RAIRO-Oper. Res. 58 (2024) 2321–2337
https://doi.org/10.1051/ro/2024088

RAIRO Operations Research
www.rairo-ro.org

DYNAMIC INVESTMENT STRATEGIES FOR A FOLK SPORTS TOURISM DESTINATION UNDER UNCERTAIN DEMAND

Xinjiao Lv[1], Bowen Deng[2] and Kui Deng[1]

Abstract. More research should shed light on discovering the optimal investment strategy for folk sports tourism destination (FSTD) projects. Therefore, in this paper, we develop a dynamic game model of FSTD considering the dynamic characteristics of FSTD investment, the mode of division of labor and cooperation between public and private operators, and the uncertainty of consumer demand. Public capital is responsible for constructing infrastructures such as venues, and private capital is responsible for services such as catering and accommodation. To promote the development of the FSTD project, the higher-level government subsidizes public investment. Consumer demand for the program is affected by factors such as the size of the two types of capital, the price and quality of services, and demand uncertainty. The study finds that the subsidy leads to an increase in the quantity of public investment and consumption demand, but private sector investment and the prices of both public and private projects are unaffected by the subsidy; the public sector's net return varies in an inverted U-shape with the rate of subsidy, but the private sector's net return rises monotonically. Demand disturbances widen the gap in the net returns of operators between the subsidized and unsubsidized scenarios.

Mathematics Subject Classification. 90C05.

Received January 18, 2024. Accepted April 18, 2024.

1. Introduction

Minshuku sports is a sports activity with significant national characteristics, which enables participants to strengthen their bodies and enjoy themselves mentally. At the same time, it is also a carrier for inheriting national culture. Also, it plays an increasingly important role in improving the economy of ethnic areas through the combination of tourism programs. Therefore, the Chinese government attaches great importance to the development of FSTD, issuing a series of documents and giving financial support to the construction [19,23]. In the meantime, based on the pressure to develop the economy and the guidance and support from the higher level government, many local governments have already started taking action for FSTD construction. For instance, Hunan Province alone has publicized 400 FSTD-related under-construction boutique projects in 2023.

Maybe finding the correct construction mode is paramount in building these FSTDs. Because it requires a large amount of capital and has a long construction period, local governments in ethnic areas not economically well-off cannot undertake it independently. It needs to strive for the support of the higher government on the one

Keywords. Folk sports tourism, investment decision, demand uncertainty, differential game.

[1] School of Physical Education, University of South China, Hengyang 421001, P.R. China.

[2] School of Economics, Southwest University of Political Science & Law, Chongqing 400054, P.R. China.

*Corresponding author: dk19981430@126.com

hand and to absorb private capital on the other. Therefore, FSTD projects generally adopt a public-private joint construction model. Among them, state-owned capital is responsible for constructing infrastructure and the main project, while private capital is mainly engaged in service projects, such as catering and accommodation. While the two capital types of operators play a Stackelberg game, they can also engage in some form of coordination. Studies have shown that coordination can improve the welfare of both parties in many cases [2, 7].

Furthermore, uncertainty about the outcome of a decision is specifically essential for decision-maker behavior [1, 3, 5, 15]. Mitigating the impact of unforeseen events must also be considered in the construction. For example, COVID-19, which started in 2019, has dealt a fatal blow to FSTD projects, as demand has fallen off a cliff in this scenario, making it difficult for many FSTD projects to sustain. Therefore, to ensure the smooth running of FSTD projects, it is essential to study and analyze the uncertainty events, especially the demand uncertainty.

However, very few existing related studies focus on the public-private FSTD investment model, and the literature that quantitatively analyzes the adverse impacts of uncertain demand on it is even scarcer. Therefore, this paper captures these two fundamental issues for research.

Existing literature has already shed light on the connotation and role of FSTD. The standard view is that FSTD is a place that uses folk sports as the primary carrier, comprehensively meets the cultural needs of tourists for fitness and recreation, and integrates sports, accommodation, amusement, and shopping [19, 23]. It plays a vital role in the transmission of national culture and the development of national philosophy [8, 17], the promotion of international ties and interactions [18], the suppression of social and moral slippage, and the lack of collective identity as well as the enhancement of social cohesion [19], and the promotion of the development of the economy of the region of satisfaction [23].

The construction of FSTD needs to find the right strategy [9, 13]. Caffyn and Lutz [4], in their empirical study on the structure of FSTD programs in Birmingham, found that taking into account factors such as social inclusiveness, the needs of ethnic minorities, and the inheritance of the colonial heritage had a significant positive impact on the success of the program. Jun [12] focuses on the crucial role of cooperation in FSTD project construction and analyzes the FSTD project construction in Fujian-Taiwan cooperation as an example, from which he analyzes the factors constraining association and proposes countermeasures such as building a platform and expanding paths. Through a questionnaire survey, Zhou *et al.* [23] found that the construction of FSTDs with ball games as one of the sports programs is most favored by consumers. Hence, the authors suggested that the investors and builders of FSTDs should consider consumers' psychology and try to incorporate ball games into FSTDs as much as possible.

In addition, like many other investments, FSTD construction investment has a distinctly dynamic character, with project capital formation stemming from investments that continue accumulating over time. Thus, analyzing such issues requires constructing dynamic models. However, most studies on FSTD project construction follow a static research approach, which is not conducive to providing project builders with investment strategies at different times. This year, many scholars, such as [6, 20, 22] developed their respective dynamic apparent to analyze economic problems with dynamic characteristics, which provides a reference for the study in this paper.

Compared with the existing relevant literature, this paper has the following differences:

- Previous literature focuses on macro-level analysis such as purpose, significance, and principles. On the other hand, this paper delves into the optimal investment in FSTD construction from the micro level and analyzes the co-investment decision of public and private capital.
- It analyzes the subsidy strategy of the higher-level government for FSTD construction. It gives a decision model of the optimal subsidy rate by which the higher-level government can instantly optimize it according to the change of (conditional) parameters.
- Our demand function is of great practical significance. Demand depends not only on prices and marketing efforts but also on the scale of public and private capital, the quality of services provided by both types of businesses, and random factors that influence it. It can better reflect objective reality than the demand function provided by previous research.

TABLE 1. Comparison table, based on various assumptions related to the proposed problem.

	Demand dependent on				framework of analysis		Policy	Considering	Stackelberg	Public and
	Price	Sales effort	Capital scale	Service quality	Static	Dynamic	incentives	uncertainty	game	private
Tan [19]	$\times$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\times$	$\times$	$\times$
Zhou *et al.* [23]	$\times$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\times$	$\times$	$\times$
Ghosh *et al.* [7]	$\surd$	$\surd$	$\times$	$\times$	$\surd$	$\times$	$\times$	$\times$	$\times$	$\times$
Barman *et al.* [2]	$\surd$	$\surd$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\surd$	$\surd$	$\times$
Dey *et al.* [6]	$\surd$	$\surd$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\times$	$\times$
Bai *et al.* [1]	$\surd$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\times$	$\times$	$\times$	$\times$
Delaney [5]	$\surd$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\times$	$\times$	$\times$
Barman *et al.* [3]	$\surd$	$\surd$	$\times$	$\times$	$\surd$	$\times$	$\times$	$\surd$	$\times$	$\times$
Kim and Kim [13]	$\times$	$\times$	$\times$	$\times$	$\surd$	$\times$	$\surd$	$\times$	$\times$	$\times$
Yi *et al.* [22]	$\surd$	$\surd$	$\times$	$\times$	$\times$	$\surd$	$\surd$	$\times$	$\surd$	$\times$
Yi *et al.* [22]	$\surd$	$\surd$	$\times$	$\times$	$\times$	$\surd$	$\surd$	$\times$	$\surd$	$\times$
Present paper	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

TABLE 2. Parameters and variables symbol descriptions.

Parameters	Description
$u_1(t), k_1(t)$	Amount of investment and capital of the project's public operator at time t
$u_{2i}(t), k_{2i}(t)$	Amount of investment and capital of the project's private operator i at time t
$q(t)$	Number of consumers of the FSTD project
$p_1(t), p_{2i}(t)$	Net prices of services of the public operator and private operator i
$\pi_1(t), \pi_{2i}(t)$	Long-term discounted profits for the public operator and private operator i
ε	Stochastic perturbations in the quantity demanded
n	Total number of private operators in the project
τ	Rate of government subsidy for public sector investment in the project
δ_1, δ_2	Depreciation rates of capital for public and private operators
a	Potential market demand
α_1, α_2	Marginal effect of capital on demand for public and private operators

– It incorporates demand uncertainty factors into the model and analyzes the impact of demand perturbations on optimal investment decisions and countermeasures. As a result, FSTD's investment operators' decisions based on the results of this study could improve risk resilience.

Next, let us use Table 1 to further elaborate on this paper's novelty by comparing it to the prior closely related primary literature.

The remainder of the article progresses as follows: Section 2 describes the notation and assumptions of the model. And the equilibrium results under several government support modes are located in Section 3. Section 4 compares the equilibrium results. Section 5 gives the conclusions of the study. In addition, we put some tedious computational procedures in the study in the appendix.

