RAIRO-Oper. Res. 58 (2024) 1771–1788
https://doi.org/10.1051/ro/2024014

RAIRO Operations Research
www.rairo-ro.org

RESEARCH ON OPTIMIZATION OF SUPPLY CHAIN INVENTORY SYSTEM UNDER CONTINGENCY CONDITIONS

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Abstract. A solution combining state observer (SOB) with Discrete Sliding Mode Control (DSMC) is presented to solve the problem of inventory shortage or backlog in upstream and downstream enterprises of the supply chain caused by contingency conditions. Firstly, based on the specific sales mode of the supply chain system and the logical relationship of inventory parameters, the corresponding dynamic model of the inventory system is established under reasonable assumptions, and relevant parameters are introduced to represent the impact of contingencies, and the dynamic model of inventory system under contingencies is established with the bullwhip effect. Then, the discrete sliding mode control (SOB-DSMC) based on the state observer is designed to compensate for the impact of emergencies on the supply chain inventory system adaptively while ensuring the stability of the system. Finally, based on market research and reasonable assumptions, simulation experiments are carried out. The simulation results show that the designed control system can better cope with emergencies in different situations, and can significantly improve the supply chain within 5 days after emergencies occur. The inventory situation of upstream and downstream enterprises in the system has reduced the maximum inventory fluctuations of manufacturers, distributors, and retailers by 77.13%, 60.67%, and 44.61% respectively, and fully restored to normal operation within 20 days, effectively solving the inventory backlog of upstream and downstream enterprises in the supply chain and out-of-stock issues. It can be seen that the SOB-DSMC method can guide the upstream enterprises in the supply chain to formulate reasonable production, shipping, and purchasing strategies, and provide theoretical reference and coping methods for the supply chain system to smoothly survive emergencies.

Mathematics Subject Classification. 93C95.

Received October 23, 2023. Accepted January 14, 2024.

1. Introduction

A supply chain is a complex network system comprising suppliers, manufacturers, distributors, retailers, and other interconnected links [20]. These upstream and downstream enterprises are intricately interconnected through the flow of materials, products, and capital, all to efficiently meet market demand in the shortest possible time and at the lowest cost. In the modern supply chain system, supply chains face emergencies not only from natural disasters such as earthquakes, tsunamis, and volcanic eruptions [7,9,17], but also from financial crises, wars, and other man-made factors [14,18,25], as well as novel coronavirus (COVID-19) epidemics, plagues,

Keywords. Contingency conditions, supply chain, inventory system, state observer, discrete sliding mode control.

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and other major public health events [3, 10, 15, 26]. The inventory system plays a crucial role in the supply chain system, directly impacting its normal operations. This underscores the significance of inventory system management and optimization as a key research focus within the field of supply chain management [6,13,16,22]. In recent years, especially in light of unforeseen circumstances, significant damage has been inflicted on inventory management systems. In August 2023, Japan began discharging treated nuclear wastewater into the ocean from the Fukushima Daiichi nuclear power plant, an action that forced China to stop importing fish products from Japan. Increased interest in China's Xinjiang salmon led to increased demand for China's Xinjiang salmon and stock shortages at many retailers. In March 2022, sauerkraut produced by Flagstaff Vegetables was exposed for hygiene issues on CCTV's "3.15" program in China. As a supplier of instant noodles to companies like Master Kong and other instant noodle manufacturers, "Flagstaff Vegetables" directly contributed to a sharp decline in demand for products from these relevant companies. This, in turn, led to varying degrees of inventory accumulation both upstream and downstream. These examples underscore the importance of optimizing supply chain inventory management in response to unforeseen circumstances. This paper aims to optimize the supply chain inventory system in the face of unexpected events. It seeks to guide upstream and downstream enterprises within the supply chain in developing production, procurement, and shipping strategies that can mitigate the impact of contingency events on supply chain inventory. This research provides a theoretical foundation and coping strategies for upstream and downstream enterprises within the supply chain to navigate contingency conditions while meeting market demands.

With the development and wide application of automated control theory, more and more scholars applied automated control ideas to the field of inventory management to dynamically control and optimize the inventory system. Simon [19] firstly applied the control ideas to inventory management, and solved the inventory optimization problem by constructing the transfer function of the inventory system. Yu *et al.* [23] utilized the Ant Colony Optimization and the basic idea of the fuzzy model to propose a more systematic and improved supply chain inventory optimization model. Fu *et al.* [8] based on distributed model predictive control effectively sought optimal solutions for the problems of input/output constraints and cross-coupling of interacting nodes of the supply chain to meet the market demand. Zaher *et al.* [24] by establishing stochastic differential equations for inventory and converting the inventory problem into an optimal control problem. Bartoszewicz *et al.* [1] and other sliding mode control ideas solved the inventory problem with finite size. The literature in this category is all about applying automation techniques in the field of inventory management but lacks solutions for managing and optimizing the supply chain inventory system under contingency conditions. As it should be, some scholars have also conducted research on supply chain inventory management under contingency conditions. For the supply chain inventory system under the role of demand perturbation, Wu *et al.* [21] designed an optimization model of the supply chain inventory system that combines an improved sliding mode controller with a perturbation observer. For the manufacturer segment, the distributor segment, For the retailer segment, Zhao *et al.* [27] designed a supply chain retailer inventory system optimization model under the effect of random perturbation based on the self-imposed perturbation control, and through a series of model design and parameter optimization, the impact of random perturbation on the retailer inventory system is compensated by controlling the random perturbation under the premise of guaranteeing the stability of the system. This kind of literature seeks to solve the inventory management problem of supply chain systems under uncertain demand or disturbance from unexpected situations, but it only considers and analyzes a certain link in the supply chain system, and gives the coping strategy of a certain link. There is no comprehensive consideration and research on the complete supply chain, and there is a lack of optimization solutions for the management of the complete supply chain inventory system under contingency conditions.

