamma]}{(2-\gamma^2)(8-5\gamma^2)}$$
$$- \frac{a_H[5\gamma^4\theta - 5\theta\gamma^6 + 4\gamma - 4\gamma\theta + 16\theta - 6\gamma^3 + 6\gamma^3\theta - 26\gamma^2\theta + 2\gamma^5 + 8\theta\gamma^5]}{2(2-\gamma^2)(8-5\gamma^2)}$$

$$q_R\left(w^{\mathrm{RZ}^*}, a_L\right) = 0$$

$$q_M(w, a_i) = \frac{(V\eta - c)\left(1-\gamma^2\right) + (\gamma - \gamma\theta + \theta)a_L + be(1+\gamma)}{2}.$$

References

[1] C. Farronato, J. Fong and A. Fradkin, Dog eat dog: balancing network effects and differentiation in a digital platform merger. *Manag. Sci.* **70** (2023) 464–483.

[2] Y. Wang, Q. Gong, T. Wang and X. Tian, The effect of online channel addition on store performance: empirical evidence from Chinese chain retailers. *J. Syst. Sci. Inf.* **9** (2021) 356–377.

[3] Y. Yuan, J. Yang, Y. Li and W. Li, Necessary conditions for coordination of dual-channel closed-loop supply chain. *Technol. Forecast. Soc. Change.* **151** (2020) 119823.

[4] S. Balasubramanian, Mail versus Mall: a strategic analysis of competition between direct marketers and conventional retailers. *Mark. Sci.* **17** (1998) 181–195.

[5] B. Wang, C. Ren, X. Dong, B. Zhang and Z. Wang, Determinants shaping willingness towards on-line recycling behaviour: an empirical study of household e-waste recycling in China. *Resour. Conserv. Recycl.* **143** (2019) 218–225.

[6] L.T.T. Doan, Y. Amer, S.-H. Lee, P.N.K. Phuc and T.T. Tran, Optimizing a reverse supply chain network for electronic waste under risk and uncertain factors. *Appl. Sci.-BASEL* **11** (2021) 1946.

[7] B. Wang and N. Wang, Decision models for a dual-recycling channel reverse supply chain with consumer strategic behavior. *Sustainability* **14** (2022) 10870.

[8] J.A. Plaza-Ubeda, E. Abad-Segura, J. de Burgos-Jimenez, A. Boteva-Asenova and L.J. Belmonte-Urena, Trends and new challenges in the green supply chain: the reverse logistics. *Sustainability* **13** (2021) 331.

[9] I. Masudin, A literature review on green supply chain management adoption drivers. *J. Ilm. Tek. Ind.* **18** (2019) 103–115.

[10] B. Liu, M. Holmbom, A. Segerstedt and W. Chen, Effects of carbon emission regulations on remanufacturing decisions with limited information of demand distribution. *Int. J. Prod. Res.* **53** (2015) 532–548.

[11] A. Yenipazarli, Managing new and remanufactured products to mitigate environmental damage under emissions regulation. *Eur. J. Oper. Res.* **249** (2016) 117–130.

[12] F. Yang, M. Wang and S. Ang, Optimal remanufacturing decisions in supply chains considering consumers' anticipated regret and power structures. *Transp. Res. Part E Logist. Transp. Rev.* **148** (2021) 102267.

[13] S. Nandan and A. Kushwaha, Interventions of non-governmental organisations for women's empowerment through micro-entrepreneurship: evidences from India. *Int. J. Indian Cult. Bus. Manag.* **29** (2023) 96.

[14] G. Anandh, S. PrasannaVenkatesan, M. Goh and K. Mathiyazhagan, Reuse assessment of WEEE: systematic review of emerging themes and research directions. *J. Environ. Manage.* **287** (2021) 112335.

[15] L. Zuo, C. Wang and Q. Sun, Sustaining WEEE collection business in China: the case of online to offline (O2O) development strategies. *Waste Manag.* **101** (2020) 222–230.

[16] Z. Li, S.M. Gilbert and G. Lai, Supplier encroachment under asymmetric information. *Manag. Sci.* **60** (2014) 449–462.

[17] P. Jawla and S.R. Singh, A reverse logistic inventory model for imperfect production process with preservation technology investment under learning and inflationary environment. *Uncertain Supply Chain Manag.* **4** (2016) 107–122.

[18] A.F. Gukaliuk and V.I. Katsma, Logistic resource management as a part of logistic management of enterprise. *Ekon. Anal.* **27** (2017) 130–138.

[19] C.-H. Chuang, C.X. Wang and Y. Zhao, Closed-loop supply chain models for a high-tech product under alternative reverse channel and collection cost structures. *Int. J. Prod. Econ.* **156** (2014) 108–123.

[20] S. Kumar Jena and S.P. Sarmah, Price competition and co-operation in a duopoly closed-loop supply chain. *Int. J. Prod. Econ.* **156** (2014) 346–360.