2. Notation and assumptions

This section will develop the research model of this paper through several assumptions for investigating the investment decision of the FSTD program.

2.1. Notation

To describe the model, let us apply Table 2 to summarize the symbols and definitions of the variables used.

2.2. Assumptions

Let us consider a local government to build an FSTD project. However, it is still underfunded despite borrowing from financial institutions and applying for partial subsidies from higher levels of government. Therefore, it decides to transfer part of the project to private capital, thus creating a pattern of co-investment by public and private capital. It may be a typical pattern in China. For example, FSTD projects have adopted this model, such as the Taiji Sports and Cultural Tourism in Henan Province and the Dragon Dance Sports and Culture in Zhangjiajie, Hunan Province. In this case, public capital is generally responsible for infrastructure construction, event organization, and advertising, while private capital is engaged in service areas such as catering and accommodation.

Thus, there are two types of investment operators in this FSTD program. The first group is the public investor with a unified operation, which derives its income mainly from sports, tourist tickets, and other services. The second category is the private sector, a group of n self-accounting individual households that derive their income primarily from food and beverage, lodging, commercial retailing, and other services. For research convenience, we assume that self-employed households are homogeneous.

According to the basics of microeconomics, the investment and capital of the two types of operators obey the following dynamic relationship:

$$\dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \tag{1}$$

$$\dot{k}_{2i}(t) = u_{2i}(t) - \delta_2 k_{2i}(t), \tag{2}$$

where $k_1(t)$ and $k_2i(t)$ show the level of public capital and private capital of individual households i at time t, $i = 1, 2...n$. And, we assume the initial value $k_1(0) = 0$, $k_{2i}(0) = 0$. $u_1(t)$ and $u_{2i}(t)$ are the amount of investment by the local government and individual households i at time t, $i = 1, 2...n$. And $\delta_j > 0 (j = 1, 2)$ is the natural capital depletion rate. These dynamic equations appear in many studies, such as [11, 21] The investment in the project incurs certain costs. Referring to related studies, such as Yi *et al.* [21] and Ma *et al.* [14], we assume they are convex quadratic standard functions, *i.e.*, $u_1^2/2$, $u_{2i}^2/2$.

The program provides a full range of services to consumers. Therefore, in addition to the price factor, the size of both public and private capital and the quality of service impact consumer demand. In addition, some uncertainties, such as epidemics, sudden changes in consumer preferences, etc., produce unavoidable perturbations in consumer demand. Therefore, we can give the following demand function:

$$q(t) = [a - p_1(t) - p_{2i}(t) + \varepsilon](\alpha_1 k_1(t) + \alpha_2 n k_{2i}(t)). \tag{3}$$

where a is the potential market size, $p_1(t)$ and $p_{2i}(t)$ represent the net price of services, *i.e.*, the price after subtracting costs, for public and private programs, respectively. α_1 and α_2 specify the sensitivity of demand to the size of the public and private capital, respectively. To a certain extent, α_1 and α_2 can represent the service quality of the two types of capital, and the larger one of them is, the more consumer demand is incurred per unit of capital and the better its service quality.

In addition, in this paper, we consider uncertainty in consumption demand. Uncertainty may take many forms. It may be random [1, 5] or fuzzy and rough [3, 15]. Considering the effects of stochastic events, such as epidemics, we use a stochastic disturbance factor, ε, to represent the disturbance of consumer demand by stochastic factors. Its density function is $f(\varepsilon)$ with mean x and follows a uniform distribution on $[\varepsilon_{min}, \varepsilon_{max}]$.

The demand function above is a high-level generalization of reality. On the one hand, an increase in the price of the service will dampen consumer demand. On the other hand, If the destination is more capitalized, the more complete the service facilities and the more services it offers, the more attractive it is to consumers; the quality of the service of the program, both public and private, is also an important factor for consumers to consider, because higher quality of service is more pleasant to consumers. Consumers are more willing to pay for it, and it improves the destination's goodwill and attracts more followers. In addition, demand will be affected by unpredictable and random factors.

Related studies inspire the above demand function (3) molding. One is that according to Jiang and Xu [10] and Yi *et al.* [21], price and non-price factors act linearly on demand. The second is that according to Ouardighi [16], the two types of factors impact consumer demand in the form of multiplication.

According to our survey, FSTD projects can apply for subsidies from the central and provincial governments at an average rate of about 30% of the public investment. Accordingly, we assume that the public sector operator can receive subsidies from higher levels of government at a subsidy rate of τ.

3. Equalization

In this section, we investigate the equilibrium of no subsidization and subsidizing public investment by a higher-level government to get some managerial implications.

3.1. No subsidies from higher levels of government (Scenario N)

This subsection investigates the case of no subsidy from the higher-level government for getting the baseline for comparisons. The long-run optimal decision issues for local government and the individual household i in this scenario are:

$$\begin{aligned}&\max_{p_1,u_1}\pi_1(t)=\textstyle\int_0^\infty e^{-\rho t}[p_1(t)q(t)-u_1^2(t)/2]dt,\\&s.t.\begin{cases}\dot{k}_1(t)=u_1(t)-\delta_1 k_1(t),\\\dot{k}_{2i}(t)=u_{2i}(t)-\delta_2 k_{2i}(t),\end{cases}\end{aligned}\tag{4}$$

$$\begin{aligned}&\max_{p_{2i},u_{2i}}\pi_{2i}(t)=\textstyle\int_0^\infty e^{-\rho t}[p_{2i}(t)q(t)-u_{2i}^2(t)/2]dt,\\&s.t.\begin{cases}\dot{k}_1(t)=u_1(t)-\delta_1 k_1(t),\\\dot{k}_{2i}(t)=u_{2i}(t)-\delta_2 k_{2i}(t),\end{cases}\end{aligned}\tag{5}$$

where ρ is the risk-free interest rate.

Equations (4) and (5) show that public and private operators maximize their profits by choosing the service price and the investment quantity. In this game, we assume that the public operator is the first mover in the Stackelberg game and the private operator is the follower. Because private operators decide only when they see the investment scale of the public operator and how much to invest, public capital plays a leading and dominant role in the whole operation process. According to the classical method of Stackelberg's differential game, we first solve the optimal strategy of the follower, *i.e.*, the private operator. Then, we substitute it into the decision function of the forerunner, *i.e.*, the public operator, and then solve its optimal strategy. Based on the above steps, we obtain the following proposition:

Proposition 3.1. *In the unsubsidized scenario, the optimal strategy for both public and private operator i are:* $p_1^{N^*}=\frac{a+\varepsilon}{3}$; $p_{2i}^{N^*}=\frac{a+\varepsilon}{3}$; $u_1^{N^*}=\frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)}$; $u_{2i}^{N^*}=\frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}$.

Proof: See Appendix A1.

Accordingly, we get the following Lemma:

Lemma 3.2. $\frac{\partial p_1^{N^*}}{\partial\varepsilon}=\frac{\partial p_2^{N^*}}{\partial\varepsilon}=\frac{1}{3}$, $\frac{\partial q^{N^*}}{\partial\varepsilon}>0$, $\frac{\partial u_1^{N^*}}{\partial\varepsilon}>0$, $\frac{\partial k_1^{N^*}}{\partial\varepsilon}>0$, $\frac{\partial u_{2i}^{N^*}}{\partial\varepsilon}>0$, $\frac{\partial k_{2i}^{N^*}}{\partial\varepsilon}>0$.

Proof: See Appendix A2.

It shows that uncertain perturbations in consumer demand affect prices, quantity demanded, investment, and capital. In particular, the marginal effect on prices for both operators is one-third, with a one-unit change in the disturbance term resulting in a one-third-unit price change in the same direction. In addition, favorable demand perturbations increase demand, investment, and capital, and unfavorable perturbations decrease demand, investment, and capital. The price, quantity demanded, and investment, *i.e.*, capital, of FSTD operators are all declining when COVID-19 occurs, which is consistent with the results of this study.

According to Proposition 3.1, we give the following equilibrium results for the no-subsidy scenario: $q^{N^*} = \frac{(a+\varepsilon)(\alpha_1 k_1^{N^*}(t)+\alpha_2 n k_{2i}^{N^*}(t))}{3}$; $k_1^{N^*}(t) = k_{1\infty}^N - k_{1\infty}^N e^{-\delta_1 t}$; $k_{1\infty}^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)\delta_1}$; $k_{2i\infty}^{N^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)\delta_2}$; $\pi_1^{N^*} = e^{-\rho t}(m_1^{N^*}k_1^{N^*} + m_2^{N^*}k_2^{N^*} + m_3^{N^*})$; $\pi_{2i}^{N^*} = e^{-\rho t}(r_1^{N^*}k_1^{N^*} + r_2^{N^*}k_2^{N^*} + r_3^{N^*})$; $J_{total}^{N^*} = \pi_1^{N^*} + n\pi_{2i}^{N^*}$.