Discrete Sliding Mode Control (DSMC) is widely used in engineering control field [2, 4, 5, 11, 12]. Because of its simple algorithm, strong disturbance resistance and other characteristics, but because DSMC is unable to adaptively regulate the control objectives for changing situations, it has not been applied to the field of supply chain inventory management and optimization in the changing market environment. To address the shortcomings of the above research, this paper establishes a complete supply chain inventory system model covering manufacturers, distributors and retailers based on the specific sales patterns and logical relationships of inventory parameters in the supply chain system, and proposes a SOB-DSMC solution to cope with unexpected situations. First of all, the state observer (SOB) is used to observe the supply chain inventory state in real time, when the unexpected condition occurs, the SOB can observe the impact of the unexpected condition on the supply chain inventory system in real time, and then the SOB can be used to correct the target inventory of the discrete sliding mode controller (DSMC) in real time, so as to help the supply chain inventory system to pass through the unexpected condition smoothly, and to weaken the impact of the unexpected condition on the inventory of upstream and downstream enterprises in the supply chain. The impact of unexpected conditions on the inventory of the upstream and downstream supply chain enterprises is weakened. In this paper, we mainly work on the following three aspects:

1) To establish a complete supply chain inventory system model covering manufacturers, distributors and retailers, and to establish a supply chain inventory system model under contingency conditions by combining the impact of contingency conditions and the bullwhip effect.
2) Propose SOB-DSMC method for controller design and construct Lyapunov function for stability proof.
3) On the basis of market survey, the supply chain inventory system model parameters are reasonably assumed, and simulation experiments are conducted for different contingency conditions to verify the efficacy of the proposed method.

2. Basic introduction and improvement of basic algorithm

As regard, we classify and survey studies on inventory management in Table 1. According to the results of our survey, we can see that the risk management of inventory using control theory method is limited. As a result, our research innovation and contribution present a discrete sliding mode control method is used to ensure that the company can smoothly survive contingency conditions and quickly return to the optimal state.

Our model is grounded in the currently prevalent ‘one-to-multiple’ mesh marketing model. Figure 1 illustrates this marketing model, where a manufacturer’s production base is responsible for shipping products to several distributors across various sales areas. Subsequently, these distributors employ either a ‘one-to-multiple’ or ‘one-to-one’ approach to distribute the products to retailers. Retailers, located in different market regions, must respond to varying market demands. distributors make import and shipment decisions based on the quantity of orders from retailers, while retailers determine their purchase and shipment strategies according to the specific market demands within their respective sales regions.

Drawing upon the ‘one-to-multiple’ mesh marketing model illustrated in Figure 1, this study explores the intrinsic interconnections within the supply chain system. It also takes into account factors such as manufacturers’ productivity and downstream enterprises’ shipment quantities and other parameters. To simplify the model in this paper, we focus solely on the distributor-retailer relationship for the supply of goods. Through achieving equivalence and simplification within the supply chain system, we establish the relationships depicted in the inventory schematic diagram, as presented in Figure 2.

Based on the relationship between inventory and production and demand in Figure 2, we let

$$\begin{aligned} x(k) &= [x_0(k), z_1(k), \ldots, z_n(k), m_1(k), \ldots, m_n(k)]^{\mathrm{T}} \\ u(k) &= [p_0(k), p_1(k), \ldots, p_n(k), v_1(k), \ldots, v_n(k)]^{\mathrm{T}} \\ d(k) &= [d_1(k), \ldots, d_n(k)]^{\mathrm{T}}. \end{aligned} \tag{1}$$

$x(k)$ is the inventory level of each link in the supply chain system at the moment of k, $x_0(k)$ is the inventory level of the producer at the moment of k, $z_i(k)$ is the inventory level of the distributor i at the moment of k, and $m_i(k)$ is the inventory level of the retailer i’s inventory level at the moment of k. $u(k)$ is the set of the production volume of the manufacturer and the shipments of each segment at the moment of k, $p_0(k)$ is the production volume of the manufacturer at the moment of k, $p_i(k)$ is the shipments of the manufacturer to the distributor i at the moment of k, and $v_i(k)$ is the shipments from distributors i to retailers i at the moment of k. $d(k)$ is the fixed market demand that the retailer needs to face at the moment, and $d_i(k)$ is the market

TABLE 1. Literature review comparison.

References	Components	Method	Types of inventory issues	Evaluations
Lotfi *et al.* [14]	Multiple Vendor, Multiple Retailer	Hybrid Fuzzy, Data-Driven Robust Optimization	Resilience and Sustainable Health Care Supply Chain with Vendor-Managed Inventory	This approach helps healthcare systems cope with uncertainty and adaptability to demand fluctuations, particularly in challenging situations such as the 2019 coronavirus disease (COVID-19) pandemic.
Yu *et al.* [23]	Manufacturer, Retailer	Ant Colony Algorithm, Fuzzy model	Cost optimization of inventory management	This paper is developed by establishing a simple mathematical model of joint inventory management. The conclusions of the experiment show that joint inventory management can save the inventory cost of the entire supply chain more than traditional inventory management.
Fu *et al.* [8]	Retailer, Wholesaler, Distributor, Factory	Distributed Model Predictive Control	Inventory management issues in supply chain networks	This paper develops a distributed model predictive control (DMPC) approach with minimal information exchange and communication to handle supply chain operations and management.
Zaher and Zaki [24]	Manufacturer	Weibull Distributed Deterioration Items, The Optimal Control Model	Solve Production Inventory System in Supply Chain Management	This paper describes how to control the inventory production system with Weibull distributed deterioration items and puts forward two effective solution methods. Both of these methods effectively provide the optimal production rate for the model.
Bartoszewicz and Latosiński [1]	Multiple Distributor	Sliding Mode Control Strategy	A class of inventory management systems with multiple suppliers	This paper proposes the idea is applied to design a control strategy for a class of supply chains with multiple suppliers. And it can guarantee 100% satisfaction of the consumer's demand without generating excessively small or large orders.
This paper	One Manufacturer, Multiple Retailer, Multiple Markets	Discrete Sliding Mode Control Strategy, State Observer	Supply chain network state restoration in Contingency Conditions	Our method can mitigate the impact of unforeseen circumstances on the inventory of both upstream and downstream enterprises, thereby helping the supply chain inventory system smoothly navigate through periods of contingency events

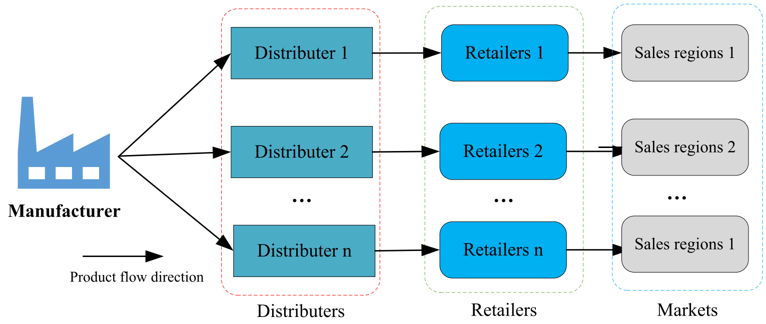

FIGURE 1. Schematic diagram of the "point-to-multiple" marketing model.