[21] H. Liu, M. Lei, H. Deng, G. Keong Leong and T. Huang, A dual channel, quality-based price competition model for the WEEE recycling market with government subsidy. *Omega* **59** (2016) 290–302.

[22] M. Zu-Jun, N. Zhang, Y. Dai and S. Hu, Managing channel profits of different cooperative models in closed-loop supply chains. *Omega* **59** (2016) 251–262.

[23] Y. Xiong, Q. Zhao and Y. Zhou, Manufacturer-remanufacturing vs supplier-remanufacturing in a closed-loop supply chain. *Int. J. Prod. Econ.* **176** (2016) 21–28.

[24] Y. Xiong and W. Yan, Implications of channel structure for marketing remanufactured products. *Eur. J. Ind. Eng.* **10** (2016) 126.

[25] M. Ullah, Impact of transportation and carbon emissions on reverse channel selection in closed-loop supply chain management. *J. Clean. Prod.* **394** (2023) 136370.

[26] Z. Li, J. Zhao and Q. Meng, Dual-channel recycling e-waste pricing decision under the impact of recyclers' loss aversion and consumers' bargaining power. *Environ. Dev. Sustain.* **24** (2022) 11697–11720.

[27] M. Theeraworawit, S. Suriyankietkaew and P. Hallinger, Sustainable supply chain management in a circular economy: a bibliometric review. *Sustainability* **14** (2022) 9304.

[28] M.-L. Tseng, M.S. Islam, N. Karia, F.A. Fauzi and S. Afrin, A literature review on green supply chain management: trends and future challenges. *Resour. Conserv. Recycl.* **141** (2019) 145–162.

[29] C. Li, L. Feng and S. Luo, Strategic introduction of an online recycling channel in the reverse supply chain with a random demand. *J. Clean. Prod.* **236** (2019) 117683.

[30] D. Wu, J. Chen, P. Li and R. Zhang, Contract coordination of dual channel reverse supply chain considering service level. *J. Clean. Prod.* **260** (2020) 121071.

[31] D. Wu, J. Chen, R. Yan and R. Zhang, Pricing strategies in dual-channel reverse supply chains considering fairness concern. *Int. J. Environ. Res. Public. Health.* **16** (2019) 1657.

[32] L. Zhang, H. Zhou, Y. Liu and R. Lu, Optimal environmental quality and price with consumer environmental awareness and retailer's fairness concerns in supply chain. *J. Clean. Prod.* **213** (2019) 1063–1079.

[33] D. Chen, J. Ignatius, D. Sun, S. Zhan, C. Zhou, M. Marra and M. Demirbag, Reverse logistics pricing strategy for a green supply chain: a view of customers' environmental awareness. *Int. J. Prod. Econ.* **217** (2019) 197–210.

[34] J. Pandey, R.K. Kaushal and S.P. Shukla, Behavioural and structural analysis of stakeholders in E-waste management. *Mater. Today Proc.* **51** (2022) 416–421.

[35] F. Zhang and R. Zhang, Trade-in remanufacturing, customer purchasing behavior, and government policy. *Manuf. Serv. Oper. Manag.* **20** (2018) 601–616.

[36] C. Fang, X. Ma, J. Zhang and X. Zhu, Personality information sharing in supply chain systems for innovative products in the circular economy era. *Int. J. Prod. Res.* **59** (2021) 5992–6001.

[37] L. Wang, X. Jiao and Q. Hao, Modeling the incentive mechanism of information sharing in a dual-channel supply chain. *Discrete Dyn. Nat. Soc.* **2021** (2021) 1–7.

[38] C. Baah, D. Opoku Agyeman, I.S.K. Acquah, Y. Agyabeng-Mensah, E. Afum, K. Issau, D. Ofori and D. Faibil, Effect of information sharing in supply chains: understanding the roles of supply chain visibility, agility, collaboration on supply chain performance. *Benchmarking Int. J.* **29** (2022) 434–455.

[39] H. Guan, H. Gurnani, X. Geng and Y. Luo, Strategic inventory and supplier encroachment. *Manuf. Serv. Oper. Manag.* **21** (2019) 536–555.

[40] A.Y. Ha, Q. Tian and S. Tong, Information sharing in competing supply chains with production cost reduction. *Manuf. Serv. Oper. Manag.* **19** (2017) 246–262.

[41] Y. Chen and Ö. Özer, Supply chain contracts that prevent information leakage. *Manag. Sci.* **65** (2019) 5619–5650.

[42] S. Nasser and D. Turcic, Temporary contract adjustment to a retailer with a private demand forecast. *Manag. Sci.* **65** (2019) 209–229.

[43] X. Zhang, Z. Lin and L. Xiao, Simulation research on information-sharing value of two-level supply chain. *Secur. Commun. Netw.* **2021** (2021) 1–8.