Where $m_1^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)}$, $m_2^{N^*} = \frac{(a+\varepsilon)^2\alpha_2 n}{9(1+\delta_2)}$, $m_3^{N^*} = (m_1^{N^*})^2/2 + m_2^{N^*}r_2^{N^*}$, $r_1^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9n(1+\delta_1)}$, $r_2^{N^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}$, $r_3^{N^*} = (r_2^{N^*})^2/2 + m_1^{N^*}r_1^{N^*}$.

3.2. Subsidizing public investment (Scenario PI)

Although FSTD construction requires public and private investments, the current practice in China is to subsidize only public investments. Therefore, the decision function of the two types of operators under the subsidy rate τ are:

$$\max_{p_1,u_1} \pi_1(t) = \int_0^\infty e^{-\rho t}[p_1(t)q(t) - (1-\tau)u_1^2(t)/2]dt,$$
$$s.t. \begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_{2i}(t) = u_{2i}(t) - \delta_2 k_{2i}(t), \end{cases} \tag{6}$$

$$\max_{p_{2i},u_{2i}} \pi_{2i}(t) = \int_0^\infty e^{-\rho t}[p_{2i}(t)q(t)/n - u_{2i}^2(t)/2]dt,$$
$$s.t. \begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_{2i}(t) = u_{2i}(t) - \delta_2 k_{2i}(t), \end{cases} \tag{7}$$

Use the inverse solution method to produce the following proposition:

Proposition 3.3. *The optimal decision functions for the public and private operator i under the scenario where the higher level government subsidizes the FSTD program are as follows:* $p_1^{PI^*} = \frac{a+\varepsilon}{3}$; $p_{2i}^{PI^*} = \frac{a+\varepsilon}{3}$; $u_1^{PI^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1-\tau)(1+\delta_1)}$; $u_{2i}^{PI^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}$.

Proof: See Appendix B1.

Proposition 3.3 leads to the following lemmas.

Lemma 3.4. *There are* $\frac{\partial p_1^{PI^*}}{\partial\varepsilon} = \frac{\partial p_2^{PI^*}}{\partial\varepsilon} = \frac{1}{3}$, $\frac{\partial q^{PI^*}}{\partial\varepsilon} > 0$, $\frac{\partial u_1^{PI^*}}{\partial\varepsilon} > 0$, $\frac{\partial k_1^{PI^*}}{\partial\varepsilon} > 0$, $\frac{\partial u_{2i}^{PI^*}}{\partial\varepsilon} > 0$, $\frac{\partial k_{2i}^{PI^*}}{\partial\varepsilon} > 0$.

Proof: See Appendix B2.

Lemma 3.4 is similar to Lemma 3.2 in that it reveals that positive (negative) stochastic perturbations have a positive (negative) effect on the prices, quantities, investments, and capital of the two business entities studied.

Lemma 3.5. *Subsidies have the following impacts:*

(1) $\frac{\partial p_1^{PI*}}{\partial\tau} = \frac{\partial p_2^{PI*}}{\partial\tau} = \frac{\partial u_{2i}^{PI*}}{\partial\tau} = \frac{\partial k_{2i}^{PI*}}{\partial\tau} = 0$, $\frac{\partial u_1^{PI*}}{\partial\tau} > 0$, $\frac{\partial k_1^{PI*}}{\partial\tau} > 0$, $\frac{\partial q^{PI*}}{\partial\tau} > 0$.

(2) If $0 < \tau < \tau_c$, $\frac{\partial \pi_1^{PI^*}}{\partial\tau} > 0$*; if* $\tau_c < \tau < 1$, $\frac{\partial \pi_1^{PI^*}}{\partial\tau} < 0$.

(3) $\frac{\partial \pi_{2i}^{PI^*}}{\partial\tau} > 0$.

Where $\tau_c = \frac{\delta_1}{1+\delta_1}$.

Proof: See Appendix B3.

The government subsidizes the public sector operators so its investment volume increases and the corresponding level of capital rises, *i.e.*, $\partial u_1^{PI^*}/\partial\tau > 0$, $\partial k_1^{PI^*}/\partial\tau > 0$. However, the private sector's investment and capital is not affected by the subsidy rate, *i.e.*, $\partial u_{2i}^{PI^*}/\partial\tau = \partial k_{2i}^{PI^*}/\partial\tau = 0$, because it gets no subsidy. In addition, the increased public capital can be used to develop new services, etc., leading to a rise in consumer demand in FSTD, $\partial q^{PI^*}/\partial\tau > 0$.

We find a fascinating phenomenon in terms of (2) and (3) in Lemma 3.5, where the profits of unsubsidized private operators rise monotonically with the subsidy rate to public operators. However, the profits of subsidized public operators do not rise monotonically but rather in an inverted U-shape, rising and then falling.

The reason for this is that, on the one hand, the public operator's investment rises with the subsidy rate, leading to a rise in the level of public capital and the quantity of consumer demand. Thus, its sales revenue is growing, but according to the basic view of microeconomics, it obeys the law of diminishing margins. On the other hand, due to the marginal increasing nature of investment costs, as the rate of investment subsidy continues to increase, the costs associated with increased investment due to higher and higher subsidy rates become higher and higher. This results in rising profits where marginal revenues are higher than marginal costs up to a certain threshold (τ_c) of subsidy rates. When the subsidy rate increases beyond that threshold (τ_c), marginal costs are higher than marginal revenues, and the level of profits declines.

Although private operators are not subsidized, the subsidy allows the public operator to monotonically increase its investment and capital, bringing about a continuous rise in consumer demand, which helps the private operators continuously improve their income. Still, they do not increase investment, so their profits rise monotonically, *i.e.*, $\partial\pi_{2i}^{PI^*}/\partial\tau > 0$.

The optimal choices of the parties in Proposition 3.3 produce the following equilibrium outcome of the game under the subsidy of the higher government: $q^{PI^*} = \frac{(a+\varepsilon)(\alpha_1 k_1^{PI^*}(t)+\alpha_2 n k_{2i}^{PI^*}(t))}{3}$; $k_1^{PI^*}(t) = k_{1\infty}^{PI} - k_{1\infty}^{PI} e^{-\delta_1 t}$; $k_{1\infty}^{PI^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1-\tau_1)(1+\delta_1)\delta_1}$; $k_{2i}^{PI^*}(t) = k_{2i\infty}^{PI} - k_{2i\infty}^{PI} e^{-\delta_2 t}$; $k_{2i\infty}^{PI^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)\delta_2}$; $\pi_1^{PI^*} = e^{-\rho t}(m_1^{PI^*} k_1^{PI^*} + m_2^{PI^*} k_{2i}^{PI^*} + m_3^{PI^*})$; $\pi_{2i}^{PI^*} = e^{-\rho t}(r_1^{PI^*} k_1^{PI^*} + r_2^{PI^*} k_{2i}^{PI^*} + r_3^{PI^*})$; $J_{total}^{PI^*} = \pi_g^{PI^*} + n\pi_{2i}^{PI^*}$.

Where $m_1^{PI^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)}$, $m_2^{PI^*} = \frac{(a+\varepsilon)^2\alpha_2 n}{9(1+\delta_2)}$, $m_3^{PI^*} = (\frac{1}{(1-\tau)} - \frac{1}{2(1-\tau)^2})(m_1^{PI^*})^2 + m_2^{PI^*} r_2^{PI^*}$, $r_1^{PI^*} = \frac{(a+\varepsilon)^2\alpha_1}{9n(1+\delta_1)}$, $r_2^{PI^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}$, $r_3^{PI^*} = (r_2^{PI^*})^2/2 + \frac{1}{(1-\tau)} m_1^{PI^*} r_1^{PI^*}$.

4. Comparison of equilibrium results

We have obtained equalization results for several scenarios. This section will concentrate on comparing these results to determine the study's managerial implications.

4.1. Comparison of results by calculation

This section compares the results in both the unsubsidized and subsidized cases.

We obtain the following inference by comparing the optimal strategies under non-subsidy (N) and subsidized public investment (PI).

Lemma 4.1. *There are* $p_1^{PI^*} = p_1^{N^*}$, $p_2^{PI^*} = p_2^{N^*}$, $q^{PI^*} > q^{N^*}$, $u_1^{PI^*} > u_1^{N^*}$, $u_{2i}^{PI^*} = u_{2i}^{N^*}$, $k_1^{PI^*} > k_1^{N^*}$, $k_{1\infty}^{PI}(t) > k_{1\infty}^{N}(t)$, $k_{2i}^{PI^*} = k_{2i}^{N^*}$, $k_{2i\infty}^{PI}(t) = k_{2i\infty}^{N}(t)$.

Proof: See Appendix C1.

Lemma 4.1 is, in a sense, a reformulation of Lemma 3.5, so it does not require much discussion here.

Lemma 4.2. *The following relationships exist:*

(1) If $\tau < \tau_a$, $V_1^{NPI} = \pi_1^{PI^*} - \pi_1^{N^*} > 0$, $\frac{\partial V_1^{NPI}}{\partial\varepsilon} > 0$;
 if $\tau > \tau_a$, $V_1^{NPI} = \pi_1^{PI^*} - \pi_1^{N^*} < 0$, $\frac{\partial V_1^{NPI}}{\partial\varepsilon} < 0$.