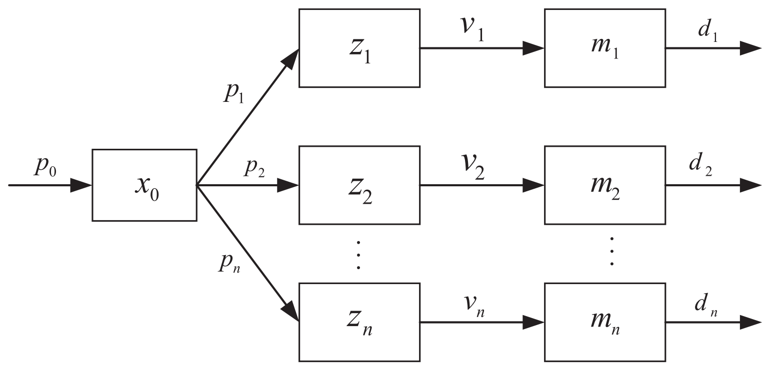

FIGURE 2. Schematic diagram of the relationship between the inventory of each link in the supply chain system.

demand of the sales area i at the moment of k, which is equal to the shipment quantity of the retailer i at the moment of k.

In order to analyze and model the supply chain inventory system more accurately to reflect the real situation of the supply chain system inventory situation more practically. we must establish the following assumptions:

Assumption 2.1. *It is assumed that under this marketing model, all the distributors pick up goods only from the designated sole manufacturer and supply only to the retailers in a specific region, and there is no crosstalk.*

Assumption 2.2. *Assume that there exists a range of values for the inventory of each segment of the supply chain system: $x_i \leq C_{\text{imax}}$, where C_{imax} is the maximum value of the warehouse inventory of the i segment of the supply chain system. Also if $x_i < 0$ represents that the inventory at this point is in out-of-stock status, which is permissible and reasonable.*

Assumption 2.3. *Assuming that the inventory products in each link of the supply chain system have the phenomenon of inventory self-loss due to the product itself or storage conditions and other factors, and assuming that the supply chain system inventory self-loss rate is represented by γ, then we can get $\gamma = [\gamma_0, \gamma_1, ..., \gamma_{2n}]^T$.*

Assumption 2.4. *Assuming that a certain degree of transportation loss exists in the upstream and downstream transportation of supply chain system products, and assuming that the transportation loss rate of each link in the supply chain system is represented by δ, then we can get $\delta = [0, \delta_1, ..., \delta_{2n}]^T$.*

Assumption 2.5. *It is assumed that the contingency situation will only affect the market demand for the products, and will not lead to the interruption of a link in the supply chain system or the change of the relevant parameters of the supply chain system.*

Based on Assumptions 2.1 to 2.4, and the schematic diagram of the relationship between the inventory of each link in the supply chain system in Figure 2, the following expression can be obtained

$$\begin{array}{l} x_0(k+1) = (1-\gamma)_0 x_0(k) + p_0(k) - p_1(k) - \ldots - p_n(k) \\ z_1(k+1) = (1-\gamma)_1 z_1(k) + (1-\delta_1)\, p_1(k) - v_1(k) \\ \vdots \\ z_n(k+1) = (1-\gamma_n) z_n(k) + (1-\delta_n)\, p_n(k) - v_n(k) \\ m_1(k+1) = (1-\gamma_{n+1}) m_1(k) + (1-\delta_{n+1})\, v_1(k) - d_1(k) \\ \vdots \\ m_n(k+1) = (1-\gamma_{2n}) m_n(k) + (1-\delta_{2n})\, v_n(k) - d_n(k). \end{array} \tag{2}$$

By integrating equation (1) with equation (4), we can obtain the following expression.

$$x(k+1) = Ax(k) + Bu(k) + Dd(k)$$

$$A = \begin{bmatrix} 1-\gamma_0 & 0 & 0\ldots & 0 & 0 \\ 0 & 1-\gamma_1 & 0\ldots & 0 & 0 \\ \vdots & \vdots & \vdots\ \ddots & \vdots & \vdots \\ 0 & 0 & 0\ldots & 1-\gamma_{2n-1} & 0 \\ 0 & 0 & 0\ldots & 0 & 1-\gamma_{2n} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 & \ldots & -1 & 0 & \ldots & 0 \\ 0 & 1-\delta_1 & 0 & \ldots & -1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0 \\ 0 & 0 & \ldots & 1-\delta_{n+1} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 1-\delta_{2n} \end{bmatrix} \tag{3}$$

$$D = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ -1 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 \\ 0 & \cdots & 0 & -1 \end{bmatrix}_{(2n+1)\times n} .$$

Thus we can get the basic expression for the state space of the inventory model of the supply chain system:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Dd(k) \\ y(k) = Cx(k). \end{cases} \tag{4}$$

where the system $y(k)$ can measure the output. The $A \in \mathbb{R}^{(2n+1)\times(2n+1)}$, $B \in \mathbb{R}^{(2n+1)\times(2n+1)}$, $C \in \mathbb{R}^{(2n+1)\times(2n+1)}$, and $D \in \mathbb{R}^{(2n+1)\times n}$ are all known constant matrices and the known matrices (A, B) is controllable and the matrix (A, C) is observable.

For the contingency situation, this paper sets the change of market demand land caused by the contingency situation as $\omega(k)$, and $\omega(k)$ is set to be $\omega(k) = [\omega_1(k), \omega_2(k), \cdots, \omega_n(k)]^T$.

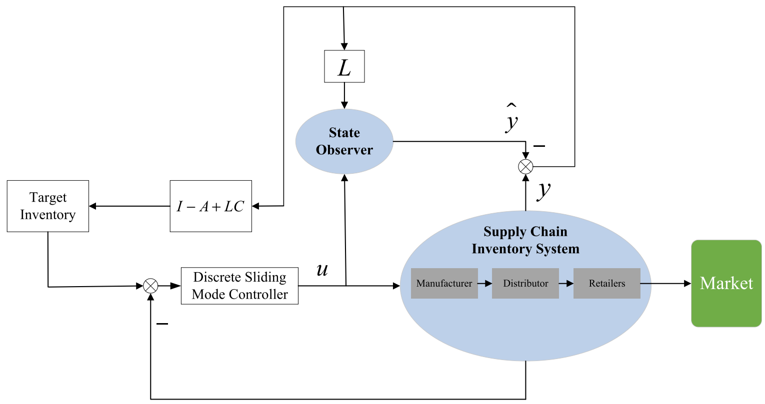

FIGURE 3. Supply Chain Inventory Optimization Scenario under contingency situation conditions.

Where $\omega_i(k)$ is the contingency situation in the sales region i caused by the amount of change in market demand, due to the existence of the supply chain system bullwhip effect, the amount of change in market demand will be against the product flow of the step by step to amplify, here let the bullwhip parameter is β, and $\beta > 1$. Then we can get the contingency situation for the supply chain inventory system impact parameter matrix $\bar{D}$ is

$$\bar{D} = \begin{bmatrix} -\beta^2 & -\beta^2 & \cdots & -\beta^2 \\ -\beta & 0 & 0 & 0 \\ 0 & \cdots & 0 & -\beta \\ -1 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 \\ 0 & \cdots & 0 & -1 \end{bmatrix}_{(2n+1)\times n}$$

.

Summarizing the above conditions as well as Assumption 2.5, the basic expression for the state space of the inventory model of the supply chain system under contingency conditions can be obtained:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Dd(k) + \bar{D}\omega(k) \\ y(k) = Cx(k). \end{cases} \tag{5}$$

3. Controller design and stability analysis

In this paper, the optimization scheme of the supply chain inventory system designed for the contingency situation is shown in Figure 3, and the SOB-DSMC method is mainly used for the design of the controller $u(k)$.

In Figure 3, L is state observer parameters, A is system state matrix, C is system measurement matrix, u is controller control quantity, y is system measurable output, $\hat{y}$ is state observer measurable output.

According to the supply chain inventory optimization scheme under the contingency situation condition shown in Figure 3, the design of the controller is divided into two main steps, which are the design of the state observer and the design of the discrete sliding mode controller.

3.1. State observer design

In this paper, the design idea of the Luenberger observer is used for state observer design, and its design principle is shown in Figure 4.

In Figure 4, x_0 is initial state of the system, A is state matrix of the system, C is measurement matrix of the system, u is control input, y is system measurable output, $\hat{x}_0$ is state observer initial state, $\hat{x}$ is state observer observed value.

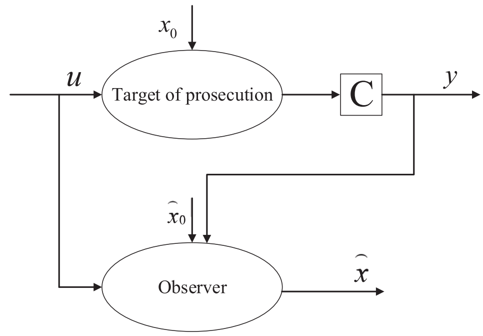

FIGURE 4. Schematic diagram of state observer.

In accordance with Figure 4 and the basic expression of the state space of the inventory model of the controlled object supply chain system, the state observer can be designed as

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + Dd(k) + L[y(k) - \hat{y}(k)] \\ \hat{y}(k) = C\hat{x}(k). \end{cases} \tag{6}$$

Taking the observation error $e_1(k) = x(k) - \hat{x}(k)$, while associating equation (5) we can get

$$\begin{aligned} & e_1(k+1) \\ & = x(k+1) - \hat{x}(k+1) \\ & = Ax(k) + Bu(k) + \bar{D}\omega(k) - A\hat{x}(k) - Bu(k) \\ & - L[y(k) - \hat{y}(k)] \\ & = A[x(k) - \hat{x}(k)] + Dd(k) - LC[x(k) - \hat{x}(k)] \\ & = (A - LC)e_1(k) + \bar{D}\omega(k). \end{aligned} \tag{7}$$

Let $\tilde{A} = A - LC$, then we can get

$$e_1(k+1) = \tilde{A}e_1(k) + \bar{D}\omega(k). \tag{8}$$

From the discrete system stability condition, we know that as long as $||\tilde{A}|| < 1$, the state observer is stable at this time. If it is the normal case $\omega(k) = 0$, when the state observer stabilizes, the observation error at this time $e_1(k) \to 0$. In case of a sudden condition $\omega(k) \neq 0$. When the state observer is stabilized, the observation error will be approximated gradually by the solution of $(I - \tilde{A})^{-1}\bar{D}\omega(k)$ at this point, we can use the state observer to obtain the amount of changes in the supply chain inventory system due to contingency situation in real time.

Discrete Sliding Mode Controller Design

The design of a discrete sliding mode controller consists of two main elements: the design of the sliding mode surface and the design of the convergence law. For the controlled system, the sliding mode surface $s = 0$ is constructed according to the state of the system, then the sliding mode surface can divide the state space of the system into two parts $s < 0$ and $s > 0$. When the state of the system is in the sliding mode surface $s = 0$, the system is stable, and there are three kinds of points on the sliding mode surface, which are the crossing point A, the starting point B, and the termination point C, as shown in Figure 5.

Firstly, the target inventory $x_d(k)$ of the inventory system at the moment of k is determined through market survey and optimization theory of the supply chain inventory system. Currently, manufacturers generally produce in real time according to the order quantity, so that the warehouse storage capacity is 0 to reduce the cost

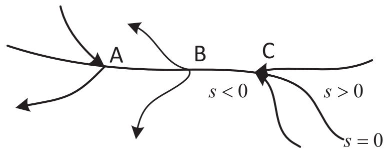

FIGURE 5. Schematic diagram of switching surface of sliding mode controller.

of storage. As for the dealer segment, to have a certain degree of risk resistance, will store a certain amount of goods in the warehouse to cope with the risk, so at the moment of k we will dealers segment the target inventory is set as $\zeta(k)$, that is, we can get $\zeta(k) = [\zeta_1(k), \zeta_2(k), \cdots .\zeta_n(k)]^T$.

For the retailers, who are directly facing the market, the target inventory of the retailers at the moment of k is set as $d(k)$ to minimize the market demand under the condition of reducing the inventory cost. In summary, this paper sets the target inventory $x_d(k)$ of the supply chain inventory system at the moment of k as follows $x_d(k) = [0, \zeta(k), d(k)]^T$.