(2) $V_{2i}^{NPI} = \pi_{2i}^{PI^*} - \pi_{2i}^{N^*} > 0$, $\frac{\partial V_1^{NPI}}{\partial\varepsilon} > 0$.

(3) If $\tau < \tau_b$, $V_{total}^{NPI} = J_{total}^{PI^*} - J_{total}^{N^*} > 0$, $\frac{\partial V_{total}^{NPI}}{\partial\varepsilon} > 0$;
 if $\tau > \tau_b$, $V_{total}^{NPI} = J_{total}^{PI^*} - J_{total}^{N^*} < 0$, $\frac{\partial V_{total}^{NPI}}{\partial\varepsilon} < 0$.

(4) If $\tau < \tau_d$, $\frac{\partial V_{total}^{NPI}}{\partial\tau} > 0$; if $\tau > \tau_d$, $\frac{\partial V_{total}^{NPI}}{\partial\tau} < 0$.

Where $\tau_{1a} = \frac{2(1-e^{-\delta_1 t})}{2(1-e^{-\delta_1 t})+\delta_1}$, $\tau_{1b} = \frac{4(1-e^{-\delta_1 t})+2\delta_1}{4(1-e^{-\delta_1 t})+3\delta_1}$, $\tau_{1c} = \frac{\delta_1}{1+\delta_1}$, $\tau_{1d} = \frac{2(1-e^{-\delta_1 t})+\delta_1}{2(1-e^{-\delta_1 t}+\delta_1)}$.

Proof: See Appendix C2.

Lemma 4.2 discusses the changes in the welfare levels of the parties as a result of the subsidies. The first of which says that the public owner's net returns do not necessarily improve due to subsidizing public investment. Moreover, contrary to common perception, the net returns improve when the subsidy rate is below a certain threshold (τ_a), while the net returns decline when the subsidy rate is above that threshold. It implies that public owner prefers to accept appropriately high (τ_c) rather than excessively high subsidy rates. Because its net benefit improvement achieves its maximum value at that point, if the subsidy rate continues to rise, its net benefit improvement value decreases. When the subsidy rate grows to (τ_a), it becomes 0. We have already analyzed the reasons for this in our explanation of Lemma 3.5, and we will not repeat them here to avoid repetition.

The (2) in Lemma 4.2 shows that subsidizing public investment improves the net returns of private operators compared to not implementing subsidies. Moreover, the improvement in their net returns rises monotonically with the subsidy rate because an increase in the number of consumers with the subsidy rate leads to an improvement in the returns of private operators without increasing their costs (their level of investment remains constant). Furthermore, the effect of this improvement is reinforced with a positive demand disturbance shock because private owners enjoy the spillover effect of a consumer rise due to subsidized public investment.

The (3) in Lemma 4.2 examined the impact of subsidized public investment on the total net income of the entire FSTD destination, including both public and private sectors. Similar to (1) in Lemma 4.2, there is also a threshold below (τ_b) in which the subsidy rate from the higher-level government for public investment leads to an improvement in total net income. Beyond this threshold, the total net income deteriorates.

The (4) in Lemma 4.2 describes the net benefit improvement of the FSTD with the subsidy rate. It shows a threshold point (τ_d) below which the value of net benefit improvement for the FSTD rises with the subsidy rate. When the subsidy rate exceeds that point, it decreases with the subsidy rate. If the subsidy rate grows to (τ_b), it becomes 0.

We obtained a more intuitive understanding of Lemma 4.2 through Figure 1. One can see that the most preferred subsidy rate for the public owner is (τ_c), and the areas within the blue (τ_a) line are win-win areas for both the public and private owners. Because the net income of two types of homeowners in the region improves due to subsidies, but when the subsidy rate is higher than (τ_a), the net income of the public homeowner deteriorates. The area below (τ_b) is the area where the net income of the entire FSTD improves due to subsidies. If the subsidy rate is higher than it, the net income decreases.

Figure 1 also shows that the maximum improvement in net returns for FSTD occurs when the subsidy rate is (τ_d), because below that line, $\partial V_{total}^{NPI}/\partial\tau > 0$; above that line, $\partial V_{total}^{NPI}/\partial\tau < 0$.

From it, we can see that the government's support for FSTD, whether for public capital or the entire FSTD, is not necessarily higher but must be within a specific limit. Using our model, governments can obtain a reasonable range of subsidy rates and the optimal subsidy rates for serving different goals. Therefore, it further highlights the significance of our research.

4.2. Numerical example

It has shown some exciting results. In this subsection, for empirical analysis, let's use the FSTD construction case in Yaoshan Ancient Village from Libo County, Guizhou Province, China. The Baiku Yao ethnic group in Yaoshan Ancient Village is recognized by UNESCO as one of the most well-preserved ethnic cultures and is regarded as the "living fossil of human civilization" and the "Indians of the East". So far, it has preserved hundreds of years of traditional ethnic sports such as gyro and archery and mainly carries out regular sports tourism programs such as gyro. Its current annual capacity is about 200,000 visits, and its expected maximum capacity is about 500,000, so we set its potential market size as $a = 50$. The useful life of the capital of the public and private operators is about ten years, so we assume the parameter of the depreciation rate of the capital, $\delta_1 = 0.1$, $\delta_2 = 0.1$. Based on the project's operation in the past years, the marginal impacts of the capital on the consumption demand per million dollars of public and private capital are, respectively, as follows: 0.03 and 0.01,

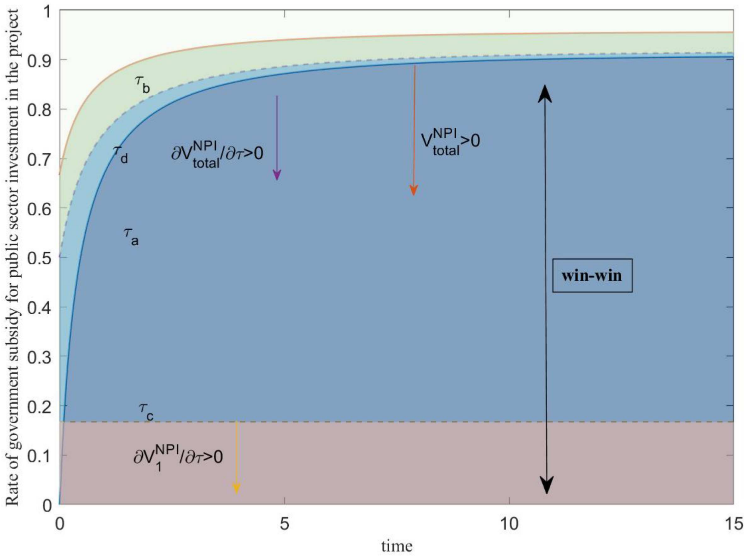

FIGURE 1. Profit improvement areas for two types of business owners and FST destinations.

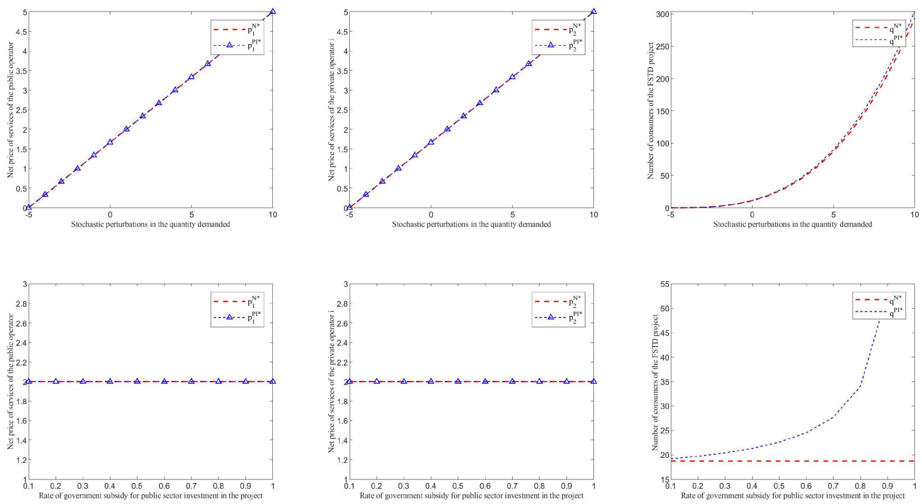

FIGURE 2. Changes of prices and number of consumers in subsidies rate and uncertain demand disturbances.

so let $\alpha_1 = 0.03$, $\alpha_2 = 0.01$. About 100 private operators are engaged in catering and lodging services in the project, so let $n = 100$. The subsidy rate of the higher level government (the central and provincial governments) for public investment in the project is about 30%, so let $\tau = 0.3$. In addition, we set the risk-free interest rate according to the current financial market condition parameter $\rho = 0.02$. We use Matlab software to visualize the results of the empirical study in Figures 2–4 based on the above parameters. Figure 2 shows that subsidies do not affect prices for the two types of operators. However, the subsidy does improve consumer demand because the interval of variation of the consumer demand perturbation and the subsidy parameter has a higher consumer demand curve with the subsidy than without it. The above results validate Lemma 4.1.