However, due to the occurrence of contingency situation conditions, the market demand will also change, which makes the target inventory under normal conditions can no longer be applied to the supply chain inventory system under contingency situation conditions. In this paper, we adaptively adjust the target inventory of the supply chain inventory system based on state observer to achieve the purpose of supply chain system adaptive response to contingency situation conditions. From it is known that when the state observer is stable, the observation error $e_1(k)$ at the moment of k will be gradually approximated to $(I - \tilde{A})^{-1}\bar{D}\omega(k)$, where the $\tilde{A}$ matrix and Matrix are both known as the constant matrices, it can be obtained to use. After adaptive compensation of the observation error at the $k-1$-moment $e_1(k-1)$, at the moment of k,the target inventory $\tilde{x}_d(k)$ is

$$\tilde{x}_d(k) = x_d(k) + (I - \tilde{A})e_1(k-1). \tag{9}$$

Finally, the discrete sliding mode controller design is carried out based on the target inventory $\tilde{x}_d$ obtained from the adaptive adjustment of the state observer, and the inventory error at the moment of k is defined to be $e(k) = \tilde{x}_d(k) - x(k)$, which defines the sliding mode surface at the moment of k to be

$$s(k) = ce(k) = c[\tilde{x}_d(k) - x(k)]. \tag{10}$$

where c is a positive definite diagonal matrix and $c = \mathrm{diag}(c_0, c_1, \cdots, c_{2n})$. we can get the sliding mode surface at the moment of $k+1$ as

$$\begin{aligned} s(k+1) &= ce(k+1) \\ &= c[\tilde{x}_d(k+1) - x(k+1)]. \end{aligned} \tag{11}$$

In order to meet the requirement of supply chain system to quickly adapt to the market demand, this paper adopts the exponential convergence law for controller design,

$$\begin{aligned} \dot{s}(k) &= [s(k+1) - s(k)]/\Delta t \\ &= -\varepsilon \mathrm{sgn}(s(k)) - \tau s(k). \end{aligned} \tag{12}$$

$\mathrm{sgn}(s(k))$ is a sign function, ε and τ are known positive definite diagonal matrices and $\varepsilon = \mathrm{diag}(\varepsilon_0,\varepsilon_1, \cdots, \varepsilon_{2n})$, $\tau = \mathrm{diag}(\tau_0, \tau_1, \cdots, \tau_{2n})$. Combining equation (11) with equation (12) we can get

$$\begin{aligned} &s(k+1) \\ &= -\varepsilon \Delta t \mathrm{sgn}(s(k)) - \tau \Delta t s(k) + s(k) \\ &= (I - \tau \Delta t)s(k) - \varepsilon \Delta t \mathrm{sgn}(s(k)). \end{aligned} \tag{13}$$

By associating equation (10) with equation (12), we can get

$$\begin{aligned} &x(k+1) \\ &= \tilde{x}_d(k+1) + c^{-1}\varepsilon\Delta t\mathrm{sgn}(s(k)) \\ &\quad - c^{-1}(I-\tau\Delta t)s(k). \end{aligned} \tag{14}$$

Also from the state space equations of the supply chain inventory system under known contingency conditions as shown in equation (5), the controller designed using the SOB-DSMC method can be obtained as follows

$$\begin{aligned} u(k) =& B^{-1}[\tilde{x}_d(k+1) - Ax(k) - Dd(k) - \bar{D}\omega(k)] \\ &+ (cB)^{-1}[\varepsilon\Delta t\mathrm{sgn}(s(k)) - (I-\tau\Delta t)s(k)]. \end{aligned} \tag{15}$$

3.2. Stability analysis

In this section, the stability of the designed controller will be proved by constructing the Lyapunov function $V(k)$. Where the constructed Lyapunov function $V(k)$ is shown in equation (16)

$$V(k) = \frac{1}{2}s^2(k). \tag{16}$$

Then we can get

$$\begin{aligned} \dot{V}(k) &= \frac{V(k+1)-V(k)}{\Delta t} \\ &= \frac{s^2(k+1)-s^2(k)}{2\Delta t} \\ &= \frac{[s(k+1)-s(k)][s(k+1)+s(k)]}{2\Delta t} \end{aligned} \tag{17}$$

according to equation (17) it can be shown that it is only necessary to show that

$$\begin{aligned} &(s(k+1)-s(k))\cdot\mathrm{sgn}(s(k)) < 0 \\ &(s(k+1)+s(k))\cdot\mathrm{sgn}(s(k)) > 0. \end{aligned} \tag{18}$$

Also from equation (12), we can get

$$\begin{aligned} &(s(k+1)-s(k))\cdot\mathrm{sgn}(s(k)) \\ &\quad= [(I-\tau\Delta t - I)s(k) - \varepsilon\Delta t\mathrm{sgn}(s(k))]\cdot\mathrm{sgn}(s(k)) \\ &\quad= [-\tau\Delta t s(k) - \varepsilon\Delta t\mathrm{sgn}(s(k))]\cdot\mathrm{sgn}(s(k)) \\ &\quad= -\tau\Delta t\,|s(k)| - \varepsilon\Delta t < 0. \end{aligned} \tag{19}$$

$$\begin{aligned} &(s(k+1)+s(k))\cdot\mathrm{sgn}(s(k)) \\ &\quad= [(I-\tau\Delta t)s(k) - \varepsilon\Delta t\mathrm{sgn}(s(k)) + s(k)]\cdot\mathrm{sgn}(s(k)) \\ &\quad= [(2I-\tau\Delta t)s(k) - \varepsilon\Delta t\mathrm{sgn}(s(k))]\cdot\mathrm{sgn}(s(k)) \\ &\quad= (2I-\tau\Delta t)\,|s(k)| - \varepsilon\Delta t. \end{aligned} \tag{20}$$

When $|s(k)| > \varepsilon\Delta t/(2I-\tau\Delta t)$,$(s(k+1)+s(k))\cdot\mathrm{sgn}(s(k)) > 0$ where $\dot{V}(k) < 0$, the system will converge to the equilibrium point $s(k) = 0$, then the inventory of each segment of the supply chain inventory system will converge to the desired target inventory.

3.3. Numerical simulation study

To verify the effectiveness of the SOB-DSMC method, this paper is based on the MATLAB 2021a platform to carry out the optimization research of supply chain inventory systems under contingency situations. Firstly, the relevant parameters of the supply chain warehouse system are reasonably assumed through the market survey. Secondly, the control parameters of the condition observer and the discrete sliding mode controller are set according to the control requirements of the supply chain warehouse system. Finally, the simulation software is used to simulate the contingency situation under different circumstances, and the simulation results are comprehensively analyzed to validate the effectiveness of the SOB-DSMC method.

In the simulation experiment, we take have chosen $n = 3$, representing the number of downstream distributors and retailers within the supply chain system. Through market survey and reasonable assumptions, we set the parameters of the inventory system model of the supply chain system as follows

$$A = \begin{bmatrix} 0.98 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.985 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.976 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.99 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.995 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.996 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.997 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0.98 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.99 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0.97 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0.96 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.94 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$C = I_{7\times 7} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

At the same time, to meet the rapidity and effectiveness of the supply chain system in response to contingency situations, this paper optimizes the parameters of the state observer and the discrete sliding mode controller through many pre-simulation experiments, to achieve the best control effect. Finally, the relevant parameters of the state observer and discrete sliding mode controller are shown in Table 2.

Firstly, the simulation is carried out for the supply chain inventory system in the normal situation, when only the discrete sliding mode controller is utilized for control. In this paper, the relevant parameters of the supply chain inventory system under normal conditions are set as shown in Table 3 below through reasonable assumptions.

In the simulation experiment, the initial value of the supply chain inventory system is set as follows

$$x(1) = [10\ ;\ 8\ ;\ 9\ ;\ 7\ ;\ 5.5\ ;\ 2.5\ ;\ 3.5]$$

$$u(1) = [1.5\ ;\ 0.4\ ;\ 0.5\ ;\ 0.6\ ;\ 0.45\ ;\ 0.35\ ;\ 0.37]$$

TABLE 2. Parameter settings related to state observer and discrete sliding mode controller.

Parameters	Value
Sampling time Δt	1
The matrix of the state observer L	AC^{-1}
$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6$	20
$\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6$	2.1
$\tau_0 = \tau_1 = \tau_2 = \tau_3 = \tau = \tau_5 = \tau_6$	0.1

TABLE 3. Parameters related to the supply chain inventory system under normal situations.

Parameters	Value (ton)
Target inventory $\zeta_1(k)$ of Distributor 1	1.52
Target inventory $\zeta_2(k)$ of Distributor 2	0.85
Target inventory $\zeta_3(k)$ of Distributor 3	2.53
Constant market demand $d_1(k)$ of Sales Area 1	3.762
Constant market demand $d_2(k)$ of Sales Area 2	2.956
Constant market demand $d_3(k)$ of Sales Area 3	5.236

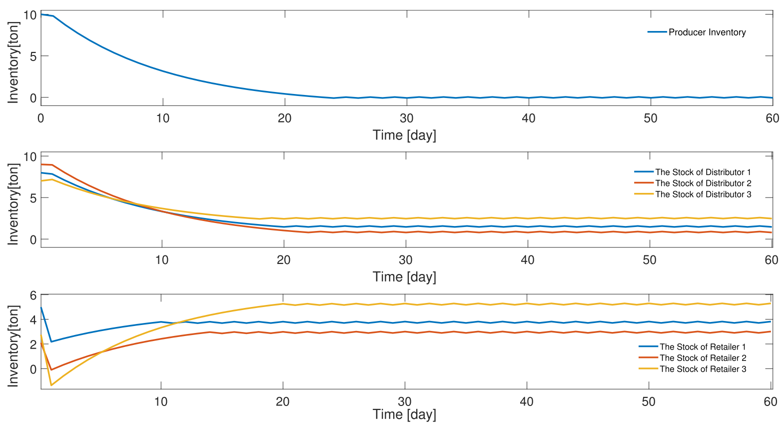

FIGURE 6. The stock in each segment under normal condition (DSMC).

During the experiment, the sampling interval was set to $\Delta t = 1s$, with the assumption that each sampling interval corresponds to 1 day of actual supply chain system operation. Therefore, the total duration of the experiment was $T = 60s$, which corresponds to a period of 60 days of actual supply chain system operation. The inventory level of the supply chain inventory system under normal conditions can be obtained through simulation experiments as shown in Figure 6, and the production and shipment quantities of each segment of the supply chain system are shown in Figure 7.

By analyzing Figures 6 and 7, it can be found that the supply chain inventory system can achieve the desired target inventory while responding to the market demand under normal conditions only by the discrete sliding mode controller. in order to carry out the comparison experiment, through the simulation experiment to simulate only in the discrete sliding mode controller under the role of the two different emergent conditions for the supply chain inventory system. (1) the contingency conditions for the supply chain market demand to play a positive role, making the market demand surge, this paper uses the contingency conditions 1 to indicate;

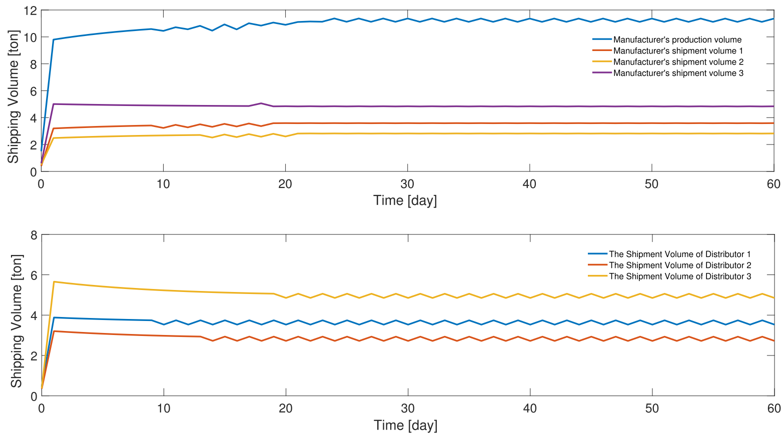

FIGURE 7. Production and shipments under normal condition(DSMC).