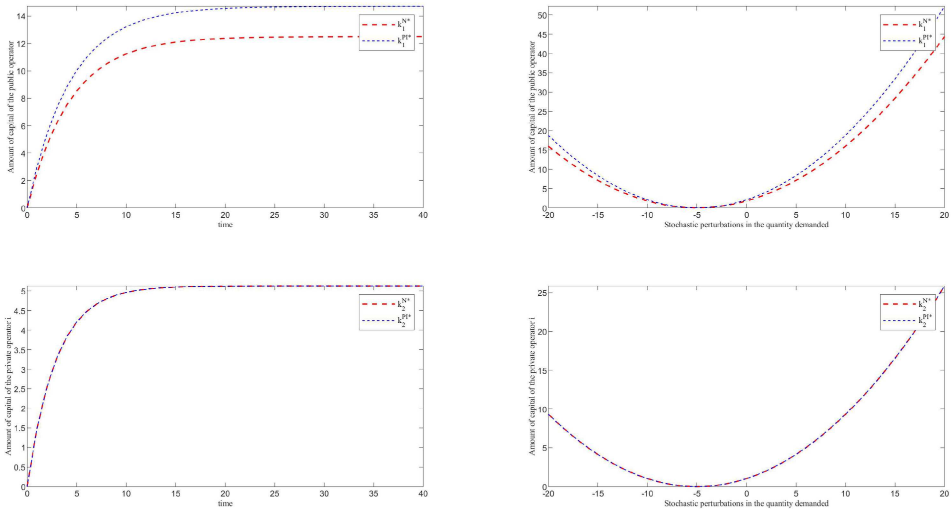

FIGURE 3. Changes in capital levels over time, demand disturbances, and subsidy rates.

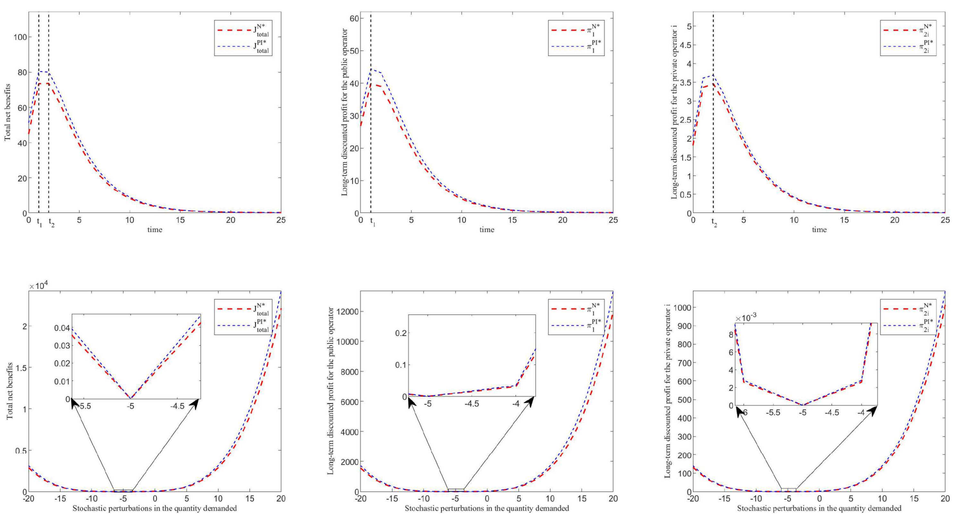

FIGURE 4. Variation of parties' and total net benefits over time and demand perturbation parameters.

Figure 3 shows that investment rises over time and eventually reaches a steady state for both public and private operators (a steady state occurs around period 25 for public operators and around period 15 for private operators). Second, subsidies increase investment in the public sector but do not affect private sector investment. Third, stochastic variation in consumer demand leads to an inverted U-shaped change in investment for both types of operators. These results confirm the relevant assertions in Lemma 3.2–4.1.

From the first line of Figure 4, one can see that the net benefits to all parties and the total net benefits rise over time, then remain relatively stable, and finally fall to zero, reflecting the fact that the construction of FSTD destinations, similar to the construction of other projects, undergoes an upward phase, a period of stable

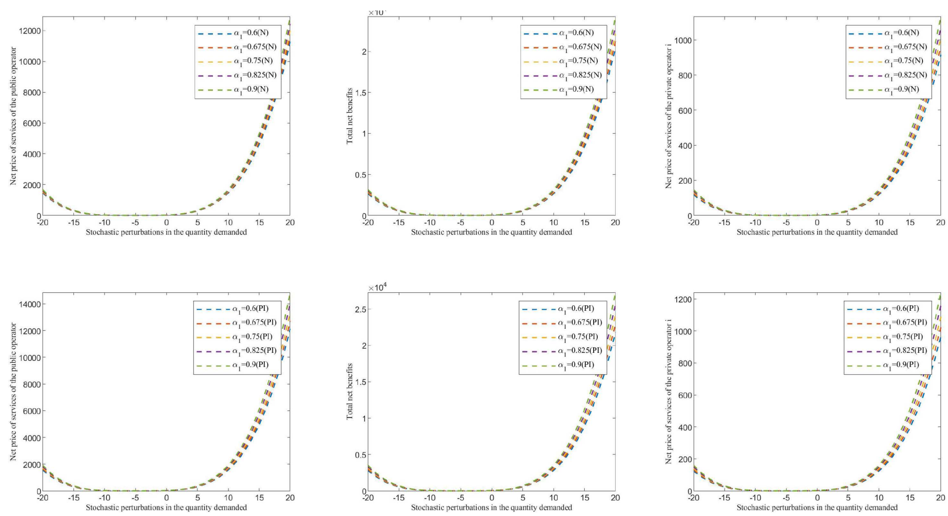

FIGURE 5. Variation of members' profits with demand disturbances for different values of α_1.

development, and a period of decline in its life cycle. Second, if higher levels of government subsidize the project, this will improve the net benefits to both public and private operators.

The second line of Figure 4 shows the effect of demand perturbations on the net returns of each party. We find that similar to the impact of demand perturbation on investment, the net returns of the parties change in an inverted U-shape under its effect.

The above results corroborate the assertions in Lemma 4.1.

On the one hand, our parameter assignments come from the field survey; on the other hand, our data examples validate the findings of the modeling study. This mutual corroboration shows the robustness of this paper's conclusions.

4.3. Sensitivity analysis

This subsection performs sensitivity analysis to verify the robustness of the study of the above numerical examples. To this end, we change two critical parameters: the impact of service quality on demand for two types of businesses (or the marginal effect of two types of capital on demand), up and down by 10% and 20%, respectively. At the same time, we examine the relationship between demand disturbance parameters and time parameters with the profits of all parties based on the changes above. From this, we obtain sensitivity analysis Figures 5–8.

Observing Figures 5–8, we can observe that two critical parameters, a and b, vary up and down by up to 20% but do not alter the trend of profit changes for all parties involved. Therefore, we believe the data example analysis conducted in this article is robust and can withstand significant parameter perturbations.

4.4. Managerial implications

(1) Public and private operators of FSTDs should determine their optimal strategies based on market demand conditions, government subsidy policies, and existing conditions. Suppose the higher-level government subsidizes the project. In that case, the public operator should increase investment and use the government subsidy to lay a good foundation for the project's future development. In contrast, the private operator should take advantage of the increased public investment to improve the quality of service to improve consumer demand. (2) For the higher level government of the FSTD project, the subsidy rate should be reasonably selected according to the development potential and stage of development of the FSTD as well as the investment level of the public

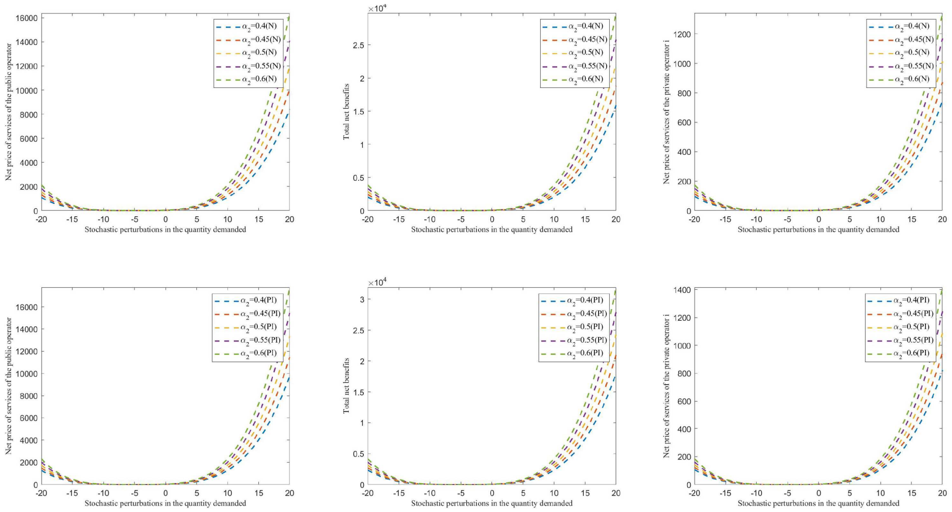

FIGURE 6. Variation of members' profits with demand disturbances for different values of α_2.