(2) the contingency conditions for the supply chain market demand to play a negative role, making the market demand plummeting, this paper uses the contingency conditions 2 to indicate. Which simulation experiment process sampling time $\Delta t = 1s$, the total sampling time is $T = 35s$, representing the supply chain system actually run 35 days. At the same time, assuming the supply chain system bullwhip coefficient $\beta = 1.21$, the impact parameter matrix $\bar{D}$ can be obtained as

$$\bar{D} = \begin{bmatrix} -1.4641 & -1.4641 & -1.4641 \\ -1.21 & 0 & 0 \\ 0 & -1.21 & 0 \\ 0 & 0 & -1.21 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

The experimental parameters of the simulation experiment under the sudden condition are the same as those under the normal condition, and the difference lies in the assumption that the sudden condition occurs at the time of $t = 30s$ during the simulation experiment, and the corresponding market demand also changes to a certain extent. Then we can get under the action of discrete sliding mode controller, the inventory changes in each segment of the supply chain inventory system when contingency 1 occurs are shown in Figure 8, and the inventory changes in each segment of the supply chain inventory system when contingency 2 occurs are shown in Figure 9.

By analyzing Figures 8 and 9, we can get that if the supply chain system is only under the action of discrete sliding mode controller, the impact of both contingency 1 and contingency 2 on the supply chain inventory system is huge, especially due to the existence of the bullwhip effect, the more in the upstream of the supply chain enterprises will be affected by the more, and accordingly the economic losses caused by the more will be greater(where MD denotes the demand in the market).

Finally, for the solution of the SOB-DSMC method proposed in this paper, a state observer is introduced to conduct simulation experiments to verify the effectiveness of the proposed method. The initial parameters of the supply chain system and the initial conditions of the simulation experiment with the same state observer are the same as the previous experiment. In the simulation experiment, the sampling time $\Delta t = 1s$, the total sampling time is $T = 50s$, representing the supply chain system is actually running for 50 days, the same is assumed that the supply chain system in the $t = 30s$ when the contingency condition occurs, then can be obtained in the SOB-DSMC conditions, the inventory changes in each segment of the supply chain inventory system when

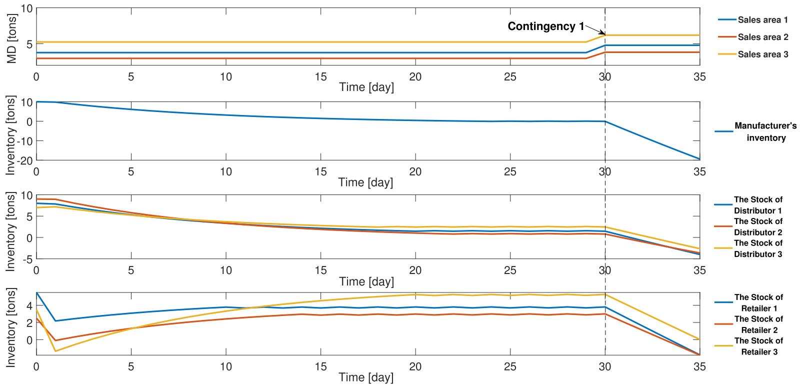

FIGURE 8. The stock in each segment under contingency 1 (DSMC).

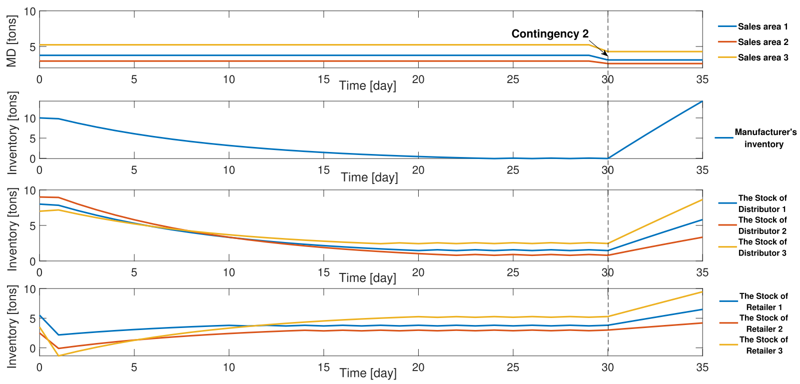

FIGURE 9. The stock in each segment under contingency 2 (DSMC).

contingency 1 occurs are shown in Figure 10, and the inventory changes in each segment of the supply chain inventory system when contingency 2 occurs are shown in Figure 11.

By analyzing the trend of the inventory curves of each link in the supply chain in Figures 10 and 11, it can be found that under the SOB-DSMC conditions, the impact of the two contingencies on the supply chain inventory system can be suppressed better. In order to compare the experimental results more intuitively, the inventory change curves of the supply chain inventory system under different conditions are shown in Figures 12 and 13, and the maximum inventory fluctuation in 5 days under the contingency condition is shown in Table 4.

in Figures 12 and13, inventory less than 0 indicates the out-of-stock condition, while the running time indicates the number of days of supply chain system operation after the occurrence of the unexpected condition. Inventory fluctuation in Table 3 refers to the difference between the inventory of each segment in the adjacent two days, take the average of the maximum inventory fluctuation within 5 days of the occurrence of the unexpected condition 1 and unexpected condition 2 as the maximum inventory fluctuation under the DSMC condition, and similarly, the maximum inventory fluctuation under the SOB-DSMC condition can be obtained, and calculate the reduction rate λ through equation (21).

$$\lambda = \frac{\Delta_{1\max} - \Delta_{2\max}}{\Delta_{1\max}} \times 100. \tag{21}$$

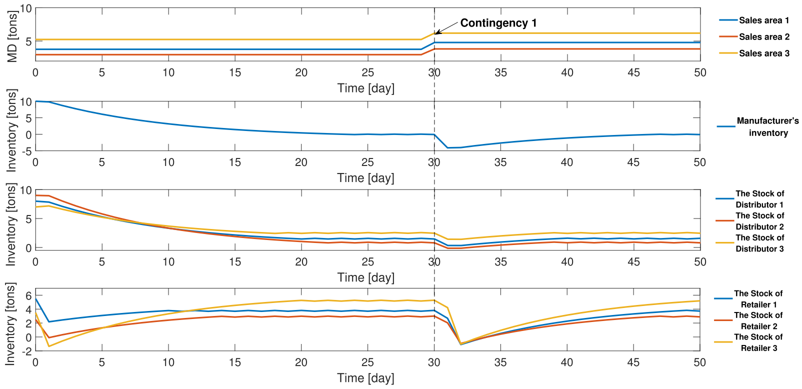

FIGURE 10. The stock in each segment under contingency 1 (SOB-DSMC).