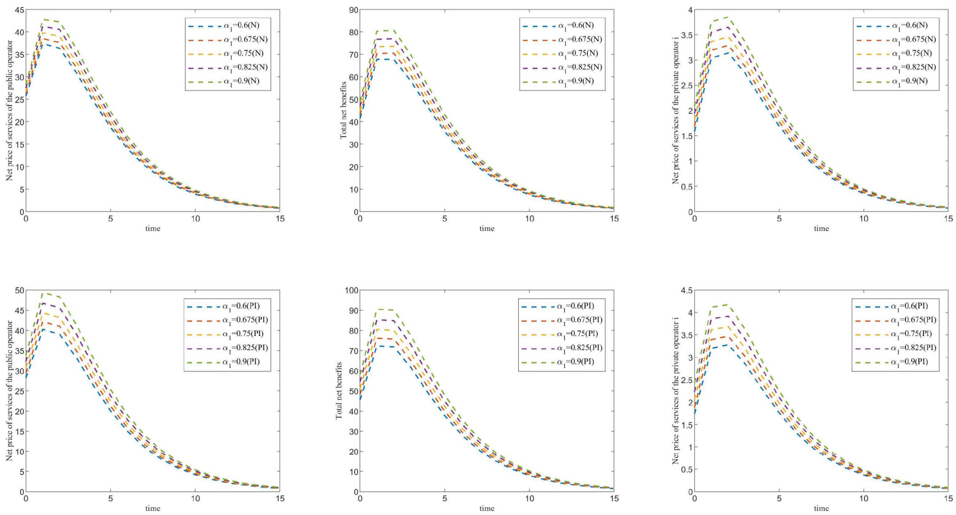

FIGURE 7. Variation of members' profits over time for different values of α_1.

operator, and the subsidy rate should be adjusted promptly according to the changes in various conditions. Secondly, the higher the subsidy rate, the better. An excessively high subsidy rate will lead to uprooting because the net income of public sector operators under an excessively high subsidy rate will fall as the subsidy rate rises. Public sector net income rises due to the subsidy only at moderate subsidy rates.

5. Conclusions and future research

This paper constructs a decision model of joint investment and operation of FSTD by both public and private capitals, takes into account the effect of consumer demand uncertainty, and studies the optimal strategies of each party under the two scenarios of no subsidy and subsidy by the higher level government for the project,

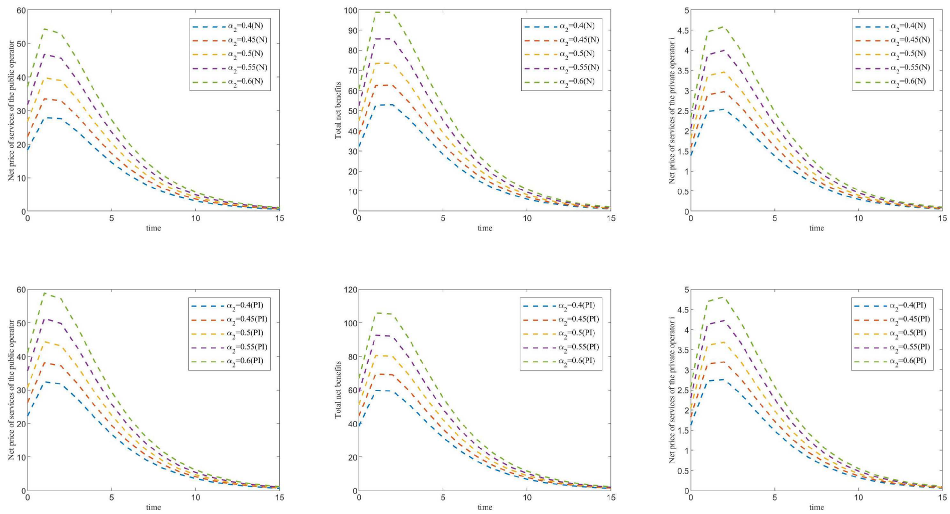

FIGURE 8. Variation of members' profits over time for different values of α_2.

and arrives at the following main conclusions: (1) Investment by public operators in the FSTD program rises if the higher level of government subsidizes the program compared to no subsidy. However, the prices of both types of operators and the amount of private investment remain unchanged. (2) The net income of private operators increases with the rate of subsidization. However, the public operator's net income rises and then falls. If the subsidy rate is above a certain threshold, its net income worsens rather than improves as the subsidy increases. (3) Stochastic perturbations in demand magnify the gap between the net incomes of the parties in both the subsidized and unsubsidized scenarios, and the gap between the public operator's investment in both the subsidized and unsubsidized scenarios is also magnified by perturbations in demand. (4) There are three periods of rapidly rising, stabilizing, and gradually declining net returns for both types of operators.

In the future, this paper can be extended in the following two ways. One is to analyze the case when the demand function is nonlinear. This paper considers a linear demand function, but it may be nonlinear. The second is to examine other uncertainties in consumption demand. In this paper, we have considered that consumption demand faces stochastic uncertainty, so is it fuzzy, rough, and other uncertainties? Also, this question may be left to future research.

Appendix A.

Time t is omitted below for ease of writing.

Appendix A1

Proof of Proposition 3.1

The following value function at time t can represent profit for each party in FST destinations:

$$\pi_1^{N^*} = e^{-\rho t} V_1^N(k_1^N, k_{2i}^N), \tag{A1}$$

$$\pi_{2i}^{N^*} = e^{-\rho t} V_{2i}^N(k_1^N, k_{2i}^N), \tag{A2}$$

$$s.t. \begin{cases} \dot{k}_1(t) = u_1(t) - \delta_1 k_1(t), \\ \dot{k}_2(t) = u_2(t) - \delta_2 k_{2i}(t), \end{cases} \tag{A3}$$

For any $k_1^N \geq 0$ and $k_2^N \geq 0$, $V_1^N(k_1^N, k_{2i}^N)$ and $V_{2i}^N(k_1^N, k_{2i}^N)$ satisfy the Hamilton-Jacobi-Bellman function as

$$\begin{aligned}\rho V_1^N(k_1^N, k_{2i}^N) = &\max[p_1(a - p_1 - p_2 + \varepsilon)(\alpha_1 k_1 + \alpha_2 n k_2) - u_1^2(t)/2 \\ &+ \frac{\partial V_1^N}{\partial k_1}(u_1 - \delta_1 k_1) + \frac{\partial V_1^N}{\partial k_{2i}}(u_{2i} - \delta_2 k_{2i})]\end{aligned} \tag{A4}$$

$$\begin{aligned}\rho V_{2i}^N(k_1^N, k_{2i}^N) = &\max[p_2(a - p_1 - p_2 + \varepsilon)(\alpha_1 k_1 + \alpha_2 n k_2)/n - u_2^2(t)/2 \\ &+ \frac{\partial V_{2i}^N}{\partial k_1}(u_1 - \delta_1 k_1) + \frac{\partial V_{2i}^N}{\partial k_{2i}}(u_{2i} - \delta_2 k_{2i})].\end{aligned} \tag{A5}$$

Thus, we obtain the following necessary conditions:
$\frac{\partial(\rho V_1^N)}{\partial p_1} = 0$, $\frac{\partial(\rho V_1^N)}{\partial u_1} = 0$, $\frac{\partial(\rho V_{2i}^N)}{\partial p_2} = 0$, $\frac{\partial(\rho V_{2i}^N)}{\partial u_2} = 0$.
which respectively imply

$$u_1 = \frac{\partial V_1^N}{\partial k_1}, u_2 = \frac{\partial V_{2i}^N}{\partial k_2}. \tag{A6}$$

$$p_1 = \frac{a - p_2 + \varepsilon}{2}, p_2 = \frac{a - p_1 + \varepsilon}{2}. \tag{A7}$$

By associating the two formulas for price in (A7), we get

$$p_1^{N^*} = \frac{a+\varepsilon}{3}, p_2^{N^*} = \frac{a+\varepsilon}{3}. \tag{A8}$$