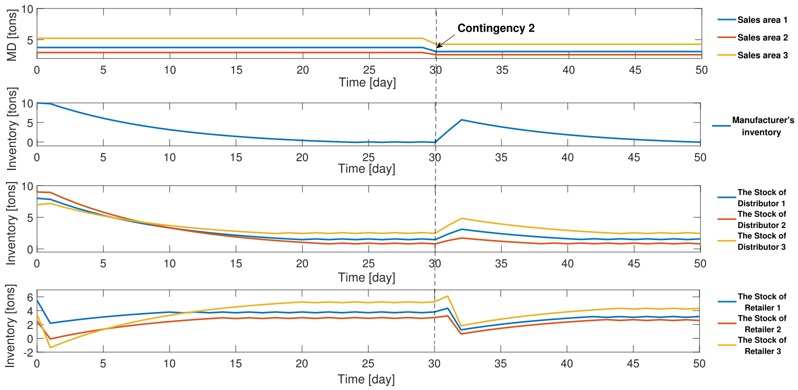

FIGURE 11. The stock in each segment under contingency 2 (SOB-DSMC).

TABLE 4. Maximum inventory fluctuations in each segment within 5 days under contingency conditions.

Parameters	Volatility of maximum stockpile $\Delta_{1\,\max}$ (DSMC)	Volatility of maximum stockpile $\Delta_{2\,\max}$ (SOB-DSMC)	Reduction rate λ
Manufacturer	3.455	0.790	77.13%
Distributor 1	1.003	0.388	61.32%
Distributor 2	0.702	0.275	60.83%
Distributor 3	1.14	0.457	59.91%
Retailer 1	0.862	0.453	47.44%
Retailer 2	0.612	0.352	42.48%
Retailer 3	0.902	0.506	43.90%

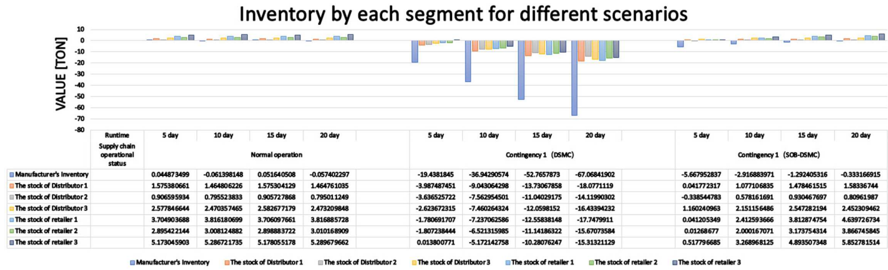

FIGURE 12. Inventory by segment under different scenarios (contingency 1).

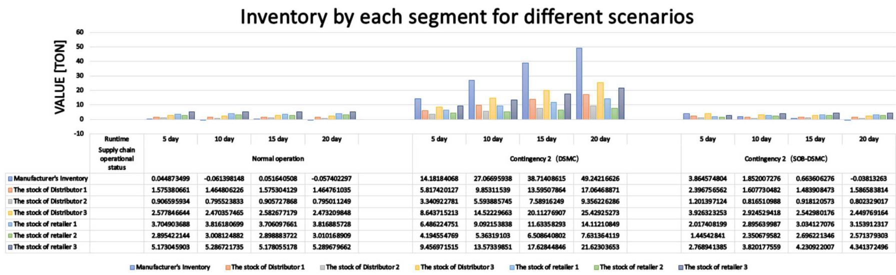

FIGURE 13. Inventory by segment under different scenarios (contingency 2).

By analyzing Figure 12 as well as Figure 13 we can find that only under the role of DSMC, both contingency 1 and contingency 2 have a greater impact on the inventory of each segment of the supply chain system, especially due to the existence of the bullwhip effect, the inventory of the producer is affected the most. However, under the SOB-DSMC, it can be found through the simulation results that whether it is contingency 1 or contingency 2, the controller designed by the SOB-DSMC method has a better inhibition effect on the impact of contingency, which can help the upstream and downstream enterprises in the supply chain effectively control the fluctuation of the inventory, and pass through the period of the contingency in a smooth manner.

Meanwhile, Table 3 shows that the SOB-DSMC method can significantly improve the inventory fluctuation problem of the upstream and downstream enterprises in the supply chain system within 5 days after being subjected to contingency conditions, which reduces the maximum inventory fluctuation of 77.13% in the producer segment, 60.67% in the distributor segment (the average of the three distributors' reduction rates), and 44.61% in the retailer segment (the average of the three retailers' reduction rates), and the maximum inventory fluctuation of 44.61% in the retailer segment. 44.61% (the average of the three retailer reduction rates) of the maximum inventory volatility. This shows that the SOB-DSMC method is able to reduce the inventory fluctuation in the supply chain in a short period of time, help the supply chain inventory system to cope with the contingency situation, and give a reasonable production, shipment, and purchase strategy for the upstream and downstream of the supply chain system.

4. Conclusion

In this paper, we propose a state observer and discrete sliding mode control (SOB-DSMC) method to effectively solve the problem of out-of-stock or backlog in the inventory of upstream and downstream enterprises in the supply chain in the face of contingency conditions. Therefore, the method in this paper has the following advances:

1. We examine the bullwhip effect within the supply chain system and develop a comprehensive inventory model for manufacturers, distributors, and retailers in response to unforeseen circumstances. We conduct simulation experiments to validate the efficacy of the proposed method under various contingency conditions.
2. From the simulation results, Whether facing positive or negative disruptive situations, this method aids both upstream and downstream enterprises in the supply chain system to develop rational production, procurement, and shipment strategies. Furthermore, it can mitigate the impact of unforeseen circumstances on the inventory of both upstream and downstream enterprises, thereby helping the supply chain inventory system smoothly navigate through periods of unexpected events.

In the simulation experiments conducted for the optimized supply chain inventory system model designed in this paper, the model utilized is a simplified version, which is one of the shortcomings of our research. Consequently, our future work will carry out a complete mesh supply chain inventory system. Furthermore, it's essential to note that the supply chain inventory system model presented in the paper is linear, whereas real-world environments involve numerous nonlinear factors. Therefore, in subsequent research, we plan to incorporate nonlinear relationships and conduct in-depth analyses to enhance the model's applicability to complex real-world scenarios.

Acknowledgements

The authors sincerely thank the editors and the anonymous reviewers for their valuable comments and suggestions, which help improve this research work.

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