By substituting (A6) and (A8) into (A4) and (A5) , we have

According to (A6) and (A7), we further infer that $\rho V_{mNS}^A(x_{NS}^A, G_{NS}^A)$ and

$$\begin{aligned}\rho V_1^N(k_1^N, k_{2i}^N) = &[(\frac{a+\varepsilon}{3})^2\alpha_1 - \delta_1\frac{\partial V_1^N}{\partial k_1}]k_1 + [(\frac{a+\varepsilon}{3})^2\alpha_2 n - \delta_2\frac{\partial V_1^N}{\partial k_{2i}}]k_{2i} \\ &+ (\frac{\partial V_1^N}{\partial k_1})^2/2 + \frac{\partial V_1^N}{\partial k_{2i}}\frac{\partial V_{2i}^N}{\partial k_{2i}}.\end{aligned} \tag{A9}$$

$$\begin{aligned}\rho V_{2i}^N(k_1^N, k_{2i}^N) = &[(\frac{a+\varepsilon}{3})^2\frac{\alpha_1}{n} - \delta_1\frac{\partial V_{2i}^N}{\partial k_1}]k_1 + [(\frac{a+\varepsilon}{3})^2\alpha_2 - \delta_2\frac{\partial V_{2i}^N}{\partial k_{2i}}]k_{2i} \\ &+ (\frac{\partial V_{2i}^N}{\partial k_{2i}})^2/2 + \frac{\partial V_1^N}{\partial k_1}\frac{\partial V_{2i}^N}{\partial k_1}.\end{aligned} \tag{A10}$$

According to (A9) and (A10), we further infer that $V_1^N(k_1^N, k_{2i}^N)$ and $V_{2i}^N(k_1^N, k_{2i}^N)$ are both linear about k_1^N and k_{2i}^N , thus, we set:

$$\begin{cases} V_1^N = m_1^N k_1^N + m_2^N k_{2i}^N + m_3^N, \\ V_{2i}^N = r_1^N k_1^N + r_2^N k_{2i}^N + r_3^N, \end{cases} \tag{A11}$$

where m_1^N,m_2^N,r_1^N,r_2^N are constants. Obviously, $\frac{\partial V_1^N}{\partial k_1} = m_1^N$, $\frac{\partial V_1^N}{\partial k_2} = m_2^N$, $\frac{\partial V_{2i}^N}{\partial k_1} = r_1^N$, $\frac{\partial V_{2i}^N}{\partial k_2} = r_2^N$. By substituting (A11) into (A9) and (A10), we get $m_1^{N^*}$, $m_2^{N^*}$, $r_1^{N^*}$, $r_2^{N^*}$, $m_3^{N^*}$, $r_3^{N^*}$.

$$\begin{cases} m_1^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)}, \\ m_2^{N^*} = \frac{(a+\varepsilon)^2\alpha_2 n}{9(1+\delta_2)}, \\ m_3^{N^*} = (m_1^{N^*})^2/2 + m_2^{N^*} r_2^{N^*}, \end{cases} \tag{A12}$$

$$\begin{cases} r_1^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9n(1+\delta_1)}, \\ r_2^{N^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}, \\ r_3^{N^*} = (r_2^{N^*})^2/2 + m_1^{N^*} r_1^{N^*}, \end{cases} \tag{A13}$$

Substituting (A12) and (A13) into (A6), we get the optimal equilibrium strategies: $u_1^{N^*} = \frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)}$, $u_2^{N^*} = \frac{(a+\varepsilon)^2\alpha_2}{9(1+\delta_2)}$.

Appendix A2

The proof for Lemma 3.2

By taking the partial derivatives of the stochastic perturbation term of demand through the net price of services for public and private programs, consumer demand, local government investment, individual households investment and the size of public capital and private capital of individual households respectively in Scenario N, we get: $\frac{\partial p_1^{N^*}}{\partial\varepsilon} = \frac{\partial p_2^{N^*}}{\partial\varepsilon} = \frac{1}{3}$, $\frac{\partial q^{N^*}}{\partial\varepsilon} = \frac{1}{9}(\frac{(a+\varepsilon)^2{\alpha_1}^2}{(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})+\frac{(a+\varepsilon)^2 n{\alpha_2}^2}{(1+\delta_2)\delta_2}(1-e^{-\delta_2 t}))$, $\frac{\partial u_1^{N^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_1}{9(1+\delta_1)}$, $\frac{\partial u_{2i}^{N^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_2}{9(1+\delta_2)}$, $\frac{\partial k_1^{N^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_1(1-e^{-\rho t})}{9(1+\delta_1)\delta_1}$, $\frac{\partial k_{2i}^{N^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_2(1-e^{-\rho t})}{9(1+\delta_2)\delta_2}$.

Where α, ε, α_1, δ_1, α_2, n, $1-e^{-\delta_1 t}$ and $1-e^{-\delta_2 t}$ are greater than zero to ensure that the above deviation results are positive.

Appendix B1

The proof for Proposition 3.3 is similar to the proof for Proposition 3.1. Therefore, we omit it.

Appendix B2

The proof for Lemma 3.4

By taking the partial derivatives of the stochastic perturbation term of demand through the net price of services for public and private programs, consumer demand, local government investment, individual households investment, the size of public capital and private capital of individual households respectively in Scenario PI, which is similar to the proof for Lemma 3.2, we get: $\frac{\partial p_1^{PI^*}}{\partial\varepsilon} = \frac{\partial p_{2i}^{PI^*}}{\partial\varepsilon} = \frac{1}{3}$, $\frac{\partial q^{PI^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_1}{9(1-\tau_1)(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})+\frac{2(a+\varepsilon)\alpha_2}{9(1+\delta_2)\delta_2}(1-e^{-\delta_2 t})$, $\frac{\partial u_1^{PI^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_1}{9(1-\tau_1)(1+\delta_1)}$, $\frac{\partial u_{2i}^{PI^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_2}{9(1+\delta_2)}$, $\frac{\partial k_1^{PI^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_1}{9(1-\tau_1)(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})$, $\frac{\partial k_{2i}^{PI^*}}{\partial\varepsilon} = \frac{2(a+\varepsilon)\alpha_2(1-e^{-\rho t})}{9(1+\delta_2)\delta_2}$

Where α, ε, α_1, δ_1, α_2, δ_2, n, $1-e^{-\delta_1 t}$, $1-e^{-\delta_2 t}$ and $1-\tau_1$ are greater than zero to ensure that the above deviation results are positive.

Appendix B3

The proof for Lemma 3.5

(1) By taking the partial derivatives of the subsidy rates for public investment through the net price of services for public and private programs, consumer demand, local government investment, individual households investment and the size of public capital, private capital of individual households and the profit of individual household respectively in Scenario PI, we get: $\frac{\partial p_1^{PI^*}}{\partial\tau_1} = \frac{\partial p_2^{PI^*}}{\partial\tau_1} = \frac{\partial u_{2i}^{PI^*}}{\partial\tau_1} = \frac{\partial k_{2i}^{PI^*}}{\partial\tau_1} = 0$, $\frac{\partial u_1^{PI^*}}{\partial\tau_1} = \frac{(a+\varepsilon)^2\alpha_1}{9(1-\tau_1)^2(1+\delta_1)}$, $\frac{\partial k_1^{PI^*}}{\partial\tau_1} = \frac{(a+\varepsilon)^2\alpha_1}{9(1-\tau_1)^2(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})$, $\frac{\partial q^{PI^*}}{\partial\tau_1} = \frac{(a+\varepsilon)^3{\alpha_1}^2}{27(1-\tau_1)^2(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})$, $\frac{\partial\pi_{2i}^{PI^*}}{\partial\tau_1} = \frac{(1-\delta_1)(a+\varepsilon)^4{\alpha_1}^2 e^{-\rho t}}{\delta_1 81 n(1-\tau_1)^2(1+\delta_1)^2}$.

Similarly, α, ε, α_1, δ_1, α_2, δ_2, n, $1-e^{-\delta_1 t}$, $1-e^{-\delta_2 t}$ $1-\delta_1$ and $1-\tau_1$ are greater than zero. Therefore, $\frac{\partial u_1^{PI^*}}{\partial\tau_1} > 0$, $\frac{\partial k_1^{PI^*}}{\partial\tau_1} > 0$, $\frac{\partial q^{PI^*}}{\partial\tau_1} > 0$, $\frac{\partial\pi_{2i}^{PI^*}}{\partial\tau_1} > 0$.

(2) By taking the partial derivatives of the subsidy rates for public investment through the profit of local government in Scenario PI, we get: $\frac{\partial\pi_1^{PI^*}}{\partial\tau_1} = [\frac{(1-\tau_1)(1+\delta_1)-1}{(1-\tau_1)^2\delta_1}](m_1^{PI^*})^2 e^{-\rho t}$.

If $(1-\tau_1)(1+\delta_1)-1>0$, *i.e.*, $0<\tau_1<\frac{\delta_1}{1+\delta_1}$, we get $\frac{\partial\pi_1^{PI^*}}{\partial\tau_1}>0$; if $\frac{\delta_1}{1+\delta_1}<\tau_1<1$, we have $\frac{\partial\pi_1^{PI^*}}{\partial\tau_1}<0$.

Appendix C1

The proof for Lemma 4.1

From Propositions 3.1 and 3.3, we get $p_1^{PI^*} = p_1^{N^*}$, $p_2^{PI^*} = p_2^{N^*}$, $u_{2i}^{PI^*} = u_{2i}^{N^*}$, $k_{2i}^{PI^*} = k_{2i}^{N^*}$, $u_1^{PI^*} - u_1^{N^*} = \frac{\tau_1(a+\varepsilon)^2\alpha_1}{9(1-\tau_1)(1+\delta_1)}$, $k_1^{PI^*} - k_1^{N^*} = \frac{\tau_1(a+\varepsilon)^2\alpha_1}{9(1-\tau_1)(1+\delta_1)\delta_1}(1-e^{-\delta_1 t})$.

Similarly, the corresponding parameter variables in the above equation are all positive. Therefore, $u_1^{PI^*} > u_1^{N^*}$ and $k_1^{PI^*} > k_1^{N^*}$.

Appendix C2

The proof for Lemma 4.2

(1) The difference between $\pi_1^{PI^*}$ and $\pi_1^{N^*}$ is:

$V_1^{NPI}=\pi_1^{PI^*} - \pi_1^{N^*} = [\frac{2(1-e^{-\delta_1 t})-\tau_1(2(1-e^{-\delta_1 t})+\delta_1)}{2(1-\tau_1)\delta_1}](\frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)})^2\frac{\tau_1}{(1-\tau_1)}$.

Obviously, $V_1^{NPI}> 0$ depends on $2(1-e^{-\delta_1 t})-\tau_1(2(1-e^{-\delta_1 t})+\delta_1)$: when $[2(1-e^{-\delta_1 t})-\tau_1(2(1-e^{-\delta_1 t})+\delta_1) > 0$, *i.e.*, $\tau_1 < \tau_{1a}$, we have $\pi_1^{PI^*} > \pi_1^{N^*}$; when $2(1-e^{-\delta_1 t})-\tau_1(2(1-e^{-\delta_1 t})+\delta_1) < 0$, *i.e.*, $\tau_1 > \tau_{1a}$, we get $\pi_1^{PI^*} < \pi_1^{N^*}$.

(2) The difference between $\pi_{2i}^{PI^*}$ and $\pi_{2i}^{N^*}$ is:

$V_{2i}^{NPI} = \pi_{2i}^{PI^*} - \pi_{2i}^{N^*} = \frac{\tau_1(1-e^{-\delta_1 t}+\delta_1)}{n(1-\tau_1)\delta_1})(\frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)})^2$.

The corresponding parameter variables in the above equation are all positive. Therefore, $\pi_{2i}^{PI^*} > \pi_{2i}^{N^*}$.

(3) The difference between $J_{total}^{PI^*}$ and $J_{total}^{N^*}$ is:

$V_{total}^{NPI}=J_{total}^{PI^*} - J_{total}^{N^*} = [\frac{4(1-\tau_1)(1-e^{-\delta_1 t})+2\delta_1-3\tau_1\delta_1}{2(1-\tau_1)\delta_1}](\frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)})^2\frac{\tau_1}{(1-\tau_1)}$.

If $4(1-\tau_1)(1-e^{-\delta_1 t})+2\delta_1-3\tau_1\delta_1 > 0$, *i.e.*, $\tau_1 < \tau_{1b}$, we get $V_{total}^{NPI} = V_{total}^{PI} - V_{total}^{N} > 0$; if $4(1-\tau_1)(1-e^{-\delta_1 t})+2\delta_1-3\tau_1\delta_1 < 0$, *i.e.*, $\tau_1 > \tau_{1b}$, we get $V_{total}^{NPI} = V_{total}^{PI} - V_{total}^{N} < 0$.

(4) The effect of the subsidy rate on net income of the FST destination is:

$\frac{\partial V_{total}^{NPI}}{\partial \tau_1} = \frac{1}{2(1-\tau_1)^2}\frac{(a+\varepsilon)^2\alpha_1}{9(1+\delta_1)})^2[\frac{4(1-e^{-\delta_1 t})}{\delta_1} + \frac{2-4\tau_1}{(1-\tau_1)}]$.

Similarly, if $\tau_1 < \frac{2(1-e^{-\delta_1 t})+\delta_1}{2(1-e^{-\delta_1 t}+\delta_1)}$, we obtain $\frac{\partial V_{total}^{NPI}}{\partial \tau_1} > 0$; if $\tau_1 > \frac{2(1-e^{-\delta_1 t})+\delta_1}{2(1-e^{-\delta_1 t}+\delta_1)}$, we obtain $\frac{\partial V_{total}^{NPI}}{\partial \tau_1} < 0$.
Finishing the proof.

Acknowledgements

This work was supported by the Science Research Project of Hunan Provincial Department of Education (Youth Project) (21B0423), the Scientific Research Project of Hunan Provincial Department of Education (Key project) (22A0298).

References

[1] Y. Bai, J. Meng, F. Meng and G. Fang, Stochastic analysis of a shale gas investment strategy for coping with production uncertainties. *Energy Policy* **144** (2020) 111639.

[2] H. Barman, M. Pervin, S.K. Roy and G.W. Weber, Analysis of a dual-channel green supply chain game-theoretical model under carbon policy. *Int. J. Syst. Sci. Oper. Logist.* **10** (2023) 224–277.

[3] H. Barman, S.K. Roy, L. Sakalauskas and G.W. Weber, Inventory model involving reworking of faulty products with three carbon policies under neutrosophic environment. *Adv. Eng. Inform.* **57** (2023) 102081.

[4] A. Caffyn and J. Lutz, Developing the heritage tourism product multiethnic cities. *Tour. Manag.* **20** (1999) 213–227.

[5] L. Delaney, The impact of operational delay on irreversible investment under knightian uncertainty. *Econ. Lett.* **215** (2022) 110494.

[6] D. Dey, S. Kar and D.K. Bhattacharya, Analytical solution of a modified single item continuous production inventory model under constant deterioration, goal levels and penalties. *OPSEARCH* **61** (2024) 399–420.

[7] P.K. Ghosh, A.K. Manna and J.K. Dey, Supply chain coordination model for green product with different payment strategies: A game theoretic approach. *J. Clean. Prod.* **290** (2021) 125734.

[8] M.H. González-Serrano, J. Crespo-Hervás, C. Pérez-Campos and F. Calabuig, Entrepreneurial ecosystems for developing the sports industry in european union countries. *J. Bus. Res.* **136** (2021) 667–677.

[9] J.P. Guironnet, Competitive intensity and industry performance of professional sports. *Econ. Model.* **126** (2023) 106441.

[10] Z. Jiang and C. Xu, Policy incentives, government subsidies, and technological innovation in new energy vehicle enterprises: Evidence from china. *Energy Policy* **177** (2023) 113527.

[11] S. Jørgensen and G. Zaccour, Time consistent side payments in a dynamic game of downstream pollution. *J. Econ. Dyn. Control* **25** (2001) 1973–1987.
[12] X. Jun, On the cooperation between fujian and taiwan folk sports cultural creative industry marketing. *Sports Sci. Res.* **2** (2018) 3–6.
[13] C. Kim and J. Kim, Spatial spillovers of sport industry clusters and community resilience: Bridging a spatial lens to building a smart tourism city. *Inf. Process. Manage.* **60** (2023) 103266.
[14] Y. Ma, Y. Sha, Z. Wang and W. Zhang, The effect of the policy mix of green credit and government subsidy on environmental innovation. *Energy Econ.* **118** (2023) 106512.
[15] A. Mondal, B.K. Giri, S.K. Roy, M. Deveci and D. Pamucar, Sustainable-resilient-responsive supply chain with demand prediction: An interval type-2 robust programming approach. *Eng. Appl. Artif. Intell.* **133** (2024) 108133.
[16] F.E. Ouardighi, Supply quality management with optimal wholesale price and revenue sharing contracts: a two-stage game approach. *Int. J. Prod. Econ.* **156** (2014) 260–268.
[17] S.R. Pitchford, Ethnic tourism and nationalism in wales. *Ann. Tour. Res.* **22** (1996) 35–52.
[18] S. Prideaux, An investigation of the relationship between south korean domestic public opinion, tourism development in north korean and a role for tourism in promoting peace othe korean peninsula. *Ann. Tour. Res.* **27** (2006) 124–137.
[19] Z. Tan, Q. Tan and Q. Nguyen, The influence of folk sports culture on rural governance based on computer sensor network. *Wirel. Commun. Mob. Comput.* **9** (2022) 7606174.
[20] Y. Yi, Z. Wei and C. Fu, An optimal combination of emissions tax and green innovation subsidies for polluting oligopolies. *J. Clean. Prod.* **284** (2021) 124693.
[21] Y. Yi, Y. Wang, C. Fu and Y. Li, Taxes or subsidies to promote investment in green technologies for a supply chain considering consumer preferences for green products. *Comput. Ind. Eng.* **171** (2022) 108371.
[22] Y. Yi, M. Zhang, A. Zhang and Y. Li, Can "dual credit" replace "subsidies" successfully? -based on analysis of vehicle supply chain decisions under the digital transformation of technology. *Energy Econ.* **130** (2024) 107303.
[23] L. Zhou, C. Wang and Q. Zhang, The construction of folk sports featured towns based on intelligent building technology based on the internet of things. *Appl. Bionics Biomech.* **8** (2022) 4541533.